Modeling of dissipation of vibration energy in problems of structural dynamics

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Abstract. This paper has proposed a model that reflects a process of energy dissipation in dynamic oscillatory systems. According to the hereditary theory of viscoelasticity, a relationship between stress and strain is realized in an integral form. At the same time, the weakly singular kernel of heredity simultaneously describes both internal friction and deformation of aftereffects (creep and relaxation). Application of the developed model is shown through the description of free and forced vibrations of dissipative mechanical systems. A new opportunity can optimize the damping properties of materials of vibrating structures, which is important in solving the problem of introducing new materials. In mechanics, a new approach was proposed, which consists of the fact that by knowing the rheological parameters of material, one can find the damping coefficients directly without resorting to experiment.

1 Introduction

In many areas of modern construction and engineering, one often has to deal with the vibrational motions of various mechanical systems. In this case, resulting oscillations (vibrations) of engineering structures and their elements which they consist of, can cause considerable deformations and stresses under adverse circumstances that lead to rapid deterioration of the structure and even to its destruction. The system oscillations can occur both in the equilibrium position and concerning some definite motion, particularly the stationary motion. They can be undamped or damped if the internal friction of the structure's material is considered. As is well known, in dynamic calculations, one of the most important factors that must be considered is the energy dissipation within the vibrational system itself, the so-called internal friction. Often when considering elastic systems, the internal friction of the material is taken into account with the help of the Voigt model. However, it is known that in systems with a finite number of degrees of freedom, more than one, it leads to incorrect results. Hence, as an internal friction of most materials is virtually independent or at least weakly depends on the speed of vibrations in a sufficiently wide frequency range [1, 2]. For an arbitrary matrix of damping of a multimass system with a finite number of degrees of freedom, the damping forces of one vibrations form perform work not equal to zero with other harmonics [2].The application of frequency-independent internal friction model of the standard linear viscoelastic body with the account of the time derivative of stress somewhat softens but does not eliminate

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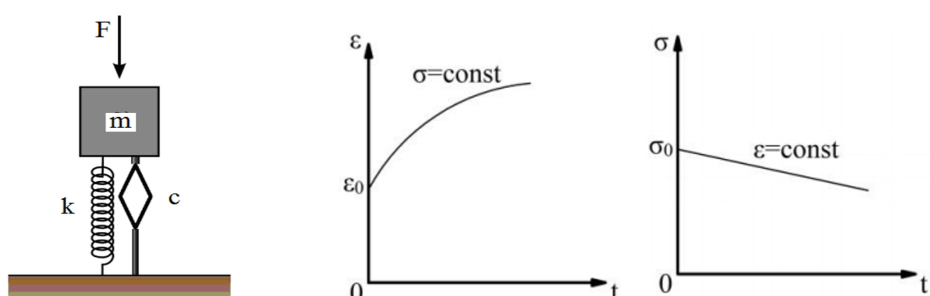
significant contradictions with experiment [3, 4]. The indicated dependences are based on one or another representation of the shape of the hysteresis curve, which gives a relationship between stress and strain in the process of harmonic oscillations. The most successful of the elementary models of frequency-independent internal friction, which has given wide application due to the simplicity of traditional solutions to problems in the theory of vibrations, is the concept of complex internal friction. However, the general shortcoming of all elementary models designed to account for frequency-independent internal friction is their inability to describe another deviation from the properties of ideal elasticity of aftereffect and relaxation [4]. In this sense, a model that reflects hereditary properties is preferable [2, 3, 5]. Properties of creep and relaxation associated with the time factor are largely possessed by all materials of engineering structures at any temperature. Recently, for describing the stress-strain state of engineering systems, much attention has been paid to the development of mathematical models where are accounted the rheological and hereditary-deformable properties of materials. This is because in the deformation process of real materials with pronounced viscoelastic properties time factor plays a significant role. On the other hand, when using hereditary models [2, 3] to describe internal damping in materials, the equations of oscillations of elastic systems are written in the same form as for viscoelastic systems. According to Volterra's principle construction of mathematical models or solutions of problems taking into account the hereditarily deformable properties of the structure's material is carried out by replacing an elastic constant in a known equilibrium equation or the solution related to an ideal elastic case, corresponding to integral operators [3, 4, 5]. The difficulty lies in decoding the algebraic and transcendental functions of integral operators. The algebra of integral operators for fractional exponential kernels was first developed by Academician Yu. N. Rabotnov. This made it possible to determine the first method for constructing exact solutions of integral and integral-differential equations (IDE) using Volterra's principle as an integral-operator method. According to hereditary Boltzmann-Volterra theory, the relationship between stress and strain is carried out in an integral form, and the weakly singular kernel of heredity can simultaneously describe both internal friction and deformations of aftereffects (creep) and relaxation. New algorithms of numerical method for solving IDE of dynamic problems has been developed together with prof. F. B Badalov. The algorithms allow studying natural and forced vibrations of structures made from dissipatively inhomogeneous materials [6, 7].

