Dynamic buckling of plate made of glass-reinforced plastic under rapidly increasing shear load

Bakhtiyor Eshmatov[1], *Mirziyod* Mirsaidov[2*], and *Rustamkhan* Abdikarimov[3]

[1]Branch of National University of Science and Technology "MISiS", 110105 Almalyk, Uzbekistan
[2]"Tashkent Institute of Irrigation and Agricultural Mechanization Engineers" National Research University, 100000 Tashkent, Uzbekistan
[3]Tashkent University of Architecture and Construction, 100194 Tashkent, Uzbekistan

Abstract. The research object of this work is a clamped rectangular plate made of glass-reinforced plastic. The dynamic problem of stability of the plate under rapidly increasing shear load is considered. Within the Kirchhoff–Love hypothesis framework, a mathematical model was built in a geometrically nonlinear formulation. By the Bubnov–Galerkin method, based on a polynomial approximation of the deflection, the problem was reduced to solving systems of nonlinear ordinary integro-differential equations. With a weakly singular Koltunov–Rzhanitsyn kernel with variable coefficients, the resulting system was solved by a numerical method based on quadrature formulas. The plate's dynamic behavior was investigated depending on the plate's geometric and physic parameters. The importance of considering the viscoelastic properties of the material is shown.

1 Introduction

During the intense development of modern industry, reducing the materials consumption of machine structures is one of the main problems of mechanical and civil engineering. To save materials, it becomes necessary to manufacture thin-walled structures. The thinner the element, and the more flexible it is, the more strongly its susceptibility to buckling and loss of stability is manifested. The latter is accompanied by a catastrophic development of deformations and, as a rule, the destruction of the structure. From this point of view, in producing lightweight, durable, and reliable structures, it is advisable to use materials that improve their operating characteristics and, in some cases, create structures that are not feasible using traditional materials. Here, structures' calculation procedure and design, including consideration of their actual properties, are rather complicated. Today, the most pressing issue is the development of effective algorithms for solving nonlinear problems of dynamic stability of shells, panels, and plates made of composite materials.

The buckling analysis on simply supported rectangular plates and stiffened panels is given in the article [1]. The thickness of the plates, longitudinal stringers, and sub-stiffeners varied. The marine grade steel was used as the construction material. The dependence of

*Corresponding author: theormir@mail.ru

the load on the displacement curve and the total energy was measured. The buckling analysis results were studied using the finite element method (FE) calculation.

Paper [2] presents a finite element formulation to study the mechanical buckling of stiffened functionally graded material (FGM) plates. The approach is based on a third-order shear deformation theory (TSDT) introduced by Guangyu Shi. The material properties of the plate were assumed to be varied in the thickness direction by a power law distribution. Still, the material of the stiffener was the same as that of one of the bottom surfaces where the stiffener was placed. A parametric study was carried out to highlight the effect of material distribution, the thickness-to-width ratio, and stiffener parameters on the buckling characteristics of the stiffened FGM plates.

The article [3] investigates the reactions to the buckling of a plate made of functionally graded material (FGM) subjected to uniform, linear, and nonlinear loads in the plane. The paper proposes new nonlinear models of load in the plane based on trigonometric and exponential functions. Dimensionless critical bending loads were estimated using a higher-order polynomial shear strain theory. To obtain an exact explicit solution, the Navier method was used, which provides a minimum numerical error. The equilibrium conditions were determined using the principle of virtual displacements, and the material's properties were estimated in the direction of thickness using a simple Voigt model or exponential law.

Since structures made of FG material are often operated at high temperatures, it is extremely important to understand the mechanical behavior of these structures, taking into account temperature. In [4], a phase field model was used to study the thermal bending of plates made of fractured functionally graded material (FGM). In addition, this study demonstrates the difference between the reaction of the static stability of the plate to thermal load in the case of temperature-dependent mechanical properties of the material (TD) and the case of temperature-independent mechanical properties of the material (TID), illustrating that the calculation for these two cases will look very distorted. This provides scientists with a scientifically sound basis for adopting a more realistic computational model. The equations are based on Reddy's third-order shear deformation Plate Theory (TSDT).

