# The effect of a seismic wave on a two-span beam on rigid supports interacting with the ground

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**Abstract.** The transverse vibrations of a two-span beam bridge, the shore supports of which interact with the surrounding soil during seismic action, are considered. The condition is accepted that the deformations of structures do not go beyond the limits of elasticity and vibrations are linear in nature. The bridge supports are assumed to be submerged in the ground and interacting with a rigid body under the action of non-stationary dynamic influences. The case is considered when the left, middle and right supports have equal masses and interact with the surrounding soil in the same way. The symmetry condition is applied, and it is sufficient to consider the equation of the right half of the beam. The problems are solved by the analytical Fourier method under the given boundary conditions. The results obtained are presented in the form of stress distribution over time and length of bridge structures, and their analysis is also presented.

## 1 Introduction

The Republic of Uzbekistan does not have direct access to seaports, there are no navigable rivers, and there are several mountain ranges in the country, impeding the development of railway and pipeline transportation systems. Uzbekistan is one of two countries in the world that needs to cross through two country borders to reach a sea or ocean shore. It is located in the middle of Central Asia and has the largest population among all the countries of Central Asia. Due to the reasons mentioned above and the geographical location of the country, the importance of the automotive road network (ARN), railways, and pipelines is hard to underestimate. According to the results of the inventory of the country, there are 14331 bridge structures as of the beginning of 2019. Of these, 7628 (53.2%) are located on public roads, and 6703 (46.8%) are located on city roads, villages and on-farm roads [1].

Seismic vibrations of structures are of a very complex spatial nature. With an intense seismic impact, leading to damage, the deformations of the structure go beyond the elastic limits and vibrations, as a rule, are not linear. However, in order to simplify the problem,

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the normalized method for determining seismic forces is based on linear theory and allows independent consideration of three mutually perpendicular (vertical and horizontal) vibration components.

Analysis of data on seismic damage showed that the impact of earthquakes of magnitude 7-9 on roads built according to normal standards leads to substantial damage to structures and serious disruptions to traffic, up to a complete cessation of traffic for a period of several days to several weeks. The failure of bridges during a possible earthquake can lead not only to the costs of restoration or construction of a new structure. The lack of transport access in emergency cases may complicate the work of rescuers and may lead to increased loss of human lives due to the delayed response [2-5].

## 2 Methods

The purpose of this article is to study the transverse vibrations of a two-span beam bridge, the pile part of which, under seismic action, interacts with the surrounding soil. Academician T. Rashidov developed a dynamic theory of seismic resistance of complex systems of underground structures, which is based on taking into account the difference in the deformations of the structure and the soil. On the problems of seismic resistance of underground and surface structures interacting with the surrounding soil in our country and abroad, a number of scientists worked [6-10].

Based on the abovementioned, in the study of seismic vibrations of beam bridges in the first approximation, one limits themselves to considering only the transverse deformations of spans and supports. This is all the more acceptable since it is transverse deformations that play a decisive role in the formation of horizontal seismic forces. Consider a two-span reinforced concrete road bridge (Fig. 1).

Consider a road two-span beam with three supports. As a first approximation, the bridge supports can be assumed to be immersed in the ground and interacting with a rigid body, and under the influence of non-stationary dynamic influences. Set the origin at point $O$ and direct the $Ox$ axis along the neutral axis of the beams, and the axis and $Oy_1$ (with origin at point $O_1$) is perpendicular to it (Fig. 2). Let a longitudinal wave flow around the supports, behind the front of which the movement of soil particles depends on the coordinate $y_1$ and time $t$ according to the law $u_0=u_0(t-z/c_0)$. The lateral surfaces of the extreme supports are in contact with the ground, and the middle support interacts with the ground only with the lower section [11].

