# Exposure to acoustic waves on viscoelastic cylinder 

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#### Abstract

The problem of the impact of acoustic waves in a homogeneous viscoelastic cylinder is considered. The investigation aims to investigate the diffraction of acoustic harmonic waves in a viscoelastic cylinder. The body is assumed to be in an infinite acoustic space filled with an ideal fluid. Numerical calculations of the angular and frequency characteristics of the scattered field for viscoelastic cylinders under the action of harmonic acoustic waves are carried out. In the case of steady waves, the Helmholtz equation describes the propagation of small disturbances in an acoustic medium. And in a viscoelastic homogeneous isotropic cylinder, scalar and vector Helmholtz equations with complex coefficients, the solution of which is described by Bessel and Neumann functions with complex arguments. A technique and algorithm have been developed for solving the problem of diffraction of acoustic harmonic waves in a viscoelastic cylinder. It has been established that the stress and displacement of a point of a viscoelastic cylinder take on a maximum value in the region of long waves. It was also found that considering the material's viscous properties reduces the stress components to $10 \%$.


## 1 Introduction

Questions investigating the diffraction of monochromatic waves by cylindrical bodies filled with an ideal fluid and in an infinite space were considered in [1-4]. In [1], the problem of sound wave diffraction on an absolutely rigid cylinder with an inhomogeneous elastic coating was considered. And the problem of scattering an incident sound wave on an elastic cylinder with an inhomogeneous coating was considered in [2, 3]. In [2], an inhomogeneous coating was assumed to be radially inhomogeneous; in [3], it was assumed to be discrete-layered. The diffraction of cylindrical sound waves by a rigid cylinder with an inhomogeneous elastic coating was studied in [4]. Modeling of a continuously inhomogeneous coating of an elastic cylinder with given sound-reflecting properties was carried out in $[5,6]$. The diffraction of plane waves by elastic cylinders with an inhomogeneous coating was considered in [7].

[^0]The scattering of plane waves in homogeneous and inhomogeneous cylindrical bodies (anisotropic) was studied in [8, 9]. In this case, in [10], the incident wave was assumed to be plane, and the wave front was assumed to be parallel to the longitudinal axis [10]. From a mathematical point of view, the problem of wave diffraction on cylinders (viscoelastic) is much more complex than the problem of diffraction in elastic cylindrical bodies [11,12]. The diffraction of elastic waves in inhomogeneous cylindrical elastic bodies placed in a deformable medium is discussed in [13,14].

In $[15,16]$, the problem of elastic wave diffraction in a viscoelastic cylindrical body was considered. At the contact of the cylindrical body with the medium, the conditions of rigid contact are set. At infinity, the Somerfeld radiation conditions are set. In [17], the problem of the scattering of a plane sound wave incident arbitrarily on an elastic cylinder with a radially inhomogeneous elastic layer in the presence of an underlying plane was considered. Direct and inverse wave diffraction problems on a rigid cylinder located in a waveguide are considered in [18-20].

In contrast to the works listed above, here we study the problem of diffraction of waves emitted by a linear source on an isotropic (viscoelastic) cylinder on which a radially inhomogeneous elastic coating is deposited. Scattering of acoustic waves in an ideal infinite circular cylinder is the simplest diffraction problem with an exact solution. In addition, problems that have exact solutions are the problems of scattering of acoustic waves in a sphere, an ellipsoid, and in other bodies whose surfaces are the coordinate surfaces of the corresponding curvilinear coordinate systems. A solution technique and an algorithm have been developed. Numerical results are obtained.

## 2 Methods

### 2.1 Problem statement and solution technique

Consider an infinite isotropic viscoelastic cylinder of radius a, whose material is characterized by density $\rho$ and instantaneous elastic constants $\lambda_{0}$ and $\mu_{0}$. The liquid surrounding the cylindrical body is ideal and is characterized by density $\rho_{1}$, and the speed of sound $-c$. Let a monochromatic plane wave fall from outer space onto a cylindrical body.

Let a harmonic wave with frequency $\omega$ propagate in a homogeneous medium. We will call it an incident wave and denote its complex pressure amplitude $p$.

Let us consider the problem of acoustic wave diffraction in a viscoelastic cylindrical body in cylindrical coordinates [21].