The study of the behavior of various rectangular thin isotropic plates having a free edge in inelastic bending is devoted to the work [5]. Various combinations of boundary conditions are subjected to uniaxial compression in the plane, and each rectangular plate is bounded by an unloaded free edge. The characteristic deflection function of each plate is formulated using a polynomial function in the form of a Taylor–Maclaurin series. An approach based on the plasticity of deformation is used, and the bending load equation is modified using the principle of operation method. The buckling coefficients of the plates are calculated for different aspect ratios and modules.

The analysis of many experimental and fundamental studies shows that most composite materials have pronounced viscoelastic properties [6].

Most researchers who have attacked the class of problems mentioned above have considered the solution of problems with such a mathematical statement in an elastic case. In these works, only some properties of construction materials were considered: the problems were solved either in a linear context or in the elastic formulation. Even if the problems were solved using a viscoelastic formulation, in many cases, the viscoelastic characteristics of the material were only taken into account in a restricted context. In these cases, viscoelastic properties were addressed using the Voight model or exponential relaxation kernels. However, mathematical models of problems of viscoelastic systems based on these assumptions cannot describe real processes occurring in shell constructions in early times [6]. The choice of an exponential kernel in calculations was not incidental. The systems of integro-differential equations obtained from the calculations were reduced by way of differentiation to the solution of ordinary differential equations, which in most cases used to be solved by the well-known Runge-Kutta's numerical method. To the present day, none of the existing methods has allowed one to solve such problems with weakly singular kernels of the type of Koltunov's, Rzhanitsyn's, Abel's, Rabotnov's, and others [6].

Badalov, Eshmatov, and Yusupov [7] developed the numerical method based on the use of quadrature rules, which makes it possible to solve a system of nonlinear integro-differential equations with weakly singular kernels of the type of Koltunov-Rzhanitsyn's, Abel's, and Rabotnov's. Verlan, Abdikarimov, and Eshmatov [8] modified this method. The method provides reasonably high accuracy of results, is universal, enables one to solve a wide class of dynamic problems in viscoelasticity, and is economical from computer time consideration. Based on this method, many numerical results have been obtained by Mirsaidov, Abdikarimov, Khodzhaev [9], and other researchers [10-11] that generally agree with evaluable experiments' data.

2 Material and methods

Consider the case when a rectangular fiberglass plate with sides a and b is subjected to the dynamic action of shear forces evenly distributed along its edges. Assume that the shear forces increase in proportion to time according to the law $P(t) = P_0 t$ (P_0 is the loading rate) (Fig.1). Problems of this type are found, for example, in ship structures, when dynamic loads occur on the vessel's hull, arising from the impact of waves, during the passage of an acoustic wave, etc. Similar loads may occur in the aircraft skin panels [12].

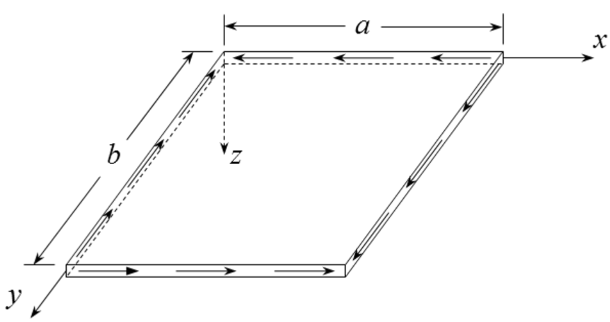

Fig. 1. Rectangular plate under shear load

To construct a mathematical model of the dynamic behavior of a plate made of a material with anisotropic viscoelastic properties under the influence of various external loads, we use the classical Kirchhoff-Love theory. In this case, following [13-19], the stress and moment resultants have the form:

$$
\begin{aligned}
N_x &= A_{11}^*\varepsilon_x + A_{12}^*\varepsilon_y + A_{16}^*\gamma_{xy} + B_{11}^*\chi_x + B_{12}^*\chi_y + B_{16}^*\chi_{xy},\\
N_y &= A_{12}^*\varepsilon_x + A_{22}^*\varepsilon_y + A_{26}^*\gamma_{xy} + B_{12}^*\chi_x + B_{22}^*\chi_y + B_{26}^*\chi_{xy},\\
N_{xy} &= A_{16}^*\varepsilon_x + A_{26}^*\varepsilon_y + A_{66}^*\gamma_{xy} + B_{16}^*\chi_x + B_{26}^*\chi_y + B_{66}^*\chi_{xy},\\
M_x &= B_{11}^*\varepsilon_x + B_{12}^*\varepsilon_y + B_{16}^*\gamma_{xy} + D_{11}^*\chi_x + D_{12}^*\chi_y + D_{16}^*\chi_{xy},\\
M_y &= B_{12}^*\varepsilon_x + B_{22}^*\varepsilon_y + B_{26}^*\gamma_{xy} + D_{12}^*\chi_x + D_{22}^*\chi_y + D_{26}^*\chi_{xy},\\
M_{xy} &= B_{16}^*\varepsilon_x + B_{26}^*\varepsilon_y + B_{66}^*\gamma_{xy} + D_{16}^*\chi_x + D_{26}^*\chi_y + D_{66}^*\chi_{xy},
\end{aligned} \tag{1}
$$

where A_{ij}^*, B_{ij}^*, D_{ij}^*, $i,j = 1,2,6$ are the operators having the following form:

$$
A_{ij}^*\varphi = \sum_{k=1}^{K}\left(\bar{Q}_{ij}^*\varphi\right)_k (z_k - z_{k-1}), \quad B_{ij}^*\varphi = \frac{1}{2}\sum_{k=1}^{K}\left(\bar{Q}_{ij}^*\varphi\right)_k (z_k^2 - z_{k-1}^2),
$$

$$D_{ij}^{*}\varphi = \frac{1}{3}\sum_{k=1}^{K}\left(\bar{Q}_{ij}^{*}\varphi\right)_k\left(z_k^3 - z_{k-1}^3\right),$$
$$\bar{Q}_{11}^{*}\varphi = \left[Q_{11}cos^4\theta + \frac{1}{2}(Q_{12} + 2Q_{66})sin^2 2\theta + Q_{22}sin^4\theta\right](1-\Gamma^*)\varphi,$$
$$Q_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}},$$
$$\bar{Q}_{12}^{*}\varphi = \left[\frac{1}{4}[Q_{11} + Q_{22} - 4Q_{66}]sin^2 2\theta + Q_{12}\left(1-\frac{1}{2}sin^2 2\theta\right)\right](1-\Gamma^*)\varphi$$
$$Q_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}},$$
$$\bar{Q}_{22}^{*}\varphi = \left[Q_{11}sin^4\theta + \frac{1}{2}(Q_{12} + 2Q_{66})sin^2 2\theta + Q_{22}cos^4\theta\right](1-\Gamma^*)\varphi,$$
$$\bar{Q}_{16}^{*}\varphi = Q_{11}\,cos^3\theta\,sin\,\theta + \frac{1}{4}(Q_{12}+2Q_{66})\,sin\,4\,\theta - Q_{22}\,cos\,\theta\,sin^3\theta(1-\Gamma^*)\varphi,$$
$$\bar{Q}_{26}^{*}\varphi = Q_{11}\,sin^3\theta\,cos\,\theta + \frac{1}{4}(Q_{12}+2Q_{66})\,sin\,4\,\theta - Q_{22}\,sin\,\theta\,cos^3\theta(1-\Gamma^*)\varphi,$$
$$\bar{Q}_{66}^{*}\varphi = \frac{1}{4}(Q_{11} - 2Q_{12} + Q_{22})\,sin^2 2\,\theta + Q_{66}\,cos^2 2\theta(1-\Gamma^*)\varphi,$$
$$Q_{12} = \frac{E_1\mu_{21}}{1-\mu_{12}\mu_{21}} = \frac{E_2\mu_{12}}{1-\mu_{12}\mu_{21}}, Q_{66} = G_{12},\ \Gamma^*\varphi = \int_0^t \Gamma(t-\tau)\varphi(\tau)d\tau$$

Here K is the number of plate layers, E_1, E_2 are the elastic moduli, G_{12} is the shear modulus, μ_{12}, μ_{21} are the Poisson ratios, θ is the angle characterizing the direction of the fibers relative to the axis OX, Γ^* is the integral operator with the relaxation kernel $\Gamma(t)$.