**Fig. 1.** Reinforced concrete road beam bridge

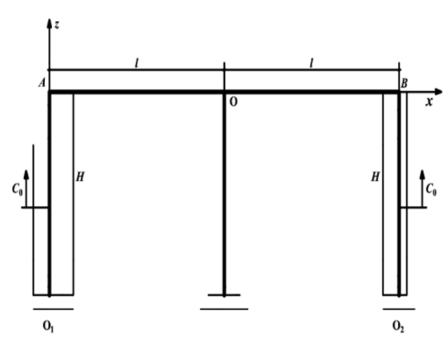


**Fig. 2.** Calculation scheme for a road two-span beam bridge with movable supports

The deflections of the beams $y_i=y_i(x, t)$ satisfy the equations

$$m_b \frac{\partial^2 y_1(x,t)}{\partial t^2} + EJ_z \frac{\partial^4 y_1(x,t)}{\partial x^4} = 0 \text{ for } -l < x < 0 \tag{1}$$

$$m_b \frac{\partial^2 y_2(x,t)}{\partial t^2} + EJ_z \frac{\partial^4 y_2(x,t)}{\partial x^4} = 0 \text{ for } 0 < x < l\,. \tag{2}$$

The deflections $y_i=y_i(x, t)$ satisfy the boundary conditions
where $m_b$ – is the linear weight of the girder, $E$ – is the Young's modulus of the material of the girder, $J_z$ – is the moment of inertia of the section, $l$ – is the length of the span

$$\frac{\partial y_1}{\partial x} = 0 \text{ for } x = -l, \ \frac{\partial^3 y_1}{\partial x^3} = 0 \text{ for } x = -l - 0 \tag{3}$$

$$\frac{\partial y_1}{\partial x} = 0 \text{ for } x = 0, \ \frac{\partial^3 y_1}{\partial x^3} = 0 \text{ for } x = +0 \tag{4}$$

$$\frac{\partial y_2}{\partial x} = 0 \text{ for } x = 0, \ \frac{\partial^3 y_2}{\partial x^3} = 0 \text{ for } x = -0 \tag{5}$$

$$\frac{\partial y_2}{\partial x} = 0 \text{ for } x = l, \ \frac{\partial^3 y_2}{\partial x^3} = 0 \text{ for } x = l + 0 \tag{6}$$

We denote by $u_1=y_1(-l, t)$, $u_0=y_1(0, t)=y_2(0, t)$, $u_2=y_2(l, t)$ the displacements of the supports interacting with the ground according to Winkler's law, which are the equations of motion

$$M_1 \frac{\partial^2 y_1(-l.t)}{\partial t^2} = -EJ_z \frac{\partial^3 y_1(-l,t)}{\partial x^3} - k_{11}[y_1(-l,t) - u_0(t)] - k_{12}[y_1(l.t) - u_1(t)] + M_1 g\,, \tag{7}$$

$$M_2 \frac{\partial^2 y_2(l,t)}{\partial t^2} = EJ_z \frac{\partial^3 y_2(l,t)}{\partial x^3} - k_{21}[y_2(-l,t) - u_0(t)] - k_{22}[y_2(l.t) - u_1(t)] + M_2 g \tag{8}$$

$$\begin{aligned} M_0 \frac{\partial^2 y_1(0.t)}{\partial t^2} &= -EJ_z\varsigma \frac{\partial^3 y_1(0,t)}{\partial x^3} + EJ_z(1-\varsigma)\frac{\partial^3 y_2(0,t)}{\partial x^3} - \\ &- k_0\varsigma[y_1(0,t) - u_0(t)] - k_0(1-\xi)[y_2(0,t) - u_0(t)] + M_0 g. \end{aligned} \tag{9}$$

$M_1$, $M_0$, $M_2$ – are the weights of the left, middle and right supports, respectively, $\zeta_1=h_1/h$, $h=h_1+h_2$, $h_1$, $h_2$ – are the contact lengths of the end sections of the left and right beams with the middle supports, $k_{12}$ and $k_{22}$ – are the stiffness coefficients of the longitudinal shear on the contact surface, respectively, of the right and left supports with the ground, $k_{11}$, $k_{12}$ and $k_0$ – are the coefficients of the elastic resistance of the soil in the lower sections of the supports, $u_0(t)$ – is the movement of soil particles behind the front of the incident longitudinal wave

$$u_1 = \frac{1}{H}\int_0^{c_0 t} u_0(t - y_1/c_0)\,dy_1 \text{ for } t < H/c_0, \tag{10}$$

$$u_1 = \frac{1}{H}\int_0^H u_0(t - y_1 / c_0)dy_1 \text{ for } t > H / c_0 . \tag{11}$$

$H$ – is the height of the supports.