Fig. 1. Calculation scheme of an incident plane wave on a cylinder

Suppose that a plane wave is an incident on this body. A cylindrical body is represented by a system of differential equations in partial derivatives (the Lame equations) [16]:

$$
\begin{equation*}
\tilde{\mu} \nabla^{2} \vec{u}+(\tilde{\lambda}+\tilde{\mu}) \text { grad dig } \vec{u}=\rho \frac{\partial^{2} \vec{u}}{\partial t^{2}} \tag{1}
\end{equation*}
$$

where $\rho$ is the density of the cylinder, $\overrightarrow{\boldsymbol{u}}\left(\boldsymbol{u}_{1}, \boldsymbol{u}_{2}, \boldsymbol{u}_{3}\right)$ is the displacement vector of the medium,

$$
\begin{equation*}
\tilde{\lambda} f(t)=\lambda_{0}\left[f(t)-\int_{-\infty}^{t} R_{\lambda}(t-\tau) f(\tau) d \tau\right] ; \tilde{\mu} f(t)=\mu_{0}\left[f(t)-\int_{-\infty}^{t} R_{\mu}(t-\tau) f(\tau) d \tau\right] \tag{2}
\end{equation*}
$$

$f(t)$ is an arbitrary function of time, $R_{\lambda}(t-\tau)$ and $R_{\mu}(t-\tau)$ are the relaxation kernels, and $\lambda_{0}, \mu_{0}$ are the instantaneous moduli of elasticity. The Koltunov-Rzhanitsyn kernel $R_{k}(t)=A_{k} e^{-\beta_{k} t} / t^{1-\alpha_{k}}$ is taken as the relaxation kernel [17].

The path that the plane wave front takes is determined by the scalar product of the vectors nr , where n is the unit vector $(|\mathrm{n}|=1)$ of the normal to the surface of the wave front, and $r$ is the radius vector from the origin $O$ to a point on the front surface. It is clear that nr $=\operatorname{rcos} \psi(r=|r|, \psi$ is the angle between n and r$)$.

Let an elastic cylinder of infinite length and radius $a$, placed in an unlimited medium, be affected by a wave with a front perpendicular to the cylinder axis $z$ [18]:

$$
\begin{equation*}
\varphi^{(p)}=e^{i k_{01} t} e^{-i \omega t}=e^{i k_{01} r \cos \theta} e^{-i \omega t} \tag{2}
\end{equation*}
$$

where $\omega$ is the frequency; $t$ is time.
Field speed potential

$$
\varphi_{c}^{(1)}=\Phi_{c}^{(1)} e^{-i \omega t} .
$$

The propagation of small disturbances in an acoustic (in an ideal fluid) medium (steady oscillations) is described by the Helmholtz equations [22]

$$
\Delta \Phi_{c}^{(1)}+k_{01}^{2} \Phi_{c}^{(1)}=0,
$$

where $\Phi_{c}^{(1)}=\Phi_{p}^{(1)}+\Phi_{s}^{(1)}$ is the acoustic field velocity potential, $\Phi_{s}^{(1)}$ is the scattered wave velocity potential. Then the particle velocity ${ }^{\vec{v}}$ and acoustic pressure in the liquid $p$ :

$$
\vec{v}=\operatorname{grad} \Phi_{c}^{(1)}, p=i \rho_{0} \omega \Phi_{c}^{(1)}
$$

where $\omega$ is the frequency, $\rho_{0}-$ is the density of the acoustic medium material, $\lambda_{01}$ is the elastic characteristics of the acoustic medium.

The mixing of a cylindrical body is described using the Green-Lemb expansion. Then the longitudinal and transverse displacements satisfy the following relations:

$$
\varphi^{(2)}=\Phi^{(2)} e^{-i \omega t}, \psi^{(2)}=\Psi^{(2)} e^{-i \omega t}
$$

Oscillations of a viscoelastic homogeneous isotropic cylinder, in the case of harmonic motion, are described by scalar and vector Helmholtz equations [22]:

$$
\begin{array}{r}
\frac{\partial^{2} \Phi_{0}^{(2)}}{\partial r^{2}}+\frac{1}{r} \frac{\partial \Phi_{0}^{(2)}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \Phi_{0}^{(2)}}{\partial \theta^{2}}+k_{1}^{2} \Phi=0 \\
\frac{\partial^{2} \Psi_{0}^{(2)}}{\partial r^{2}}+\frac{1}{r} \frac{\partial \Psi_{0}^{(2)}}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \Psi_{0}^{(2)}}{\partial \theta^{2}}+k_{2}^{2} \Psi_{0}^{(2)}=0 \tag{3}
\end{array}
$$

