Features of spectral vibration diagnostics of traction power transformers in high-speed motion

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Abstract. This paper uses spectral analysis methods to develop a mathematical model and algorithms for evaluating the diagnosed signals corresponding to certain defects of windings and magnetic circuits of traction power transformers. A distinctive feature of the study is at certain frequencies of spectral dependences, taking into account the effects associated with defect formation that have a different cause of their manifestation, for example, wide-field fluctuations and changes in the shape and width of spectral peaks. The presence of random fluctuations caused the application of the normal Gaussian distribution law of the measured values of the spectra. To solve the problem of identifying the diagnosed spectra corresponding to certain defects of the power transformer, algorithms for identifying the vector of diagnostic features based on the method of statistical recognition theory for large amounts of information are proposed.

1 Introduction

One of the important ways to diagnose power transformers (PT) of traction power supply is vibration diagnostics, which ensures its reliable operation.

New design solutions underlying the developed PT [1, 2, 3] make it possible to introduce into their operation practice the optimal modes of regulation of the voltage of the PT under load to determine their technical condition using the methods of spectral analysis of technical means of vibration diagnostics.

The diagnostic parameters of power transformers (PT) of traction substations are the parameters of the technical condition of the elements of the windings and the magnetic circuit, which practically reach more than 100 items of electrical and non-electrical characteristics. Among the methods for diagnosing expensive high-voltage equipment, spectral analysis of vibration diagnostics, acoustic noise, external electromagnetic radiation, electric currents, and voltages, ultrasonic magnetic detection, chromatographic analysis of dissolved gases in oil, and so on are widely used. [4,5].

A review of the international literature (for such countries as Germany, France, Japan, China, South Korea, etc.) shows that in the functional control and diagnosis of PT under operating voltage, diagnostic monitoring is used based on the spectral method, which is a priority form of diagnosis [1-5].

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A common disadvantage of the above spectral analysis processing methods is:

- insufficient information integrity

- the need to improve processing algorithms

- ensuring invariance to disturbances of various natures, for example, the shift of the spectral peak in frequency, shape, and amplitude.

The reasons for such changes are the influence of the external environment, changes in the parameters of measuring instruments, pre-processing, or the gradual development of defects due to relatively slow processes.

This paper substantiates one of the possible estimates of the parameters of spectral peaks according to the dispersions of their amplitude, shape, and frequency, which provide the current state of the PT, the dangers, and risks associated with its use.

The accumulated experience of operation and testing of traction power supply PT shows that the problem of insufficient electrodynamic resistance of both windings and magnetic circuits under extreme loads and short circuits remains relevant today.

2 Methods

Vibration diagnostics sensors are installed on the wall of the PT tank, and by analyzing their spectral characteristics, the technical condition of the crimping of the windings and the magnetic circuit, which are the main elements, is determined. The process of diagnosing is carried out during its operation. The technical data of the sensors of vibration installations for PT diagnostics must be within the following limits, taking into account the magnetostriction of the magnetic circuit and electrodynamic processes in the windings:

vibration acceleration - $10-15 m/s^2$; vibration speed - 10-15 mm/s; vibration displacement - 100 mkm;

frequency - from zero to 700 Hz.

At the Department of Power Supply of the Tashkent State Transport University, new design solutions for vibration sensors have been developed based on magnetoelastic magnetic circuits with high metrological static and dynamic characteristics.

An analysis of the results of experimental vibration tests of traction power supply PT showed that obvious factors for the appearance of peaks in the spectrograms at certain frequencies, several effects associated with defect formation, or another reason for their manifestation are observed on the spectral dependences. For example, such factors include broad-band fluctuations, changes in the shape and width of spectral peaks, spectral doublets, etc., which appear during high-speed electric locomotives.

To formalize the technology for constructing a vector of real diagnostic features, the spectral analysis method of amplitude-modulated PT output currents is used, with preliminary processing and analysis, which ensure the smoothing of random fluctuations [6, 7]. It is also necessary to use the approximation method of spectral peaks, identify doublets and determine the boundaries of spectral peaks. The purpose of processing is to determine the peak or integral intensities of the spectral components [8, 9, 10], the positions of the maxima, i.e., "centers of gravity" of peaks, and ultimately for more correct, reliable definitions of diagnostic signs for subsequent prediction of the detection of the parameter of spectral curve models (Fig. 1. a, b, c).