The relationship between the strains in the median surface ε_x, ε_y, γ_{xy}, χ_x, χ_y, χ_{xy} and displacements u, v, w in directions x, y, z, written by considering the von Kármán type of geometric nonlinearity, in the form [12-19]:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\quad \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y},$$
$$\chi_x = -\frac{\partial^2 w}{\partial x^2},\quad \chi_y = -\frac{\partial^2 w}{\partial y^2},\quad \chi_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}. \tag{2}$$

By substituting (1) and (2) into the equations of motion

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + p_x = \rho\frac{\partial^2 u}{\partial t^2},\quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + p_y = \rho\frac{\partial^2 v}{\partial t^2},$$
$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}\right) +$$
$$+ \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\right) + q = \rho\frac{\partial^2 w}{\partial t^2}, \tag{3}$$

the system of nonlinear integro-differential equations in partial derivatives is obtained.

If a dynamic process is considered without considering the propagation of elastic waves, then in the first two equations of the set (3) it is possible to discard the inertial terms concerning u and v [12-19]. In this case, the solution of the resulting system is sought in the form [20]:

$$w(x,y,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\gamma_m\cos\frac{\lambda_m x}{a}-\gamma_m\cosh\frac{\lambda_m x}{a}+\sin\frac{\lambda_m x}{a}-\sinh\frac{\lambda_m x}{a}\right)* \\ *\left(\gamma_n\cos\frac{\lambda_n y}{b}-\gamma_n\cosh\frac{\lambda_n y}{b}+\sin\frac{\lambda_n y}{b}-\sinh\frac{\lambda_n y}{b}\right)w_{mn}(t), \quad (4)$$

where $w_{mn}(t),\ m,\ n=1,\ 2,\ 3,\ \ldots$ are the unknown functions of time; the λ_m and λ_n are the roots of the frequency equation:

$$\cos\lambda_m\cosh\lambda_m=1$$

And

$$\gamma_m=\frac{\cos\lambda_m-\cosh\lambda_m}{\sin\lambda_m+\sinh\lambda_m}$$

Substituting the approximating functions (4) into the resulting system of equations and performing the Bubnov–Galerkin method procedure, a system of nonlinear ordinary partial integro-differential equations is obtained. Further, this system is integrated using the numerical method based on quadrature formulas [7,8].

3 Results and discussion

The calculations use the simplest and, at the same time, quite common weakly-singular Koltunov–Rzhanitsyn kernel having the form $\Gamma(t)=Ae^{-\beta t}t^{\alpha-1}(0<\alpha<1)$ as the relaxation kernel, where $A,\ \alpha,\ \beta$ are the rheological viscosity parameters determined from the experiments [6]. Experimental studies have shown that weak singular functions most accurately describe the rates of relaxation processes. A detailed description of the numerical method based on quadrature that allows solving such systems of equations is given in [7,8]. The method allows solving systems of nonlinear integro-differential equations, preliminarily transforming singular kernels into regular ones.

In the calculations, the following parameters for plastic (KAST-V): E_1 = 25.5 GPa, E_2 = 14.91 GPa, G_{12} = 4.41 GPa, μ_{12} = 0.2, ρ = 1900 kg/m^3 have been used.

After this, the following parameters of the plate are used in the calculations (unless otherwise specified): $a=b=0.5m, h=0.5sm, \theta=45^0, P_0=5MPa/s.$

Figure 2 illustrates the form of the solution obtained using many terms in the series (4). Note that the buckled shape possesses nodal lines (lines of zero deflection) that are not parallel to the sides of the plate. Areas with both positive and negative deflections appear on the plate. Nodal lines do not occur for uniform compressive loadings of anisotropic plates.