And the problem reduces to solving equations

$$
\begin{equation*}
U_{\theta}=\frac{1}{r} \frac{\partial \varphi_{0}^{(2)}}{\partial \theta}-\frac{\partial \Psi_{0}^{(2)}}{\partial r} \quad U_{r}=\frac{\partial \varphi_{0}^{(2)}}{\partial r}+\frac{1}{r} \frac{\partial \Psi_{0}^{(2)}}{\partial \theta} \tag{4}
\end{equation*}
$$

where

$$
\kappa_{1}^{2}=\frac{\omega^{2}}{c_{p 1}^{2} \Gamma_{\lambda \mu}}, \kappa_{2}^{2}=\frac{\omega^{2}}{c_{s 1}^{2} \Gamma_{\mu}}, c_{p 1}^{2}=\left(\lambda_{0}+2 \mu_{0}\right) / \rho_{1}, c_{s 1}^{2}=\mu_{0} / \rho_{1}
$$

are
velocities of longitudinal and transverse waves;

$$
\begin{aligned}
& \Gamma_{\lambda \mu}^{\bullet}=1-\Gamma_{\lambda \mu}^{C}\left(\omega_{R}\right)-i \Gamma_{\lambda \mu}^{S}\left(\omega_{R}\right), \Gamma_{\mu}^{\bullet}=1-\Gamma_{\mu}^{C}\left(\omega_{R}\right)-i \Gamma_{\mu}^{S}\left(\omega_{R}\right) \\
& \Gamma_{\lambda}^{c}\left(\omega_{R}\right)=\int_{0}^{\infty} R_{\lambda}(\tau) \cos \omega_{R} \tau d \tau, \Gamma_{\mu}^{c}\left(\omega_{R}\right)=\int_{0}^{\infty} R_{\mu}(\tau) \cos \omega_{R} \tau d \tau \\
& \Gamma_{\lambda}^{s}\left(\omega_{R}\right)=\int_{0}^{\infty} R_{\lambda}(\tau) \sin \omega_{R} \tau d \tau, \quad \Gamma_{\mu}^{s}\left(\omega_{R}\right)=\int_{0}^{\infty} R_{\mu}(\tau) \sin \omega_{R} \tau d \tau
\end{aligned}
$$

$\Gamma_{\lambda \mu}^{C}\left(\omega_{R}\right), \Gamma_{\lambda \mu}^{S}\left(\omega_{R}\right), \Gamma_{\mu}^{C}\left(\omega_{R}\right), \Gamma_{\mu}^{S}\left(\omega_{R}\right)$ are the cosine and sine Fourier images, $\varphi=\varphi_{i}+\varphi_{s}, \varphi_{i}$ is the potential of the incident waves; $\varphi_{s}$ is the potential of the reflected waves, is the potential of the longitudinal waves in the fluid, $\Psi$ and $\psi$ are the potentials of the transverse waves in the viscoelastic cylinder and the fluid.

For $\mathrm{r}=\mathrm{a}$ (on the surface of the cylinder), the following boundary conditions are satisfied [23]:

$$
\begin{align*}
& p_{r r}=-\sigma_{r r}, V_{r}=-i w U_{r}  \tag{5}\\
& p_{r \theta}=-\sigma_{r \theta}, V_{\theta}=-i w U_{\theta}
\end{align*}
$$

where $V_{r}, V_{\theta}$ are the normal and tangential velocities of fluid particles; $U_{r}, U_{\theta}$ are the normal and district mixing of the environment; $p_{r r}, p_{r \theta}$ are the normal and tangential stresses of the liquid; $\sigma_{r r}, \sigma_{r \theta}$ are the normal and tangential stresses of the cylinder.