3 Results and Discussion

As PT malfunctions develop, corresponding changes occur in electrodynamic processes [11] and in the electrochemical state of solid and liquid insulation [12], leading to

qualitative and quantitative changes in the quantities affecting its elements. As a result, the level of their mechanical vibrations changes accordingly.

It is known that the vibration on the ST tank unambiguously correlates with the mechanical state of the winding and magnetic circuit [13, 14]. As a result, the frequency and amplitude of vibration, characterized by vibration velocity and vibration acceleration, change.



Fig. 1. Various representations of spectral information: full spectrum a); compressed b); approximated by a triangle c).

Given that PT vibration spectra have many random fluctuations, it is advisable to use a probabilistic approach to peak detection [15,16]. In this sense, the task is to approximate the peaks by a mathematical model, which can be chosen using a priori information characterizing the properties of the spectra, taking into account the type of one or another distribution law of the spectrum estimation errors. Practice shows that the normal Gaussian law corresponds to the measured values of the y(f)PT spectrum, which has a distribution law [17, 18]:

$$P[y(f)] = \frac{1}{\sqrt{2\pi\sigma_{\mathrm{III}}}y(f)} \exp\left\{-\frac{\left|y(f)-\xi(f)\right|^2}{2\sigma_{\mathrm{III}}^2y(f)}\right\}$$
(1)

where σ^2_{u} is the variance of random noise $\xi(f)$; y(f) is the current random value of the spectrum. The criterion for detecting a peak in a spectrum is usually the likelihood ratio [19]:

$$P\left[\frac{y(f)}{H_1}\right] / \left[\frac{y(f)}{H_0}\right] \ge c \tag{2}$$

where H_0 and H_1 are, respectively, the absence of a peak and the presence of a peak in the region under study; c - spectrum peak detection threshold.

Taking into account that the experimental data are presented as a discrete series $y(f_i)i = 1,2,...n$, we write the likelihood function in the form [20,21]:

$$\exp\left\{-\sum_{i=1}^{n} \frac{\left[y(f_{i})-\Pi(f_{i})\right]^{2}}{2\sigma_{\mathrm{III}}^{2}f(i)}\right\} / \left\{-\sum_{i=1}^{n} \frac{\left[y(f_{i})-\overline{\Pi}(f_{i})\right]^{2}}{2\sigma_{\mathrm{III}}^{2}f(i)}\right\} \ge c$$
(3)

where $\Pi(f_i)$ is the spectral peak model; $I(f_i) - \overline{\Pi}(f_i)$ is the mathematical expectation of the spectral peaks.

Condition (3) corresponds to the numerical detection criterion [22]:

$$\sum_{i=1}^{n} y(f_i) \cdot \prod_j (f_i) \ge c .$$

$$(4)$$

The choice of detection threshold can be calculated as follows:

$$C_{\text{of}} = (2 \div 3)\sigma_{\text{III}}^2 \sqrt{\sum_{i=1}^{n} \prod_{j=1}^{2} (f)}.$$
(5)

At a high level of noise in the elements and devices of vibration diagnostics, which have during high-speed movement of electric trains, there may be a lack of a priori information, leading to the inadequacy of the model to the $\prod_{j} (f_i)$ signal $y(f_i)$, which worsens the

performance of the spectrum peak detection algorithm. In this case, it is advisable to use a rectangular model. Having a detection criterion of the following form:

$$\sum_{i=1}^{n} y(f_i) \ge (2 \div 3)\sigma_{\text{III}} \sqrt{a} , \qquad (6)$$

where *a* is the width of the rectangular peak.

