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Forced vibrations of hereditary deformable shell elements of aircraft structures in gas flow under influence of atmospheric turbulence

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Abstract. The problem of vibrations of thin, hereditarily deformable shell elements moving in a gas under the action of atmospheric turbulence is considered. The work aims to study the flutter phenomenon of aircraft elements in a gas flow under the action of loads caused by atmospheric turbulence. Assuming that the relationship between stresses and strains for the shell material is linear-hereditary, a thin shell is used, which obeys the Kirchhoff-Love hypothesis. The aerodynamic force is written according to the linearized piston theory. A system of nonlinear integro-differential equations in partial derivatives is obtained to describe nonlinear oscillations of a thin isotropic viscoelastic shell. The system of nonlinear integro-differential equations is solved numerically by the method proposed by F. Badalov, which is based on the Bubnov-Galerkin methods, finite differences, and power series. When using an exponential kernel, the flutter rate increases to approximately 1.5%. Therefore, when using an exponential kernel, the flutter velocity of a viscoelastic shell practically coincides with the critical flutter velocity for ideally elastic plates. It was also found that the critical flutter velocity increases with an increase in the number of pinched sides of the shell.

1 Introduction

The aircraft, during its flight, is subjected to various external loads. These loads are dynamic in nature and are caused by many reasons: excess pressure from the gas side, that is, non-conservative aerodynamic pressures of the gas flow (non-conservatism is because these pressures depend on the deformation of the structure itself), random effects of atmospheric turbulence; exposure to air waves caused by explosions; dynamic loads arising from aircraft maneuvers; the tail unit is exposed to the action of a turbulent wake that occurs behind the wings, nacelles or other parts of the aircraft [1, 2]. In addition, variable forces act on the aircraft during takeoff and landing as it moves along the Earth [3, 4]. Under the action of variable loads, the aircraft is deformed and performs forced oscillations in the gas flow under the action of atmospheric turbulence [5, 6]. The problem of the

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strength and reliability of the aircraft as a whole and its individual elements and the problem of fatigue of the material and individual structural elements have always been important for designers [7,8]. Recently, due to the increase in the weight and size of machines and the speed of flight, these problems have become particularly important. When solving these problems, an important role is played by the correct consideration of the influence of vibrations and deformations of the aircraft, arising under the action of external variable loads on the strength of the machine [9,10]. Let's take into account the strength, economy, and ease of execution of the main elements of aircraft from a composite material [11,12], in addition to taking into account the pronounced rheological and hereditary-deformed properties at any temperature of these materials. The aircraft can be considered elastic and hereditarily - deformable solid body. When calculating the stress-strain state of the aircraft, accounting for these properties of the material brings the theory of calculation closer to the actual condition of the structure [13,14].

This work on studying the phenomenon of flutter of the shells of aircraft elements in the gas flow under the action of loads caused by atmospheric turbulence is devoted.

2 Objects and methods of research

Let us consider the problem of forced vibrations of hereditarily deformable shell structural elements of an aircraft in a gas flow under the influence of atmospheric turbulence. When determining the dynamic response of hereditarily deformable systems to the pressure of atmospheric turbulence near critical states (divergence, flutter), we consider this pressure as a random function of our arguments [15]. Assuming that the dependence between stresses and strains for the material of the structure is linearly hereditary, and the forces of aerodynamic action from the side of the gas flow, the streamlined shell structure, are written according to the linearized piston theory [16], we obtain, to describe the forced oscillations of the system (making the usual technical assumptions of thin shell structures) the following operator stochastic integro-differential equation (IDE) [17]

$$A\frac{\partial^2 W}{\partial t^2}+B\frac{\partial W}{\partial t}+BV\bar{n}_0\,gradW+(1-R^*)LW=q \qquad (1)$$