The deflections of the beam, except for (8) – (10), at the points of conjugation with the supports, satisfy the condition of being kept in a horizontal position

$$\frac{\partial y_1}{\partial x} = 0 \text{ for } x = -l,\ x = 0 \tag{12}$$

$$\frac{\partial y_1}{\partial x} = 0 \text{ for } x = 0,\ x = l$$

Consider the case when the right and left supports have equal masses and interact with the surrounding soil according to the same laws. Then one should set $y_1(x, t)=y_2(-x, t)=y(x, t)$, $M_1=M_2=M$, $\varsigma=1/2$, $k_{11}=k_{21}=k_1$ and $k_{12}=k_{22}=k_2$ and use symmetry condition, it is enough to consider the equation of the right half of the beam, the equation of motion of which, taking into account conditions (9) and (10), can be written in the form

$$m_б \frac{\partial^2 y(x,t)}{\partial t^2} + EJ_z \frac{\partial^4 y(x,t)}{\partial x^4} = 0,\ 0 < x < 1,\ t > 0 \tag{13}$$

Moreover, the function $y(x, t)$ satisfies the conditions

$$\frac{\partial y}{\partial x} = 0,\ M_0 \frac{\partial^2 y(0.t)}{\partial t^2} = -EJ_z \frac{\partial^3 y(0,t)}{\partial x^3} - k_0 y(0,t) + k_0 u_0(t)] + M_0 g \text{ for } x=0, \tag{14}$$

$$\frac{\partial y}{\partial x} = 0,\ M \frac{\partial^2 y(l.t)}{\partial t^2} = EJ_z \frac{\partial^3 y(l,t)}{\partial x^3} - (k_1 + k_2) y(l,t) + k_1 u_0(t) + k_2 u_1(t) + M_1 g \text{ for } x=l. \tag{15}$$

We introduce a new function by the formula

$$\bar{y} = y(\xi,t) - y_*(\xi,t)$$

where $y_* = \xi^2 (1-\xi)^2 [c_1(t)\xi + c_2(t)]$

$$c_1 = \frac{(\beta_0 - \beta_1)u_0(t) - \beta_2 u_1(t) + \overline{M}_0^* - \overline{M}_1^*}{12},$$

$$c_2 = -\frac{(\beta_1 + 4\beta_0)u_0(t) + \beta_2 u_1(t) + \overline{M}_1^* + 4\overline{M}_0^*}{12}.$$

$$\xi = x / l,\ \beta_0 = k_0 l^3 / EJ_z,\ \beta_1 = k_1 l^3 / EJ_z,\ \beta_2 = k_2 l^3 / EJ_z,\ \overline{M}_0^* = M_0 g l^3 / EJ_z,$$

$$\overline{M}_1^* = M_1 g l^3 / EJ_z.$$

The function $\bar{y}(\xi,t)$ satisfies the inhomogeneous equation $\left(\overline{m}_{в} = m_{в} l^4 / EJ\right)$

$$\overline{m}_{б}\frac{\partial^2 \bar{y}(\xi,t)}{\partial t^2}+\frac{\partial^4 \bar{y}(\xi,t)}{\partial \xi^4}=-\overline{m}_{в}\xi^2(1-\xi)^2[\ddot{c}_1(t)\xi+\ddot{c}_2(t)]-c_1(t)(120\xi-48)-24c_2(t) \quad (16)$$