For a rigid cylinder, the radial velocity on its surface is zero.
At infinity, the radiation conditions for the potentials of reflected waves must be satisfied [24]:

$$
\begin{equation*}
r\left(\frac{\partial \varphi_{c}}{\partial n}+i k_{1} \varphi_{c}\right)_{r \rightarrow \infty}=0\left(\frac{1}{r}\right),\left(\varphi_{c}\right)_{r \rightarrow \infty}=0\left(\frac{1}{r}\right) \tag{6}
\end{equation*}
$$

The solution to the problem is sought in series. Then we expand the function corresponding to the incident plane wave in a Fourier series:

$$
\Phi_{s}^{(1)}=e^{i k_{01} r \cos \theta}=\sum_{n=-\infty}^{\infty} J_{n}\left(k_{01} r\right) e^{i n \theta}
$$

We represent the velocity potential of the reflected wave as a superposition of cylindrical waves emanating from points on the axis of the cylinder:

$$
\Phi_{s}^{(1)}=\sum_{n=-\infty}^{\infty} A_{n} H_{n}^{1}\left(k_{1} r\right) e^{i n \theta}
$$

And the potential of the common field

$$
\begin{equation*}
\Phi_{c}^{(1)}=\Phi_{p}^{(1)}+\Phi_{s}^{(1)}=\sum_{n=-\infty}^{\infty}\left(J_{n}\left(k_{1} r\right)+A_{n} H_{n}^{1}\left(k_{1} r\right)\right) e^{i n \theta} \tag{7}
\end{equation*}
$$

Potentials of longitudinal and transverse (shear) waves in a cylinder

$$
\begin{align*}
& \Phi_{0}^{(2)}=\sum_{n=-\infty}^{\infty} C_{n} H_{n}^{(1)}\left(k_{1} r\right) e^{i n \theta}  \tag{8}\\
& \Psi_{o}^{(2)}=\sum_{n=-\infty}^{\infty} D_{n} H_{n}^{(1)}\left(k_{2} r\right) e^{i n \theta} \tag{9}
\end{align*}
$$

The displacements of a viscoelastic cylinder are related to the scalar potential and the only nonzero component of the vector potential by the relations [25]:

$$
U_{r}=\frac{\partial \psi}{\partial r}+\frac{1}{r} \frac{\partial \psi}{\partial \theta} ; \quad U_{\theta}=\frac{1}{r} \frac{\partial \psi}{\partial \theta}-\frac{\partial \psi}{\partial r}
$$

Then

$$
\begin{align*}
U_{r}= & -\left(k_{1} C_{n} H_{n}^{(1)}\left(k_{1} r\right)+\frac{i n}{r} D_{n} H_{n}^{(1)}\left(k_{2} r\right)\right) e^{i n \theta}  \tag{10}\\
& U_{\theta}=-\left(\frac{i n}{r} C_{n} H_{n}^{(1)}\left(k_{1} r\right)-k_{2} D_{n} H_{n}^{(1)}\left(k_{2} r\right)\right) e^{i n \theta}
\end{align*}
$$

Radial and tangential stresses in a fluid:

$$
p_{r r}=p_{0} \frac{\partial \varphi}{\partial t}-2 \mu_{0}\left(\frac{1}{r} \frac{\partial \varphi}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \varphi}{\partial \theta^{2}}+\frac{1}{r^{2}} \frac{\partial \phi}{\partial \theta}-\frac{1}{r} \frac{\partial^{2} \phi}{\partial r \partial \theta}\right)
$$

$$
p_{r \theta}=2 \mu_{0}\left(\frac{1}{r} \frac{\partial^{2} \varphi}{\partial r \partial \theta}-\frac{1}{r^{2}} \frac{\partial \varphi}{\partial \theta}+\frac{1}{r} \frac{\partial \phi}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \phi}{\partial \theta^{2}}-\frac{1}{2 \nu_{0}} \frac{\partial \phi}{\partial t}\right)
$$