where $W(x,t)$ and $q(x,t)$ are functions of *x* and time *t* with scope $x\in GCR^n, t\in R$. At a fixed *t* functions $W(x,t)$ and $q(x,t)$ are random functions of atmospheric turbulence, and are considered as elements of Hilbert spaces H_1 and H_2, respectively; *A, L* are deterministic, symmetric, and positive-definite linear operators from H_1 and H_2. The operator A will be called inertial, the operator L is elastic, and the operator R* is Volterra operator with a weakly singular heredity kernel of Abel type. $B=\frac{\gamma P_0}{c_0}, \overline{n_0}=[\cos\theta.\sin\theta]$ is unperturbed parameters; P_0, c_0 are pressure and speed of sound, respectively; γ is gas polytrope index, *V* is flow rate.

It is required to find solutions to IDE (1) for random perturbations of atmospheric turbulence near critical states (divergence, flutter) under given boundary and initial conditions.

Application of the method of generalized coordinates. As in deterministic problems, in ideally elastic systems, the method of generalized coordinates opens the way for efficient analytical and numerical solutions of various problems of random oscillations of hereditarily deformed systems. Indeed, to construct an appropriate system of coordinate functions, consider the related homogeneous problem.

$$L\varphi - \omega^2 A\varphi = 0 \tag{2}$$

Its own elements $\varphi_1, \varphi_2, \ldots$ have the meaning of eigenmodes of the corresponding elastic system. Eigenvalues $\omega_1^2, \omega_2^2, \ldots$ are equal to the squares of the natural frequencies of this system, which are related to the eigenfunctions by the Rayleigh relations

$$\omega_k^2 = \frac{(L\varphi_k, \varphi_k)}{(A\varphi_k, \varphi_k)} \tag{3}$$

The modes of oscillation are pairwise orthogonal with the weight of the operator A and with the weight of the operator L, i.e., relations

$$(A\varphi_j, \varphi_k) = (L\varphi_j, \varphi_k) = 0 \quad (j \neq k) \tag{4}$$

Any element from the domain of the operator L can be represented as a series concerning its own elements $\varphi_1, \varphi_2, \ldots$; moreover, this series converges at least in the norm of the operator L [18]. In particular, if $W(x,t)$ is an element of the corresponding phase space, i.e. $W \in D(L)$, with a parametric dependence on time t, then the expansion takes place

$$W(x,t) = \sum_k f_k(t)\varphi_k(x) \tag{5}$$

Here $f_k(t)$ is time functions (generalized coordinates).

The listed properties are formulated assuming that the spectrum of the problem (2) is discrete. These assumptions are satisfied if L^{-1} is a completely continuous operator. For elastic shell systems (rods, plates, shells, etc.) of limited dimensions L^{-1} is a completely continuous operator [18]. Various spectral problems for equation (2), in the case of a one-dimensional formulation, can be solved by the Koch method in combination with the differential sweep method, and in a multidimensional formulation, by the Koch method in combination with the variational iteration method, followed by using a modified version of the differential sweep method [19].

Generalized coordinate $f_k(t)(\kappa = 1,2...)$ satisfies the weakly singular IDE

$$\ddot{f}_k(t) + r_k \dot{f}_k(t) + V\sum_i b_{ki} f_i(x) + \omega_k^2 (1 - R^*) f_k(t) = Q_k(t) \tag{6}$$

where

$$r_k = \frac{(B\varphi_k, \varphi_k)}{(A\varphi_k, \varphi_k)}, b_{ki} = \frac{(B\overline{n_0} grad\varphi_i, \varphi_k)}{(A\varphi_k, \varphi_k)}, Q_k = \frac{(q, \varphi_k)}{(A_k\varphi_k, \varphi_k)} \tag{7}$$

$$R*f_k(t)=\int_0^t R(t-\tau)f_k(\tau)d\tau, R(t-\tau)=\varepsilon e^{-\beta(t-\tau)}(t-\tau)^{\alpha-1}, \quad \varepsilon>0, \beta>0,\ 0<\alpha<1 .$$

In (6), dots denote differentiation concerning time.