homogeneous boundary

$$\frac{\partial \bar{y}}{\partial \xi}=0,\ \frac{M_0^*}{g}\frac{\partial^2 \bar{y}}{\partial t^2}=-\frac{\partial^3 \bar{y}}{\partial \xi^3}-\beta_0 \bar{y} \text{ for } \xi=0 \quad (17)$$

$$\frac{\partial \bar{y}}{\partial \xi}=0,\ \frac{M_1^*}{g}\frac{\partial^2 \bar{y}}{\partial t^2}=\frac{\partial^3 \bar{y}}{\partial \xi^3}-(\beta_1+\beta_2)\bar{y} \text{ for } \xi=1. \quad (18)$$

and initial conditions

$$\bar{y}(\xi,0)=\xi^2(1-\xi)^2[c_1(0)\xi+c_2(0)]\frac{\partial y}{\partial t}=\xi^2(1-\xi)^2[\dot{c}_1(0)\xi+\dot{c}_2(0)] \text{ for } y=0. \quad (19)$$

The initial boundary value problem (18) – (19) for equation (16) is obtained by the Fourier method, according to which the solution of the corresponding homogeneous equation can be represented in the form

$$\bar{y}=\varphi(\xi)T(t)$$

where we put $\ddot{T}=-\omega^2 T(t)$. The function $\varphi(\xi)$ satisfies the equation:

$$\varphi^{IY}-\lambda^4\varphi=0. \quad (20)$$

and boundary conditions

$$\varphi'=0,\ \varphi'''=(\lambda^4\overline{M}_0-\beta_0)\varphi \text{ for } \xi=0, \quad (21)$$

$$\varphi'=0,\ \varphi'''=-(\lambda^4\overline{M}_1-\beta)\varphi \text{ for } \xi=1, \quad (22)$$

where $\lambda=l\sqrt[4]{\frac{m_{в}\omega^2}{EJ_z}}$, $\beta=\beta_1+\beta_2$, $\overline{M}_0=M_0/m_{в}l$, $\overline{M}_1=M_1/m_{в}l$.

We represent the solution of equation (21) in terms of the Krylov functions ($C_i$ are arbitrary constants)

$$\varphi=C_1Y_1\lambda\xi)+C_2Y_2(\lambda\xi)+C_3Y_3(\lambda\xi)+C_4Y_4(\lambda\xi)\ \ 0<\xi<1,$$

where $Y_1(z)$ Krylov functions $Y_1(z)=(chz+\cos z)/2,\ Y_2(z)=(shz+\sin z)/2,$

$$Y_3(z) = (chz - \cos z)/2, \ \ Y_4(z) = (shz - \sin z)/2.$$
$$\varphi = C_1 Y_1 \lambda\xi) + C_2 Y_2(\lambda\xi) + C_3 Y_3(\lambda\xi) + C_4 Y_4(\lambda\xi) \ \ 0 < \xi < 1.$$

From conditions (21), (22) it follows
$C_2$=0

$$f_0(\lambda)C_1 - C_4 = 0 \tag{23}$$

$$C_1[Y_4(\lambda) + f_1\lambda)Y_3(\lambda)] + C_3 Y_2(\lambda) = 0, \tag{24}$$

$$C_1[Y_2(\lambda) + f_1(\lambda)Y_1(\lambda)] + C_3[Y_4(\lambda) + f_1(\lambda)Y_3(\lambda)] + C_4[Y_1(\lambda) + f_1(\lambda)Y_4(\lambda)] = 0 \tag{25}$$

where $f_0 = \dfrac{\lambda^4 \overline{M}_0 - \beta_0}{\lambda^3}$, $f_1 = \dfrac{\lambda^4 \overline{M}_1 - \beta}{\lambda^3}$.