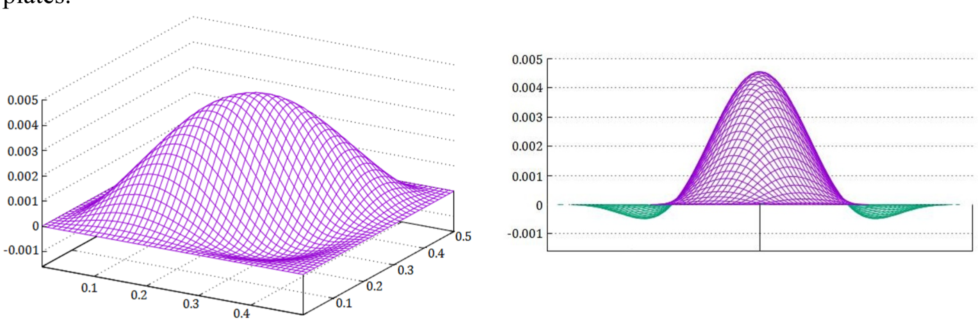

Fig. 2. Buckled shapes

When solving problems of stability of the plate under the action of compressive loads, as one of the criteria determining the critical time, it was assumed that the sag of the deflection should not exceed a value equal to the thickness of the plate [12]. Here, as a criterion determining the critical time, we will assume that the difference between the values of deflections at the uppermost and lowest point of the deformable plate should not exceed its thickness.

The following graphs correspond to the results obtained for the midpoint of the clamped plate. On the graphs, m (meter) is the dimension for the deflection, and s (second) is for time.

Figure 3 shows the effect of the viscoelastic properties of the material on the behavior of an anisotropic reinforced plate made of KAST-V. It can be seen from the figure that taking into account viscoelastic properties leads to a decrease in the critical time. The difference in critical times for elastic and viscoelastic plates reaches more than 15%. This result shows the importance of taking into account viscoelastic properties.

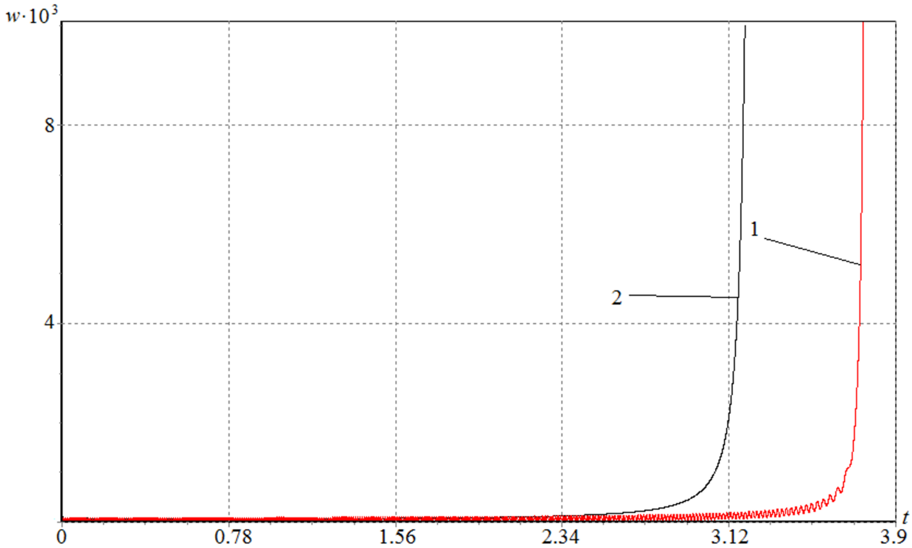

Fig. 3. Effect of viscoelastic properties on behavior of anisotropic reinforced plate: 1 is elastic and 2 is viscoelastic cases

An increase in the rigidity of the plate due to an increase in the thickness of the plate leads to a proportional increase in the critical time value. (Fig.4)