If normalized along the frequency axis (for example a=1), then the detection algorithm will be determined by the formula:

$$\sum_{i=1}^{n} y(f_i) \ge (2 \div 3)\sigma_{\text{III}},\tag{7}$$

if, in the primary spectral data, there is a superposition of their components caused by rattling and rapid random oscillations of the places of attachment to the tank of primary vibration sensors, it is advisable to use a non-parametric method for separating these lines. The essence of which lies in the method of narrowing the spectral peaks. Mathematically, this method reduces to solving the Fredholm integral equation of the 1st kind [23, 24]:

$$y(f) = \int K(f - w) \cdot y(w) \cdot dw_2 + f(w)$$

$$\Omega$$
(8)

Where K(f - w) is the kernel of the integral equation, determined by the factors leading to a blurring of the combined spectral peaks on the segment (Fig. 2) y(w); at f(w) = 0 - the equation will be homogeneous.

The Fredholm equation (8) is solved by successive approximations.

The technical implementation of the solution of the Fredholm equation is carried out by synthesizing digital filters in the frequency domain. The essence of this synthesis is based on the theory of the Fourier transform, which relates the change in the scale of a function in the time domain with the change in the scale of the frequency response.

Information about the position of spectral peaks, their peak, and integral intensity is essential for vibration diagnostics of PT. To identify diagnostic features based on them, there is a group of methods based on transformations of the original signal (see Fig. 1a), leading to distortion of the peak profiles but invariant concerning the desired parameters. For example, it is known that the Dirac delta function is represented by a linear combination of even derivatives of the Gaussian function:

$$\delta(x - x_0) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \sum_{k=0}^{\infty} (-1)^k \frac{\sigma^{2k}}{2^k \cdot k!} y^{2k}(x), \tag{9}$$

where
$$y(x) = \exp\left\{-\frac{(x-x_0)}{2\sigma_x^2}\right\}$$
. (10)

Analysis of expression (10) shows that a simpler expression can replace it:

$$y(x) = \sum_{k=0}^{n} (-1)^{k} a_{2k} \cdot y^{2k}(x), \qquad (11)$$

where a_{2k} are the uncertain coefficients determined by the Lagrange method.

The theoretical basis for recognizing diagnostic features is statistical recognition in the presence of sufficient information or the use of deterministic methods that more simply describe the essential aspects of the phenomenon [25, 26, 27].

It is known that the technical state of PT W_i - is characterized by diagnostical features

 u_{ik} in the form of *an n*-dimensional matrix vector:

$$U_i = \{ u_{i1}, u_{i2}, \dots, u_{in} \}.$$
(12)

For the most common technical states of the PT of high-speed traction power supply, several reference spectra are required that uniquely correspond to these matrix states in the form:

$$V_i = \{v_{i1}, v_{i2}, \dots, v_{in}\}.$$
 (13)

When comparing the diagnosed features in the form of identified spectra with the reference spectra in the form of a composition of peaks of a given shape in the form of triangles, the spectra are superimposed, and their overlap area is calculated ΔS_{j} , j = 1, 2, ..., N (Fig. 2).

The value of the ratio is taken as the compliance criterion V_i :

$$V_j = \frac{\Delta S_j}{S_j},\tag{14}$$

where S_{i} is the total area of the spectrum of the *j* -th standard, ΔS_{i} is the overlap area.



Fig. 2. Illustration of areas of superposition of reference spectra and diagnosed ST.

Mathematical evaluation V_j can also be performed based on the operator W, which establishes a correlation between the vector of diagnostic features and vectors X, which determine certain defects in the technical condition of the ST.

$$U = W(A, E, X) \tag{15}$$

where A is the parameters of the diagnosed ST; E is external negative influences on the vibration diagnostics system; X is a vector that determines defects in the technical condition of the ST. To reduce the error in diagnostics, it is necessary to maintain E = const.

If the difference between the defect-containing spectrum X_0 and the reference spectrum V_0 is by ΔX , then the vector of diagnostic features can be mathematically represented as:

$$U_{i} = V_{0} + \Delta X = W(A, E, X_{0}) + \frac{dW(A, E, X)}{dX} \Big|_{X = X_{0}}^{\Delta X}.$$
 (16)

Numeric value expression ΔX is defined as the value of the boundary ratio $X_{\Gamma p}$. The preliminary values of experimental vibration tests for possible defects were equal to $X_{\Gamma p} \approx 0.8 \div 0.9$.