From here

$$
\begin{align*}
& p_{r r}=\sum_{n=-\infty}^{\infty}\left(\left(i \omega \rho_{0}-\lambda_{0} k_{1}^{2}-2 \mu_{0} \frac{n^{2}}{r^{2}}\right) J_{n}\left(k_{1} r\right)-2 \mu_{0} \frac{k_{1}}{r} J_{n}\left(k_{1} r\right)\right) e^{i n \theta}+ \\
& +\sum_{n=-\infty}^{\infty}\left(\left(i \omega \rho_{0}-\lambda_{0} k_{1}^{2}-2 \mu_{0} k_{1}^{2}+2 \mu_{0} \frac{n^{2}}{r^{2}}\right) H_{n}^{(1)}\left(k_{1} r\right)-2 \mu_{0} \frac{k_{1}}{r} H_{n}^{(1)}\left(k_{1} r\right)\right) A_{n} e^{i n \theta}+ \\
& +2 \mu_{0} \sum_{n=-\infty}^{\infty}\left(\frac{i n}{r} k_{2} H_{n}^{(1)}\left(k_{2} r\right)-\frac{i n}{r^{2}} H_{n}^{(1)}\left(k_{2} r\right)\right) B_{n^{i n \theta}} ; \\
& \quad p_{r \theta}=2 \mu_{0} \sum_{n=-\infty}^{\infty}\left(\frac{n i}{r} k_{1} J_{n}\left(k_{1} r\right)-\frac{n i}{r^{2}} J_{n}\left(k_{1} r\right)\right) e^{i n \theta}+ \\
& \quad+2 \mu_{0} \sum_{n=-\infty}^{\infty}\left(\frac{n i}{r} k_{1} H_{n}^{(1)}\left(k_{1} r\right)-\frac{n i}{r^{2}} H_{n}^{(1)}\left(k_{1} r\right)\right) A_{n} e^{i n \theta}+ \\
& \quad+2 \mu_{0} \sum_{n=-\infty}^{\infty}\left(\frac{k_{2}^{2}}{2} H_{n}^{(1)}\left(k_{1} r\right)-\frac{n^{2}}{r^{2}} H_{n}^{(1)}\left(k_{2} r\right)+\frac{k_{2}}{r} H_{n}^{(1)}\left(k_{2} r\right)\right) B_{n} e^{i n \theta} . \tag{11}
\end{align*}
$$

Radial and tangential stresses in a cylinder

$$
\begin{aligned}
-\frac{\sigma_{r r}}{2 \mu_{01}} & =\frac{\bar{\lambda}+2 \bar{\mu}}{2 \mu_{01}} k_{1}^{2} \psi+\frac{1}{r} \frac{\partial \psi}{\partial \theta}+\frac{1}{r^{2}} \frac{\partial \psi}{\partial \theta^{2}}-\frac{1}{r} \frac{\partial^{2} \psi}{\partial r \partial \theta}+\frac{1}{r^{2}} \frac{\partial \psi}{\partial \theta} \\
\frac{\sigma_{\theta \theta}}{2 \mu_{01}} & =-\frac{\bar{\lambda}}{2 \mu_{01}} k_{1}^{2} \psi+\frac{1}{r^{2}} \frac{\partial \psi}{\partial \theta^{2}}+\frac{1}{r} \frac{\partial \psi}{\partial r}+\frac{1}{r^{2}} \frac{\partial \psi}{\partial \theta}-\frac{1}{r} \frac{\partial^{2} \psi}{\partial r \partial \theta} \\
\frac{\sigma_{r \theta}}{2 \mu_{01}} & =\frac{1}{2} k_{2}^{2} \Gamma_{\mu}^{\square} \psi+\frac{1}{r} \frac{\partial \psi}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \psi}{\partial \theta^{2}}+\frac{1}{r^{2}} \frac{\partial \psi}{\partial \theta}-\frac{1}{r} \frac{\partial^{2} \psi}{\partial r \partial \theta}
\end{aligned}
$$

From here

$$
\begin{align*}
& \sigma_{r r}=-2 \bar{\mu} \sum_{-\infty}^{+\infty}\left(\frac{\lambda+2 \mu}{2 \mu} k_{1}^{2} H_{n}^{(1)}\left(k_{1} r\right)+\frac{k_{1}}{r} H_{n}^{(1)}\left(k_{1} r\right)-\frac{n^{2}}{r^{2}} H_{n}^{(1)}\left(k_{1} r\right)\right) C_{n} e^{i n \theta}- \\
& -2 \bar{\mu} \sum_{-\infty}^{+\infty}\left(\frac{i n}{r} k_{2} H_{n}^{(1)}\left(k_{2} r\right)-\frac{i n}{r^{2}} H_{n}^{(1)}\left(k_{2} r\right)\right) D_{n} e^{i n \theta}, \\
& \quad \sigma_{r \theta}=2 \mu \sum_{-\infty}^{+\infty}\left(\frac{k_{4}^{2}+2 \mu}{2} H_{n}^{(1)}\left(k_{4} r\right)+\frac{k_{4}}{r} H_{n}^{(1)}\left(k_{4} r\right)-\frac{n^{2}}{r^{2}} H_{n}^{(1)}\left(k_{4} r\right)\right) D_{n} e^{i n \theta}-  \tag{12}\\
& \quad-2 \mu \sum_{-\infty}^{+\infty}\left(\frac{i n}{r^{2}} H_{n}^{(1)}\left(k_{3} r\right)-\frac{i n}{r} k_{3} H_{n}^{(1)}\left(k_{3} r\right)\right) C_{n} e^{i n \theta} .
\end{align*}
$$