2 Methods

Let's consider a standard structural model of the Kelvin-Voigt medium, which is a parallel connection of a viscous and an elastic element (Fig. 1).



a) *b)*

Fig. 1. Kelvin-Voigt medium model

Applying load in such an environment does not cause instantaneous elastic deformations. Deformations grow from zero at t = 0 to some value determined by the force F and spring elasticity. At unloading, the deformation does not drop to zero. No stress relaxation is observed in such medium. The equation of state can be obtained by summing elastic (proportional to deformation) and viscous (proportional to deformation rate) parts of stresses [8]:

$$\sigma = E\varepsilon + \eta\frac{d\varepsilon}{dt} = \sigma_S + \sigma_F \qquad (1)$$

The equation of motion for this system shown in Figure 3 is written in the following form:

$$mZ(t) + cZ(t) + kZ(t) = F(t) \qquad (2)$$

Or with the account of coefficient of inelastic resistance (dissipation coefficient) γ:

$$mZ(t) + 2\gamma\omega Z(t) + \omega^2 Z(t) = F(t)/m \qquad (3)$$

where $cZ(t)$ is dissipation force, $kZ(t)$ is restoring elastic force.

The differential equation (9) with nonzero initial conditions has a known solution:

$$Z(t) = Ae^{-\gamma\omega t}\sin(\omega t + \varphi_0) + \frac{1}{m\omega}\int_o^t F(\tau)e^{-\gamma\omega(t-\tau)}\sin\omega(t-\tau)d\tau \qquad (4)$$

$$A = \sqrt{Z_0^2 + (Z_0 + \gamma\omega Z_0)^2/\omega}\ , \qquad tg\varphi_0 = \omega Z_0/(Z_0 + \gamma\omega Z_0) \qquad (5)$$

Here, ω is frequency of damped free vibrations. The model corresponding to equations (1) and (3) is very common in rheology. However, its essential disadvantage is the absence of a singularity at a time which is strictly unacceptable for the correct description of deformation from time to time. In studying dynamic problems, choosing a relaxation kernel that sufficiently well perceives the properties of real materials is essential. As mentioned above, the basic physical equations relating to stresses and deformations of viscoelastic bodies contain a time factor. Experience shows a significant effect of loading rates: time factor on σ ~ ε, creep, and relaxation diagrams. To describe the deformation processes of viscoelastic materials, we use Boltzmann's theory of hereditary viscoelasticity [6, 9, 10]. If at any moment of time τ, the body has received deformation ε(τ) during Δτ, then the change in the force that produces this extension is proportional to ε(τ)dτ and the function of time t – τ. The effects of deformations obtained at different times are added; that is, they are combined by direct addition. The mathematical relation of dependences of stresses on deformations based on Boltzmann's hypotheses is expressed in the form of integral equations of Volterra of the second kind:

$$\sigma(t) = E(\varepsilon(t)) - E\int_0^t R(t-\tau)\varepsilon(\tau)d\tau$$
$$\varepsilon(t) = \frac{\sigma(t)}{E} + \frac{1}{E}\int_0^t K(t-\tau)\sigma(\tau)d\tau \qquad (6)$$

The kernel of heredity, as follows from physical considerations, must be a positive, monotonic, and integrable function in the interval (0, ∞). Since in the initial relatively short time interval, the creep and relaxation processes proceed very intensively so that the initial rates of the processes can be considered infinite, i.e.:

$$\left.\frac{d\varepsilon(t)}{dt}\right|_{t\to 0} \to \infty; \qquad \left.\frac{d\sigma(t)}{dt}\right|_{t\to 0} \to -\infty \tag{7}$$