CT diagnostics based on spectral analysis and the known identification algorithm involves comparing the selected reference spectra with those being diagnosed. However, in vibration diagnostics, the prevailing task is to compare the established diagnostic features with a combination of reference spectra that uniquely correspond to specific defects, such as winding deformation.

In this case, the input parameter of the algorithm is the allowable window width, determined by the positions of the lines in a narrow frequency band, which determines the diagnosis node's measurement quality.

It should be borne in mind that each of N the reference spectra is given by a known intensity $X_{ik} \in [0,1]$ and frequency $f_{ik} \in [0, f_{\max}]$.

If the diagnosed spectrum is characterized by a sequence of amplitudes U_{jk} and, respectively, frequencies f_{jk} at k = 1, 2, ..., n, then the error of the spectral lines can be estimated from the inequality [28]:

$$U_{jk} - \delta(U_k) \le \sum_{j=1}^N k_j f_{jk} \le U_k + \delta(U_k), \qquad (17)$$

where k_j is a coefficient that considers the allowable combination of reference spectra for various types of defects.

Each factor f_{jk} is the sum of all lines of *the j* -th standard located in Δf the vicinity of k the -th lines of the sample:

$$\begin{split} f_{jk} &= \sum_{S} f_{jkS} \,, \eqno(18) \\ f_{jkS} &\in \left[f_{jk} - \Delta f, f_{jk} + \Delta f \right] . \end{split}$$

where f_{ik} is the final value of the frequency.

Analysis of (17) and (18) and the conducted test spectrograms show that when (17) and (18) are performed, the diagnosed spectrum corresponds to the standard with an allowable error Δf , with relative standard deviations equal to $1 \div 5\%$.

To improve the reliability of the technology for assessing the state of the PT in vibroacoustic diagnostics by identifying the diagnosed spectra, we use the Bayes method [29].

The algorithms of this group are based on the vector representation of spectrograms, the frequency range of which $f_{\min} \div f_{\max}$ is divided into *n* intervals by the value $\Delta = (f_{\min} - f_{\max})/n$. The spectrogram is represented as a vector, *the j*-th element of which is determined by the spectrum values in the interval $[f_{\min} + (j-1)\Delta; f_{\min} + j\Delta]$, where $j \Delta$ is the current value of. In this case, if there is a peak in the spectrum in a given interval, then *the j*-th element of the vector is assumed to be equal to the intensity of this peak, and the sign is set to 1 in the absence of a peak, the sign is set to zero.

Spectra of elements, for example, a winding or a magnetic circuit of an MT with predetermined fixed, known defects, will constitute a library of standards. The elements of each vector λ_j are determined by the conditions $0 \le \lambda_i \le 100$. The diagnosed spectrum is also determined by a vector U with elements $0 \le u_i \le 100, i = 1, 2, ..., n$. If the elements of the vectors are given with additive errors $\Delta \lambda_i$, and Δu_i the diagnosed spectrum is determined by the sum, then the spectral components of the standards are determined as:

$$U - \Delta U = \sum_{j=1}^{m} a_j (\lambda - \Delta \lambda_j), \qquad (19)$$

where a_{j} are weight coefficients.

For generalized calculations of the indicated values corresponding to various types of PT, it is advisable to use matrix notation:

$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_m); A = (a_1, a_2, \dots, a_m).$$
$$U + \sum_{j=1}^m a_j \cdot \Delta \lambda_j - \Delta u = \lambda A.$$
(20)

For the right side of equality (20), setting the confidence interval reduced to the output permissible errors $-l_1 \le \Delta \le l_2$, we obtain a system of inequalities by which it is possible to determine the permissible values of the corresponding coefficients a_i :

$$U - l_1 \le \lambda A \le U + l_2. \tag{21}$$

Analysis of (20) and (21) shows that there is a possibility of having a set of spectral lines $\lambda A = \sum_{j=1}^{m} a_j \cdot \lambda_j$ containing *k* the component, with the probability:

$$p_k = p_0^k (1 - p_0)^{m-k} . (22)$$

The probability of allowable combinations of spectral components $B_1, B_2, ..., B_i, ..., B_n$ can be defined as follows:

$$p(B_i) = K p_0^{kB_i} (1 - p_0)^{m - B_i}, i = 1, 2, \dots, N$$
(23)

where $K = \left[1 - (1 - p_0)^m\right]^{-1}$ is a normalizing factor.