Arbitrary constants $A_{n}, B_{n}, C_{n}, D_{n}$ are determined from the boundary conditions (2). Then we obtain a system of linear algebraic equations

$$
\begin{equation*}
a_{k 1} A_{n}+a_{k 2} B_{n}+a_{k 3} C_{n}+a_{k 4} D_{n}=b_{k} \tag{13}
\end{equation*}
$$

here

$$
\begin{gathered}
a_{11}=k_{01} a H_{n}^{(1)}\left(k_{01} a\right), a_{12}=\operatorname{inH}_{n}^{(1)}\left(k_{02} a\right), a_{13}=i w a k_{1} H_{n}^{(1)}\left(k_{1} a\right) \\
a_{14}=-\omega n H_{n}^{(1)}\left(k_{2} a\right), a_{21}=\operatorname{inH} H_{n}^{(1)}\left(k_{01} a\right), a_{22}=-k_{02} a H_{n}^{(1)}\left(k_{02} a\right), \\
a_{23}=-\omega n H_{n}^{(1)}\left(k_{1} a\right), a_{24}=-i \omega a k_{2} H_{n}^{(1)}\left(k_{2} a\right), \\
a_{31}=\left(i \omega \rho_{0} a^{2}+2 \mu_{0} n^{2}\right) H_{n}^{(1)}\left(k_{01} a\right)-2 \mu_{0} k_{01} a H_{n}^{(1)}\left(k_{01} a\right), \\
a_{32}=2 \mu_{0}\left(i n k_{02} a H_{n}^{(1)}\left(k_{02} a\right)-i n H_{n}^{(1)}\left(k_{02} a\right)=2 \mu_{0} i n\left(k_{02} a H_{n}^{(1)}\left(k_{02} a\right)-H_{n}^{(1)}\left(k_{02} a\right),\right.\right. \\
a_{33}=2 \bar{\mu}\left(\frac{\bar{\lambda}+2 \bar{\mu}}{2 \bar{\mu}} k_{1}^{2} a^{2} H_{n}^{(1)}\left(k_{1} a\right)+k_{1} a H_{n}^{(1)}\left(k_{1} a\right)-n^{2} H_{n}^{(1)}\left(k_{1} a\right)\right) \\
a_{34}=-2 \bar{\mu} i n\left(k_{2} a H_{n}^{(1)}\left(k_{2} a\right)-H_{n}^{(1)}\left(k_{2} a\right)\right) \\
a_{41}=2 \mu_{0}\left(i n k_{01} a H_{n}^{(1)}\left(k_{01} a\right)-i n H_{n}^{(1)}\left(k_{01} a\right)\right)=2 \mu_{0} n i\left(k_{01} a H_{n}^{(1)}\left(k_{01} a\right)-H_{n}^{(1)}\left(k_{01} a\right)\right), \\
a_{42}=2 \mu \mu_{0}\left(\frac{k_{02}^{2} a^{2}}{2} H_{n}^{(1)}\left(k_{02} a\right)+k_{02} a H_{n}^{(1)}\left(k_{02} a\right)-n^{2} H_{n}^{(1)}\left(k_{02} a\right)\right) \\
a_{43}=2 \mu i n\left(H_{n}^{(1)}\left(k_{1} a\right)-k_{1} a H_{n}^{(1)}\left(k_{1} a\right)\right), \\
a_{44}=-\mu_{0}\left(\frac{k_{2}^{2} a^{2}}{2} H_{n}^{(1)}\left(k_{2} a\right)+k_{2} a H_{n}^{(1)}\left(k_{2} a\right)-n^{2} H_{n}^{(1)}\left(k_{2} a\right)\right) \\
b_{1}=-k_{01} a J_{n}\left(k_{01} a\right), b_{2}=-i n J_{n}\left(k_{01} a\right), \\
b_{3}=\left(i \omega \rho_{0} a^{2}-2 \mu_{0} n^{2}\right) J_{n}\left(k_{01} a\right)+2 \mu_{0} k_{01} a J_{n}\left(k_{01} a\right), \\
b_{4}=2 \mu \mu_{0}\left(J_{n}\left(k_{01} a\right)-k_{01} a J_{n}\left(k_{01} a\right)\right) .
\end{gathered}
$$