 https://doi.org/10.1051/e3sconf/202340103009

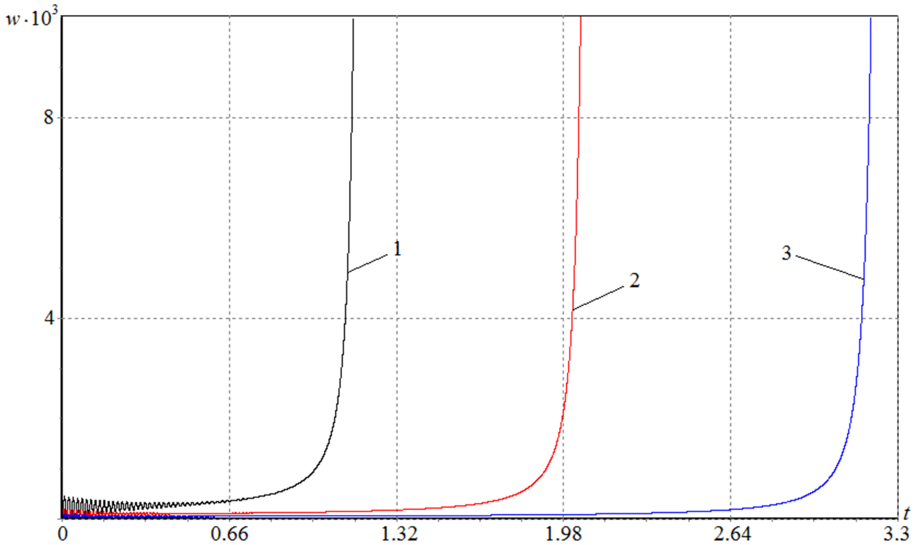

Fig. 4. Dependence of deflection on time for various values of thicknesses of the plate:
1 is $h = 0.3$ *sm*; 2 is $h = 0.4$ *sm*; 3 is $h = 0.5$ *sm*.

Figure 5 shows similar results for $\lambda = 1$; 1.2; 1.4. Here λ is the ratio of plate edges. Thus, if $\lambda = 1$, then the plate has a square shape. As can be seen from the graph, an increase in one of the edges of the plate leads to a shift of the deflection curve to the left (i.e., to a decrease in the critical time).

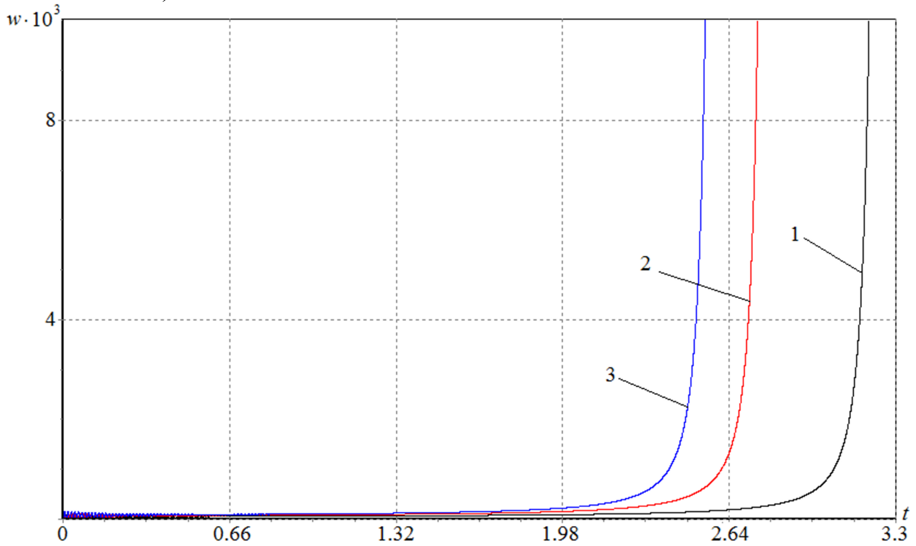

Fig. 5. Dependence of deflection on time for various values of parameter λ ($\lambda = a/b$) 1 is λ= 1; 2 is λ= 1.2; 3 is λ= 1.4

The different curves in Fig. 6 correspond to the cases of changing the reflections of the midpoint of the reinforced rectangular plate at different loading speeds. As expected, an increase in the loading speed leads to an earlier deflection increase. Similar results were obtained in the study of the stability of simply-supported plates under the influence of external compressive loads.