We equate the determinant of the system of equations to zero (23) – (25) with respect to, $C_1$, $C_3$ and $C_4$, we compose an equation to determine the eigenvalues $\lambda=\lambda_i$.
Setting $C_1$=1, we represent the expressions for the eigen functions $\varphi_i(\xi)$ in the form

$$\varphi_i = Y_1(\lambda_i\xi) - \frac{\lambda_i^3 Y_4(\lambda_i) + (\lambda_i^4 \overline{M}_0 - \beta_0)Y_3(\lambda_i)}{\lambda_i^3 Y_2(\lambda_i)} Y_3(\lambda_i\xi) + \frac{\lambda_i^4 \overline{M}_0 - \beta_0}{\lambda_i^3} Y_4(\lambda_i\xi). \tag{26}$$

It can be shown that the Eigen functions $\varphi_i(\xi)$ satisfy the generalized orthogonally condition

$$m_{\text{в}} \int_0^1 \varphi_i \varphi_k d\xi + [M_0 \varphi_i(0)\varphi_k(0) + M_1 \varphi_i(1)\varphi_k(1)]/l = 0 \ \text{ for } i \neq k. \tag{27}$$

We represent the solution of the inhomogeneous equation (16) as the sum

$$\overline{y} = \sum_{n=1}^{\infty} \varphi_n(\xi) T_n(t). \tag{28}$$

Putting expression (28) into equation (16), we obtain

$$\sum_{n=1}^{\infty} [\ddot{T}_n + \omega_n^2 T_n]\varphi_n(\xi) = \xi^2(1-\xi)^2[\ddot{c}_1(t)\xi + \ddot{c}_2(t)] - [c_1(t)(120\xi - 48) - 24c_2(t)]/\overline{m}_{\text{в}}$$

.

Using the orthogonality condition (27), we compose an equation for the expansion coefficients

$$\ddot{T}_n + \omega_n^2 T_n = F_n(t) = -b_n \ddot{c}_1 - d_n \ddot{c}_2 - [r_n c_1(t) + s_n c_2(t)]/\overline{m}_{в}, \tag{29}$$

where $b_n = \dfrac{1}{\|\varphi_n\|} \displaystyle\int_0^1 \xi^3 (1-\xi)^2 \varphi_n(\xi) d\xi$, $d_n = \dfrac{1}{\|\varphi_n\|} \displaystyle\int_0^1 \xi^2 (1-\xi) \ \varphi_n(\xi) d\xi$,

$$r_n = \frac{24}{\|\varphi_n\|}\int_0^1 (5\xi - 2)\varphi_n(\xi)d\xi\,,\ s_n = \frac{24}{\|\varphi_n\|}\int_0^1 \varphi_n(\xi)d\xi\,,$$

$$\|\varphi_n\| = \int_0^1 \varphi_n^2 d\xi + \overline{M}_0\varphi_n^2(0) + \overline{M}_1\varphi_n^2(1).$$

Equations (29) satisfy the initial conditions

$$T_n(0) = \frac{1}{\|\varphi_n\|}\int_0^1 \xi^2(1-\xi)^2[\xi c_1(0) + c_2(0)]\varphi_n(\xi)d\xi\,,$$

$$\dot{T}_n(0) = \frac{1}{\|\varphi_n\|}\int_0^1 \xi^2(1-\xi)^2[\xi\dot{c}_1(0) + \dot{c}_2(0)]\varphi_n(\xi)d\xi$$

The solution of equation (29) under the indicated initial conditions has the form:

$$T_n = \frac{1}{\omega_n}\int_0^t F_n(\tau)\sin[\,\omega_n(t-\tau)]d\tau + T_n(0)\cos(\omega_n t) + \frac{\dot{T}_n(0)}{\omega_n}\sin[\,\omega_n t) \tag{30}$$