Then the heredity kernels must have an integrable (weak) singularity at zero when the function goes to infinity and the integral of it is finite:

$$K(t) = \frac{E}{\lambda\sigma_0}\frac{d\varepsilon(t)}{dt}; \qquad R(t) = \frac{1}{\lambda E\varepsilon_0}\frac{d\sigma(t)}{dt} \tag{8}$$

To date, many functions have been analyzed that can be used as heredity kernels. As such functions, power and exponential functions are most often considered, as well as various combinations of these functions [6, 9]. As weakly singular kernels of heredity that satisfy the above conditions, a power-law Abel-Rzhanitsyn-Koltunov kernel can be chosen in the form:

$$R(t) = \bar{\varepsilon}t^{\alpha-1}, \qquad \bar{\varepsilon} > 0, \qquad (0 < \alpha < 1) \tag{9}$$

and the kernel in the combination of power and exponential functions of Abel type is the Rzhanitsyn-Koltunov kernel [6, 9]:

$$R(t) = \bar{\varepsilon}e^{-\beta t}t^{\alpha-1}, \qquad \bar{\varepsilon} > 0, \qquad \beta > 0, \qquad (0 < \alpha < 1) \tag{10}$$

Here $\bar{\varepsilon},\alpha,\beta$ are parameters of kernel where to be determined from experiments. The theory of hereditary changes in internal factors taking into account the "memory" of material about all temporary structural changes that occur during loading and assuming a linear relationship between stresses and deformations at any time, is called the theory of linear hereditary creep. The law of deformation in a stressed condition was obtained from this theory by generalizing equation (1) to the model with an infinite number of elastic and viscous elements. Then the equation of motion for the system containing a hereditary character (integral relationship between stresses and strains in the form $\sigma = E(1-R^*)\varepsilon$) will be written by the following formula:

$$Z(t) + \omega^2(1-R^*)Z(t) = F(t)/m \tag{11}$$

Based on the obtained equations given in [6] for the integro-differential equation with nonzero initial conditions, we can write the following solution:

$$Z(t) = a_0Y_1(t) + \frac{a_0Y_2(t)}{\omega} + \frac{1}{\omega}\int_o^t F(\tau)Y_2(t-\tau)d\tau \tag{12}$$

where $Y_1(t) = c\phi(\omega t)$, $Y_2(t) = s\phi(\omega t)$ are functions describing damping oscillatory processes at the set creep of a material [12-14].

The obtaining of these functions is resulted in work [6, 7, 8]. The final solution of IDE will be given to the system with one degree of freedom. It is easy to see that the general solution of (11) is the sum of the general solution of the homogeneous equation of the following IDE:

$$Z(t) + \omega^2(1 - R)Z(t) = 0 \quad (13)$$

and a partial solution. To solve it, we used sine and cosine functions of fractional order and the integral-operator method [9]. These functions describe the mechanism of internal friction of the material for many existing weakly singular heredity kernel of Abel type:

$$\begin{aligned} Y_1(t) &= c\phi(\omega t) = \cos\omega t + c\overline{\phi}(\omega, t) \\ Y_2(t) &= s\phi(\omega t) = \sin\omega t + s\overline{\phi}(\omega, t) \end{aligned} \quad (14)$$

Using transformations eliminating weakly singular features of the integral in equation (13) with account for the kernel of heredity according to formula (10) and with subsequent use of quadrature formulas [9], we obtain the solution in the following form:

$$\begin{gathered} Z_n = a_0 + a_1 t_n - \omega^2 \sum_{j=0}^{n-1} S_j(t_n - t_j)\left[Z_j - \frac{\varepsilon}{\alpha}\sum_{k=0}^{j} B_k e^{\beta t_k} Z_{j-k}\right], \quad n = 1,2....... \\ S_0 = \frac{\Delta t}{2}, \ \ S_j = \Delta t, \ \ j = 1, n-1, \ \ t_n = n\Delta t \ \ Z(t_n) = Z_n \\ B_k = \frac{(\Delta t)^\alpha}{2}, \ \ B_k = \frac{(\Delta t)^\alpha}{2}\left[(k+1)^\alpha - (k-1)^\alpha\right], \ \ k = 1, j-1 \end{gathered} \quad (15)$$

where a_0 and a_1 are initial conditions.