Suppose that $q(x,t)=q(x)\cdot Q(t)$, where $Q(t)$ is random function of time. Then from (7), we have

$$Q_k(t)=q_kQ(t), \quad q_k=\frac{(q,\varphi_k)}{(A\varphi_k,\varphi_k)} \tag{8}$$

Now the problem is to find a solution to IDE (6) for given initial conditions.

The following assertion holds: if the weight function $h_k(t)$ satisfies a homogeneous IDE of the form

$$\ddot{h}_k(t)+r_k\dot{h}_k(t)+V\sum_i b_{ki}^* h_i(t)+\omega_k^2(1-R^*)h_k(t)=0 \tag{9}$$

under initial conditions

$$h_k(0)=0\,, \dot{h}_k(0)=1 \tag{10}$$

where

$$b_{ki}^*=b_{ki}\frac{q_i}{q_k},$$

then the Duhamel integral

$$f_k(t)=\int_0^t h_k(t-\tau)Q_k(\tau)d\tau \tag{11}$$

gives a solution of IDE (6) that satisfies homogeneous initial conditions, i.e.

$$f_k(0)=0, \dot{f}_k(0)=0 \tag{12}$$

Indeed, differentiating $f_k(t)$ by *t* and taking into account the double dependence of the right side of equality (11) on the variable concerning which differentiation is performed, we obtain

$$\dot{f}_k(t)=h_k(0)Q_k(t)+\int_0^t \frac{dh_k(t-\tau)}{dt}Q_k(\tau)d\tau$$

$$\ddot{f}_k(t)=\dot{h}_k(0)Q_k(t)+\int_0^t \frac{d^2h_k(t-\tau)}{dt^2}Q_k(\tau)d\tau .$$

From the initial conditions (12), we have

$$h_k(0)=0 \tag{13}$$

$$\ddot{f}_k(t) = \dot{h}_k(0)Q_k(t) + \int_0^t \frac{d^2 h_k(\tau)}{d\tau^2} Q_k(t-\tau)d\tau \,.$$

Substituting relation (11), (13) into (6) and using the easily proven integral identity

$$R * f_k(t) = \int_0^t R(t-\tau)\int_0^\tau R(\tau-s)Q_k(s)dsd\tau = \int_0^t \left(\int_0^\tau R(\tau-s)h_k(s)ds \right) Q_k(t-\tau)d\tau \qquad (14)$$

gives

$$\int_0^t \left\{ \ddot{h}_k(t) + r_k\dot{h}_k(t) + V\sum_i b_{ki}^* h_i(t) + \omega_k^2(1-R^*)h_k(t) \right\} Q_k(t-\tau)d\tau + \dot{h}_k(0)Q_k(t) = Q_k(t) \qquad (15)$$

The resulting equality is satisfied for any $Q_k(t)$, if equalities (9) and (10) hold.

Substituting (11) into (5), we determine the dynamic responses of the hereditarily deformable aircraft structural elements in the gas flow to the pressure of atmospheric turbulence near critical states (divergences, flutter).

IDE (9) under initial conditions (10) will be called a system of weakly singular IDEs of the weight function (impulse transition function).

Numerical solution of the IDE of the weight function. In the general case, the exact analytical solution of the IDE of the weight function (8) in the presence of a weakly singular singularity of the Abel-type heredity kernel presents a significant mathematical difficulty. In this regard, we construct approximate solutions of IDE (6) under initial conditions (10) as follows: first, we reduce IDE (6) under initial conditions (10) to an equivalent integral equation of the form

$$h_k(t) = t - \int_0^t \left\{ r_k h_k(\tau) + (t-\tau)\left[V\sum_i b_{ki}^* h_i(\tau) + \omega_k^2\left(h_k(\tau) - \int_0^\tau R(\tau-s)h_k(s)ds \right) \right] \right\} d\tau \qquad (16)$$