I use the Bayes formula for a posteriori, i.e., after taking into account the new information, we obtain the expression:

$$P\left(\frac{B_i}{U}\right) = \frac{P(B_i)}{\sum\limits_{i=1}^{N} P(B_i)}$$
(24)

Analysis of (23) and (24) shows that posterior probabilities differ by a normalizing factor, among which there is a certain combination of spectra B_i that has the highest probability. Finding the maximum B_{max} is a linear programming problem. The initial data of the vibration specters are displayed in space R, where the parameters of the elements of the diagnosed object are determined in the form of a convex polyhedron, the corner points of which uniquely specify the possible combination of spectral components B_i .

Let us agree to call base combinations such combinations $(C_1...C_q)$ that are positive and are included in the admissible combination of vectors λ_j . They satisfy the following double inequality:

$$\sum_{i=1}^{0} p(C_i) \le \sum_{i=1}^{N} p(B_j) \le \sum_{i=1}^{Q} \sum_{j=1}^{N} p\Big[B_j(C_i)\Big].$$
(25)

where $p(C_i)$ are the probabilities of the basic combinations of spectra, $Bj(C_i)$ is the reference combination of spectra.

Taking into account the a priori probability p_0 , inequality (25) can be transformed to the form:

$$K\sum_{i=1}^{Q} p_0^{kC_i} (1-p_0)^{m-kC_i} \le \sum_{i=1}^{N} p(B_i) \le K\sum_{i=1}^{Q} p_0^{kC_i} .$$
⁽²⁶⁾

where p_0 is the prior probability of each component.

After mathematical transformations, for an arbitrary admissible combination, B_i the following interval boundaries are valid:

$$\frac{\frac{p_{0}^{kB_{i}}(1-p_{0})^{m-kB_{i}}}{\sum\limits_{i=1}^{Q}p_{0}^{kC_{i}}} \leq p \leq \frac{\frac{p_{0}^{kB_{i}}(1-p_{0})^{m-kB_{i}}}{\sum\limits_{i=1}^{Q}p_{0}^{kC_{i}}\left(1-p_{0}^{kC_{i}}\right)}.$$
(27)

If we take into account equations (19) and (20) expressing the spectral components, then inequality (27), which determines the lower and upper bounds for a posteriori probabilities, i.e., probabilities after the experiment, will sign the following inequality:

$$\frac{Q}{\sum_{i=1}^{k} p_0^{kC_i(v_j)} \left(1 - p_0^{kC_i(v_j)}\right) \le p \le \frac{Q}{\sum_{i=1}^{k} p_0^{kC_i(v_j)}}{\frac{Q}{\sum_{i=1}^{k} p_0^{kC_i} \left(1 - p_0^{kC_i}\right)}.$$
(28)

From inequality (28) for the a posteriori probability of individual spectral components, we obtain:

$$\lim p = \frac{q_j}{r}, j = 1, 2, ..., m.$$
(29)

where j is the spectral component, m is the final value of the spectrum component, q_{j} is the

number of combinations from the group $C_1, C_2, ..., C_r$ in which the component exists v_i .

From the above, we can conclude that the use of the Bayes method makes it possible to solve the problem of identification, i.e., establishing the identity of the probability of the calculated element PT to the known standard.

4 Conclusion

A mathematical model and algorithms for processing vibrodiagnostic signals of power transformers of traction power supply have been developed, which make it possible to quickly evaluate the technical condition of their windings and magnetic circuits based on the results of diagnostic values presented in the form of spectra. The technical requirements for vibration diagnostics sensors of power transformers have been established.

To improve the reliability of assessing the state of pressing the windings and the magnetic circuit of power transformers, the Bayes method was applied, i.e., solving the problem of identifying the diagnosable spectra of the signal of the vibration diagnostics device with reference spectra uniquely corresponding to certain defects of power transformers.

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