Putting the expressions $T_n(t)$ in series (28), we obtain solutions to the original problem in the form

$$y = y_*(\xi,t) + \sum_{n=1}^{\infty} T_n(t)\varphi_n(\xi)\,. \tag{31}$$

## 3 Results and discussion

Fig. 3, 4 and 5 shows the time variation curves of axial stresses in different sections of the beam of three types. Young's modulus and moment of inertia in all beams are the same and correspondingly equal to $E$=3.45·10$^4$ MPa, $J_z$=0.023 m$^4$. The movement of soil particles behind the wave front is determined by the law $u_0 = A\sin\left(\frac{c_0 t - y}{L}\right)$, where $c_0$=600 m/s is the propagation velocity of the longitudinal wave, $A$=0.005 m is the amplitude of the oscillations, L is the wave length. The calculations take $L$=50 м, $L$=100 м, $L$=150 м, $L$=200 м. The height of the supports for all bridges is assumed to be $H$=8 m. The coefficients of elastic compression of the soil in the lower part of the supports $k_0$=$k_1$=10$^6$ N/m, the coefficient of longitudinal displacement of the soil on the side surface of the right support $k_2$=5·10$^6$ N/m.
Fig. 3, 4 and 5 shows the dependence of the longitudinal stress on time in different sections for three types of beam
Example 1. The beam has the following characteristics: $l$=12 m, $m$=300 kg/m$^3$

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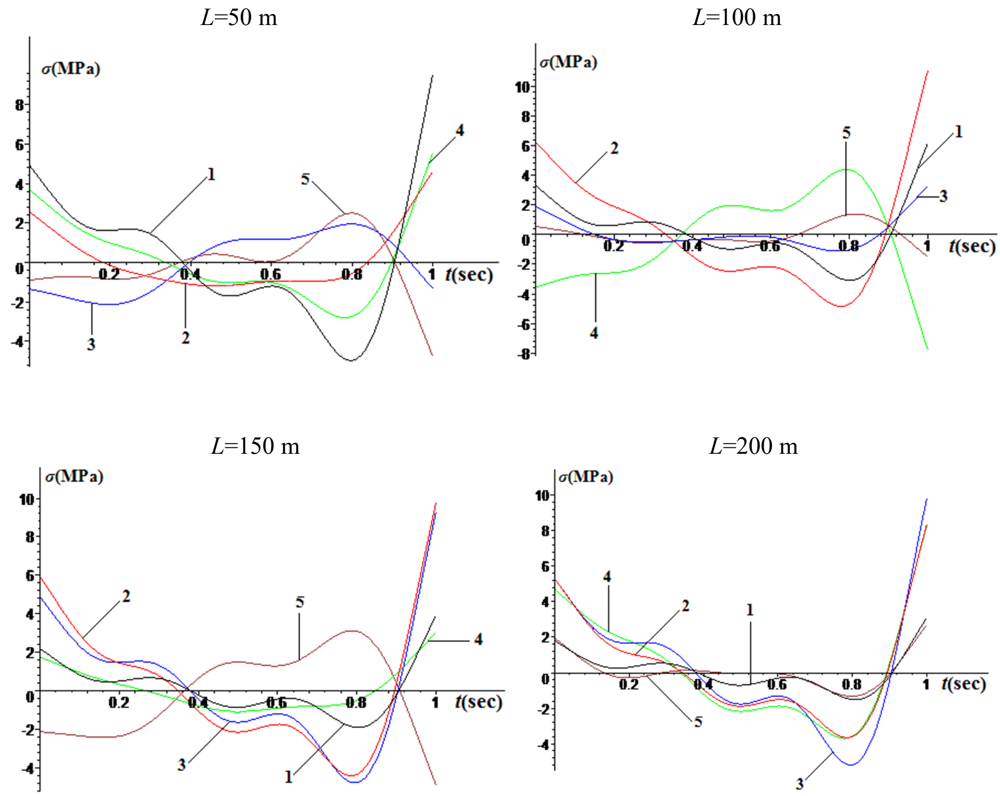
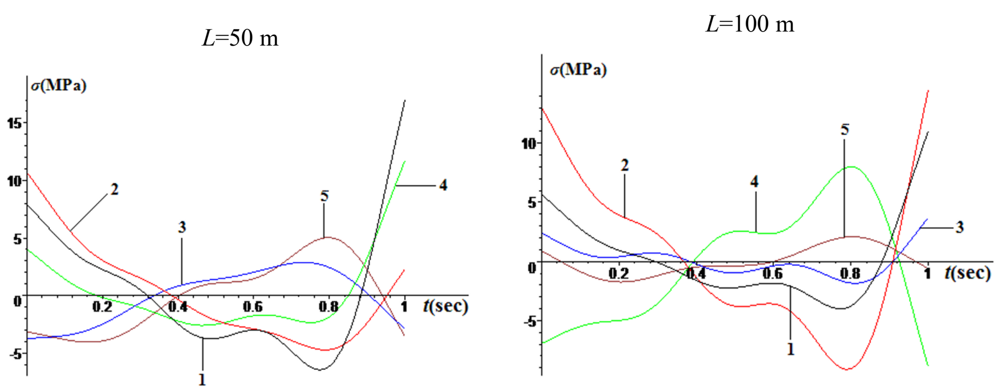