Then, using the substitution $\tau - s = z^{\frac{1}{\alpha}}$, eliminating the weakly singular features of the inner integral and then using the quadrature trapezoid formulas, we obtain the following algorithm for finding a numerical solution to the problem:

$$h_{kj} = \frac{1}{\left(1+\eta_k\frac{\Delta t}{2}\right)}\left\{ t_j - \sum_{n=0}^{j-1} D_n\left[\eta_k h_{kn} + \left(t_j - t_n\right)\left[V\sum_i b_{ki}^* h_{in} + \omega_k^2\left(h_{kn} - \frac{\varepsilon}{\alpha}\sum_{m=0}^{n} c_m e^{-\beta t_m} h_{k/n-m} \right) \right] \right] \right\} \qquad (17)$$

where

$$h_{kj} = h_k\left(t_j\right),\ t_j = j\Delta t,\ j = 0,1,1,\ldots\ D_0 = D_j = \frac{\Delta t}{2},\ D_n = \Delta t,\ n = \overline{1, j-1},\ C_0 = \frac{\left(\Delta t\right)^\alpha}{2},$$

$$C_l = \frac{\left(\Delta t\right)^\alpha}{2}\left[\left(l+1\right)^\alpha - \left(l-1\right)^\alpha\right],\ l = \overline{1, n-1},\ B_n = \frac{\left(\Delta t\right)^\alpha}{2}\left[n^\alpha - \left(n-1\right)^\alpha\right].$$

Thus, the proposed algorithm for the numerical solution of IDE (9) is universal in nature since it allows one to determine not only the numerical and graphical values of the weight function near the critical state but also the critical speed V_{cr} and flutter critical time as ideally elastic $(\varepsilon = 0)$, and hereditarily deformable formulation of the problem $(\varepsilon \neq 0)$.

Determining the critical flight speeds at which the flutter or divergence of an aircraft begins is one of the most important tasks in the problems of aero stability. Its solution is reduced to studying the oscillatory instability of the unperturbed motion of the aircraft based on the developed algorithm (17) and a special algorithm for finding the critical speed [21] based on a computational experiment for given geometric and mechanical parameters. According to this technique, the loss of dynamic stability is determined from the conditions for the existence of undamped harmonic oscillations with a constant increasing amplitude.

As you know, testing is one of the possible ways to check the accuracy of the calculation methodology and the reliability of the study results. For this purpose, we borrow from [21] the results of solving several problems related to the classical flutter.

Table 1 shows the numerical values of the critical flutter velocity for an elastically fixed elongated plate without considering the damped terms of the aerodynamic forces for various geometric θ and mechanical parameters c_2/c_1, found according to the exact and proposed calculation method.

Table 1. Values of the critical flutter velocity for an elastically fixed elongated plate without taking into account the damped terms of aerodynamic forces for various geometric θ and mechanical parameters c_2/c_1, found according to the exact and proposed calculation method

c_2/c_1 θ	1	1/2	2/3	3/4	3/2
½	0.288 (0.29)	0.5 (0.5)	0.421 (0.421)	0.39 (0.39)	0.129 (0.129)
¾	0.324 (0.34)	(0.47) (0.52)	0.425 (0.456)	0.4 (0.43)	0.165 (0.166)

A comparative analysis of the calculation results in this table shows the reliability and high accuracy of the proposed calculation method. The results of the computational experiment as ideally elastic, in the case $\varepsilon=0$, and in the hereditarily deformed case $\varepsilon=0.1$; $\alpha=0.25$; $\beta=0.05$ at $c_2/c_1=1/2$; $\theta=\frac{1}{2}$ shown respectively in Fig. 1 and Fig. 2 , in the ideal elastic case ($N_{cr}=N_0V_{cr}=0.503$; $\bar{t}_{cr}=t_0t=17$), and in the viscoelastic case - $N_{cr}=0.35$; $\bar{t}_{cr}=27$.