From the solution of system (13), the pressure in the medium is determined [23]:

$$
\begin{equation*}
p=p_{i}+p_{s} \tag{15}
\end{equation*}
$$

where $p_{i}$ is the pressure of the incident wave and $p_{s}$ is the pressure of the reflected wave. Then the total pressure

$$
p=\left(i \omega \rho_{0}-\left(\lambda_{0}+2 \mu_{0}\right) k_{01}^{2}\right) \varphi
$$

It is determined from the solutions of equations (4) and (5):

$$
\begin{gather*}
p_{i}=\left(i w \rho_{0}-\left(\lambda_{0}+2 \mu_{0}\right) k_{01}^{2}\right) \sum_{n=-\infty}^{\infty} J_{n}\left(k_{01} r\right) e^{i n \theta}  \tag{16}\\
p_{s}=\left(i w \rho_{0}-\left(\lambda_{0}+2 \mu_{0}\right) k_{01}^{2}\right) \sum_{n=-\infty}^{\infty} A_{n} H_{n}\left(k_{01} r\right) e^{i n \theta} \tag{17}
\end{gather*}
$$

## 3 Results and their discussion

Figure 2 shows the amplitude-frequency characteristics of radial displacements at various values of the amplitude of the relaxation core ( $1 . \mathrm{A}=0.0048 ; 2 . \mathrm{A}=0.0070 ; 1 . \mathrm{A}=0.048$ ). Attention should be paid, on the one hand, to the character of the curve and, on the other hand, to the magnitude of the positional scattering cross-section. As you can see, in the case of a soft cylinder (Figure 3, $k a \leq 0.3$ ), the scattering diagram is similar to a circular diagram, which is typical for monopole scattering, and for a rigid cylinder (Figure 2, a, $k a \leq$ 0.3 ) it is similar to a cardioid diagram, which is formed as a superposition of monopole and dipole scattering. Monopole scattering by a small obstacle, as already noted, is due only to the difference in the compressibility of the obstacle and the medium.


Fig. 2. Amplitude-frequency characteristics of radial displacements at different values of the amplitude of the relaxation core ( $1 . \mathrm{A}=0.0048 ; 2 . \mathrm{A}=0.0070 ; 3 . \mathrm{A}=0.048$ ).


Fig. 3. Dependences of $\sigma(\psi) / \pi a$ on $\psi$ for different values of the wave radius of the soft cylinder:

$$
1 . k a=2.9 ; 2 . k a=3.0 ; 3 . k a=1.0 ; 4 . k a=0.3 ; 5 . k a=0.1 .
$$

For a rigid cylinder, the compressibility is zero. Dipole scattering is associated with such an obstacle, which differs from the medium only in density. This result can be commented on by the following physical considerations. In a sound wave, the processes of compression and displacement of particles of the medium occur. A small obstacle in the sound field, a small particle in the form of a rigid cylinder, changes the nature of the compression and displacement of the particles of the medium near the cylinder.

Thus, a change like compression leads to monopole scattering, and a change like the motion of the medium particles leads to dipole scattering. The contribution of both types of scattering turns out to be approximately the same, which is easy to verify.

## 4 Conclusion

1. A technique and algorithm have been developed for solving the problem of diffraction of acoustic harmonic waves in a viscoelastic cylinder.
2. It is established that if $k a \gg 1$, then for both cylinders, the value $\sigma_{s} / 2 a \rightarrow 2$; if $k a \ll 1$, then the quantity $\sigma_{s} / 2 a$ tends to zero for a rigid cylinder and increases for a soft cylinder.

Also, note that an unlimited increase in the value of $\sigma_{\mathrm{s}}$ at $k a \rightarrow 0$ is related to the infinite length of the cylinder. There is no such result for a cylinder of finite length.
3. Numerical calculations were carried out, which revealed the possibility of changing the sound-reflecting properties of viscoelastic cylindrical bodies.

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