**Fig. 3.** The dependence of the longitudinal stress on time $t$ (sec) in different sections of the beam $\vec{x} = x/l$: $1-\bar{x}(black) = 0.2$, $1-\bar{x} = 0.25(red)$, $1-\bar{x}(blue) = 0.5$, $1-\bar{x}(green) = 0.75$, $1-\bar{x}(brown) = 1$. $l$=12 m, $m$=300 kg/m$^3$

Example 2. The beam has the following characteristics: $l$=15 m, $m$=350 kg/m$^3$,



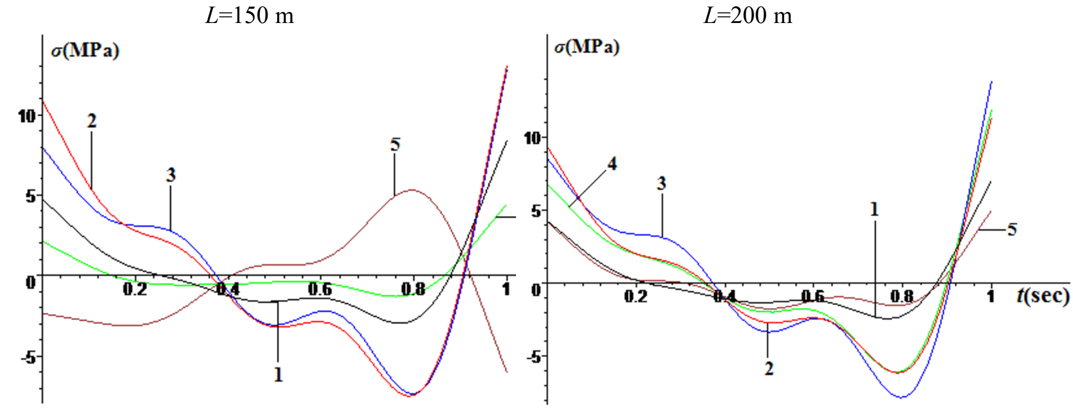



**Fig. 4.** The dependence of the longitudinal stress on time $t$ (sec) in different sections of the beam $\vec{x} = x/l$: $1-\bar{x}(black) = 0.2$, $1-\bar{x} = 0.25(red)$, $1-\bar{x}(blue) = 0.5$, $1-\bar{x}(green) = 0.75$, $1-\bar{x}(brown) = 1$. $l$=15 m, $m$=350 kg/m$^3$

Example 3. The beam has the following characteristics: $l$=18 m, $m$=350 kg/m$^3$,