Figures 1, 2 show that taking into account the viscoelastic properties of the structural material leads to a decrease in the critical velocity and an increase in the critical flutter time. Computational experiments have shown that the influence of the damping parameter β in the nuclei of heredity on the critical flutter velocity, compared with the viscosity parameter ε and singularities α, turned out to be insignificant, which once again confirms the well-known conclusions - exponential relaxation kernels are unable to fully describe the hereditary properties of the construction material. Since the strain rates proportional to the exponential kernels of heredity will be finite at the initial moment of time, which contradicts the experiment [19, 20] and, as a result, when solving not only the flutter problem but any dynamic problem, errors accumulate over time and the result will be distorted than the actual process.

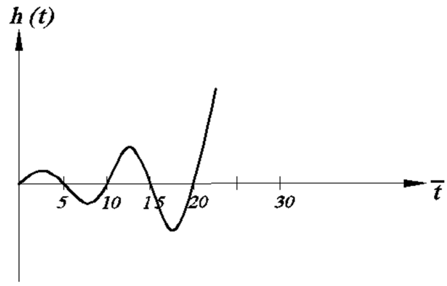

Fig.1. Results of a computational experiment for the ideal elastic case ($\varepsilon=0$) ($N_{cr}=N_0V_{cr}=0.503$; $\bar{t}_{cr}=t_0t=17$) $\varepsilon=0.1$; $\alpha=0.25$; $\beta=0.05$ at $c_2/c_1=1/2$; $\theta=\frac{1}{2}$

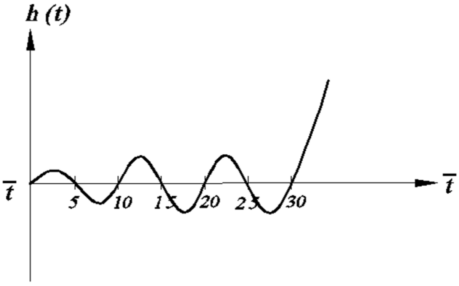

Fig. 2. Results of a computational experiment in the viscoelastic case $N_{cr}=0.35$; $\bar{t}_{cr}=27$, at $c_2/c_1=1/2$; $\theta=\frac{1}{2}$

Thus, the studies in this part of the work have once again confirmed that the main constitutive relations of viscoelastic bodies, the core of heredity, must contain weakly singular features of the Abel type.

Probabilistic-statistical characteristics of forced random vibrations of hereditarily deformable aircraft structural elements. It can be seen from formula (4.27) that with the help of the weight function, the explicit form of the solution of IDE (6) in generalized coordinates is established, which allows us to determine the mathematical expectation of the moment of the input process, correlation functions, spectral densities, i.e., all the probabilistic-statistical characteristics of the investigated IDE (6).

Indeed, the mean value of the generalized coordinates

$$\overline{f_k(t)}=\int_0^t h_k(t-\tau)\overline{Q_k}(\tau)d\tau \tag{18}$$

correlation function

$$K_{f_jf_k}(t_1,t_2)=\int_0^{t_1}\int_0^{t_2}K_{Q_j\theta_k}(\tau_1,\tau_2)h_j(t-\tau_1)h(t_2-\tau_2)d\tau_1d\tau_2 \tag{19}$$

RMS value of generalized coordinates

$$\overline{f_k^2(t)}=\int_0^t\int_0^t K_{Q_k\theta_k}(\tau_1,\tau_2)h_k(t-\tau_1)h_k(t-\tau_2)d\tau_1d\tau_2 \tag{20}$$

As is known [7], in most cases, the random function $Q(t)$ for simplicity, it is presented in the form

$$Q(t)=A(t)\varphi(t) \tag{21}$$

where A(t) is deterministic function and $\varphi(t)$ is stationary random function with an autocorrelation function $K_{\varphi\varphi}(\tau_1-\tau_2)$. In this case, the reaction of IDU (5) to $Q(t)$ those

$$f_k(t) = q_k \int_0^t h_k(t-\tau) Q(\tau) d\tau \tag{22}$$

will be a non-stationary function.