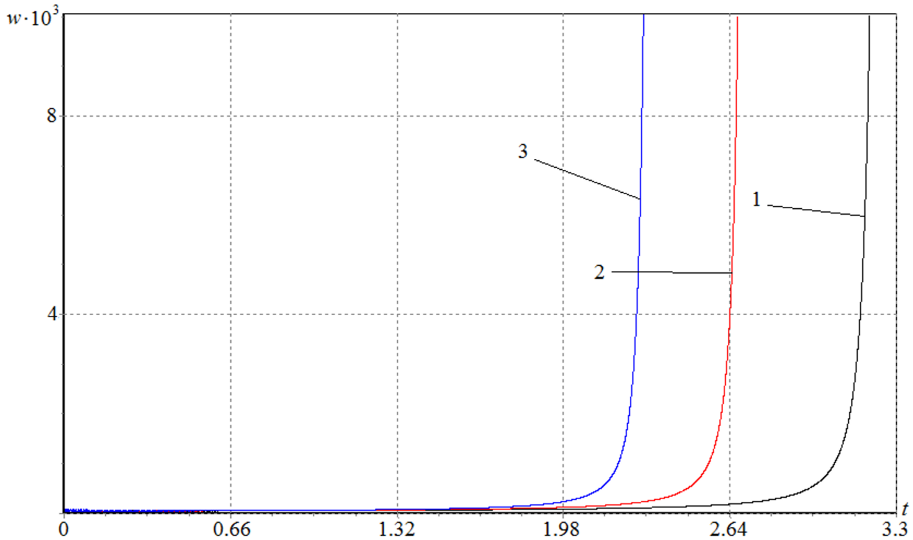

Fig. 6. Dependence of deflection on time for various values of velocities of loading
1 is $P_0 = 5$ *MPa/s*; 2 is $P_0 = 6$ *MPa/s*; 3 is $P_0 = 7$ *MPa/s.*

4 Conclusion

The study of the dynamic stability of viscoelastic anisotropic reinforced plates subjected to the dynamic action of shear forces uniformly distributed along their edges shows:

- the importance of taking into account the viscoelastic properties of the material. The results show that the difference in the critical time for solving elastic and viscoelastic problems for plates made of KAST-V is more than 15%.
- changes in the physical and geometric parameters of the plate also significantly affect the values of the critical time. This is especially noticeable when considering the viscoelastic properties of the construction material.

The results obtained from the work and their conclusions allow us to accurately predict the dynamic behavior of plates made of reinforced plastics.

References

1. A.R. Prabowo, R. Ridwan, T. Muttaqie, Designs **6,** 3 (2022)
2. H.-N. Nguyen, T.C. Tan, D.T. Luat, V.-D. Phan, D.V. Thom, P.V. Minh, Materials **12,** 8 (2019)
3. S.J. Singh, S.P. Harsha, Journal of Mechanical Science and Technology **33**, (2019)
4. T.V. Do, D.H. Doan, C.T. Nguen, D.D. Nguen, International Journal of Structural Stability and Dynamics **22,** 8 (2022)
5. U.G. Eziefula, International Review of Applied Sciences and Engineering **11,** 1 (2020)
6. M.A. Koltunov, Polymer Mechanics **2,** 4 (1969)
7. F.B. Badalov, Kh. Eshmatov, M. Yusupov, Journal of Applied Mathematics and Mechanics **51**, (1987)
8. A.F. Verlan, R.A. Abdikarimov, Kh. Eshmatov, Electronic modeling **32,** 2 (2010)
9. M.M. Mirsaidov, R.A. Abdikarimov, D.A. Khodzhaev, PNRPU Mechanics Bulletin. **2**, (2019)

10. B.Kh. Eshmatov, R.A. Abdikarimov, M. Amabili, N.I. Vatin, Magazine of Civil Engineering **118,** 2 (2023)
11. B.Kh. Eshmatov, Mechanics of Solids **53,** 5 (2018)
12. A.S. Volmir, The nonlinear dynamics of plates and shells. USA, Ohio, Foreign Technology Division Wright-Patterson Air Force, 1974. 543 p.
13. J.E. Ashton, J.M. Whitney, Theory of laminated plates. Lancaster, PA, Technomic Publishing Co., Inc., 1970. 158 p.
14. R.M. Jones, Mechanics of Composite Materials. New York, McGraw-Hill Book Co., 1975. 450 p.
15. J.M. Whitney, Structural Analysis Of Laminated Anisotropic Plates. United Kingdom, Routledge, 2018. 356 p.
16. J.N. Reddy, Mechanics of Laminated Composite Plates and Shells. CRC Press, USA, 2004. 831 p.
17. M.S. Qatu, Vibration of Laminated Shells and Plates. Elsevier, 2004. 409 p.
18. V.V. Vasiliev, Mechanics of Composite Structures. CRC Press, 1993. 517 p.
19. A. Bogdanovich, C. Pastore, Mechanics of Textile and Laminated Composites: With applications to structural analysis. Springer, 1996. 584 p.
20. A.W. Leissa, Vibration of Plates. National Aeronautics and Space Administration, USA, 1969. 363 p.