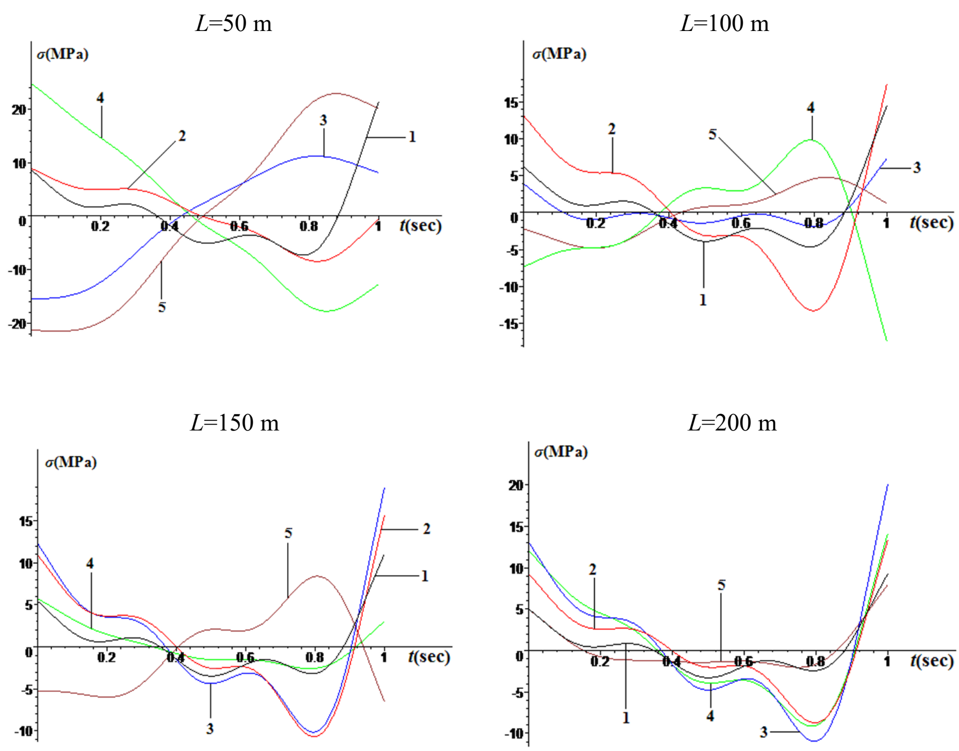



**Fig. 5.** The dependence of the longitudinal stress on time $t$ (sec) in different sections of the beam $\vec{x} = x/l$: $1-\bar{x}(black) = 0.2$, $1-\bar{x} = 0.25(red)$, $1-\bar{x}(blue) = 0.5$, $1-\bar{x}(green) = 0.75$, $1-\bar{x}(brown) = 1$. $l$=18 m, $m$=350 kg/m$^3$

From the analysis of the curves presented in Fig. 3, 4 and 5 it follows that the stress changes over time in different cross sections qualitatively for all types of beams are close to harmonic. The greatest stress values over time are achieved in sections close to the edge sections of the beams, and with increasing wave length $L$, the nature of stress changes in different sections over time acquire the same character and their maximum values decrease. This indicates the regularity of the transition to the static state of the beam in the case of the action of a wave with a long length. For all types of beams presented in Examples 1, 2 and 3, tensile stresses are acquired in cross sections, which are approximately twice as large as compressions.

Thus, under the action of a sinusoidal seismic wave with a frequency of $c_0/L$, maximum bending moments with a positive sign prevail in the beam sections and it can be expected that the destruction of the beam can mainly occur due to high values of tensile stresses in the sections of the elements in contact with the supports.

## 4 Conclusions

Bridge structures on highways are an important component of the unified transport system of Uzbekistan. Even partial destruction of several structures during earthquakes can lead to disruption of communication between settlements. Taking everything into account, the increase in the number and carrying capacity of motor vehicles, the issues of the development of earthquake-resistant structures of bridge structures are relevant.

A method for calculating a two-span beam on the action of a sinusoidal longitudinal seismic wave with a front parallel to the axis of the beam, based on the application of the Fourier method, is proposed.

According to the analysis of the results of calculations performed for three types of beams, it was found that the action of this type of seismic wave leads to the occurrence of tension and compression stresses in the cross sections, and the beams work more on tension.

By analyzing the patterns of changes in the maximum stress values in the boundary sections of the beam, it has been that the stresses in these sections have a positive sign, which may be the cause of the rupture of the beams in these sections.

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