Substituting (21) into (16), with $t_1 = t_2 = t$, we get

$$K_{f_j f_k}(t) = q_j q_k \int_0^t h_j(t-\tau_1) A(\tau_1) \int_0^t h_k(t-\tau_2) A(\tau_2) K_{\varphi\varphi}(\tau_1 - \tau_2) d\tau_1 d\tau_2 \tag{23}$$

Hence, using the substitution $t - \tau_1 = z$, $t - \tau_2 = x$, we get

$$K_{f_j f_k}(t) = q_j q_k \int_0^t h_j(z) A(t-z) \int_0^t h_k(x) A(t-x) K_{\varphi\varphi}(x-z) dx dz \tag{24}$$

Now consider the method of numerical integration of the correlation function when the weight function $h_m(t), m = j,k$ is a solution of the weakly singular IDE (9) under the initial conditions (10). The operation of double integration, which must be performed to implement formula (24) in practical applications, even in the case of differential equations, requires laborious calculations that are not always amenable to quadratures. In the case of weakly singular IDEs, this difficulty will increase even more. In this regard, one of the possible computational algorithms based on the use of the trapezoid formula, in combination with the method of eliminating the weakly singular singularity of integrals and IDEs [11], is proposed below to perform double integration in (24), suitable, both for the case of differential and weakly singular IDE. The general form of this algorithm is

$$K_{f_j f_k}(t_n) = \sum_{r=0}^{n} D_i h_{jr} A(t_n - z_n) \sum_{S=0}^{n} D_s h_{ks} A(t_n - x_s) K_{\varphi\varphi}(x_s - z_r) \tag{25}$$

Values h_{jr} and h_{ks} are calculated by formulas (7).

Thus, the proposed algorithm for calculating the double integral of the correlation function (23) is very simple, universal, and easily implemented in modern personal computers for arbitrary autocorrelation functions of the input process and various approximations of autocorrelation functions obtained as a result of processing real accelerograms, and arbitrarily changing during deterministic functions A(t).

As is known [7], the main task of the practical application of the theory of the correlation function is, using formulas (5), (25) to calculate the expected service life of a structure and develop methods for designing such structures, the probability of failure of which during the specified service life will not exceed the specified value. Defining a system of correlation functions is only the first step towards solving the problem. True, their value makes it easy to find the law of change in time of the mathematical expectation and the mean square value of the generalized coordinates. However, this is not enough for practical purposes.

The second step towards solving the problem is to determine the density of generalized coordinates with fixed integral parameters, in which the probability $P(f_k > R/\tau)$ exceeding the coordinates f_k during τ some dangerous value R at least once. Then, to analyze the damageability of the structure from rare overloads, following [7], we use the approximate conditional probability formula

$$P(f_k > R/\tau) = \frac{1}{2\pi}\int_0^{\pi} \exp\left[-\frac{R^2}{K_{f_k f_k}}\right] dt \tag{26}$$

or using formula (25) from (26) we have

$$P(f_k > R/\tau) = \frac{1}{2\pi}\sum_{n=0}^{K} D_n \exp\left[-\frac{R^2}{K_{f_k f_k}(t_n)}\right] \tag{27}$$

where

$$D_0 = \frac{\Delta\tau}{2}, D_k = \frac{\Delta\tau}{2}, D_n = \Delta\tau, n = \overline{1,k} - 1.$$

3 Conclusions

In conclusion, we note that the theoretical position discussed in this article and practical methods for studying random oscillations are suitable only for linear hereditarily deformable systems. Meanwhile, it is of considerable interest to study the dynamic response of nonlinear hereditarily deformable aircraft structural elements to random perturbations of atmospheric turbulence near the flutter critical state.
Thus, after some generalizations, the method described in this paper can be extended to nonlinear hereditarily deformable complex systems.

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