The calculation can be made for ideal elastic ($\varepsilon = 0$) systems. In the obtained solution, the damping rate depends on rheological parameters ε, α, and β of the relaxation kernel. Therefore, to study a dependence of dissipative characteristics $\psi = 2\pi\gamma$ and $\delta = 2\pi\gamma$ on ε, α, β is very relevant. The logarithmic damping decrement δ of free vibrations is calculated by the formula:

$$\Delta = ln\frac{A_{i+1}}{A_i}; \qquad \gamma = \frac{1}{2\pi}\, ln\frac{A_{ii+1}}{A_{ii}}$$

Table 1 shows the dependence of vibration decrement accordingly to the vibration damping coefficient on rheological parameters. It can be seen from this table that a decrease in the singularity parameter leads to an increase in the energy absorption coefficient of the system, and thus, free oscillations in practice will disappear after a certain period of time [17, 18].

Table 1. Inelastic resistance coefficients from rheological parameters

ε	α	β	δ	γ
0.1	0.5	0.05	0.0128	0.002038
0.1	0.4	0.05	0.0892	0.014204
0.1	0.3	0.05	0.0932	0.014841
0.1	0.2	0.05	0.1380	0.021975

So a new opportunity originates from optimizing the damping properties of materials of vibrating structures, which are very important in solving the problem of introducing new materials. A new approach is proposed in mechanics, that knowing the rheological parameters of material, one can find directly without resorting to experiment. It is clear from the above studies that the solution of IDE in the presence of internal friction is rapidly damping. Therefore, a partial solution to the equation of forced vibrations according to equation (11) is of practical interest [15, 16]. After some transformations given in [6], the second part of the formula (12) in the integral can be written in the following form:

$$Z_n = \sum_{j=0}^{n-1} S_j(t_n - t_j)\left[F_j - \omega^2\left(Z_j - \frac{\varepsilon}{\alpha}\sum_{k=0}^{j} B_k e^{\beta t_k} Z_{j-k}\right)\right], \quad n = 1,2.......$$

$$S_j = \Delta t, \quad j = 1, n-1, \quad t_n = n\Delta t \quad Z(t_n) = Z_n$$

$$B_k = \frac{(\Delta t)^\alpha}{2}\left[(k+1)^\alpha - (k-1)^\alpha\right] \tag{16}$$

$$k = 1, j-1, \quad B_j = \frac{(\Delta t)^\alpha}{2}\left[j^\alpha - (j-1)^\alpha\right], \quad k = j$$

3 Numerical results and analysis

For calculation has been chosen the dynamic design scheme of longitudinal oscillations of the bridge support as a system with one dynamic degree of freedom (Fig. 2). For the beginning, there is no external load $\ddot{\Delta}(t) = 0$ at the moment of free vibration. Since undamped and free vibrations are considered, the amplitude values of the displacements do not change over time. The diagram is constructed by solving the proposed equation for hereditarily deformable systems at $R^* = 0$ (Fig. 3).

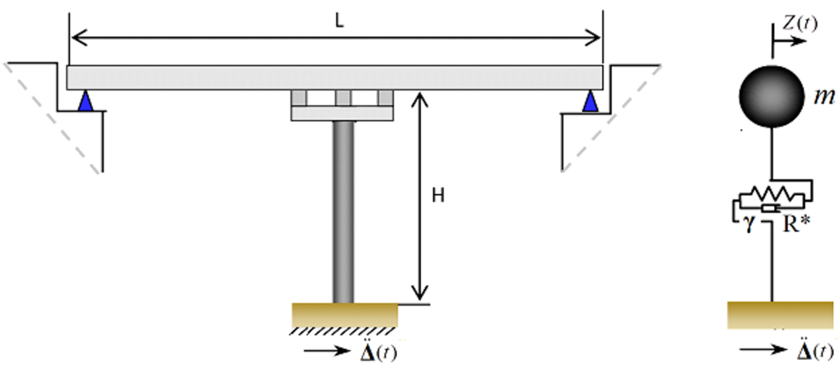


Fig. 2. Dynamic design scheme of longitudinal oscillations of the bridge support, $L = 24\ m$, $H = 12\ m$.

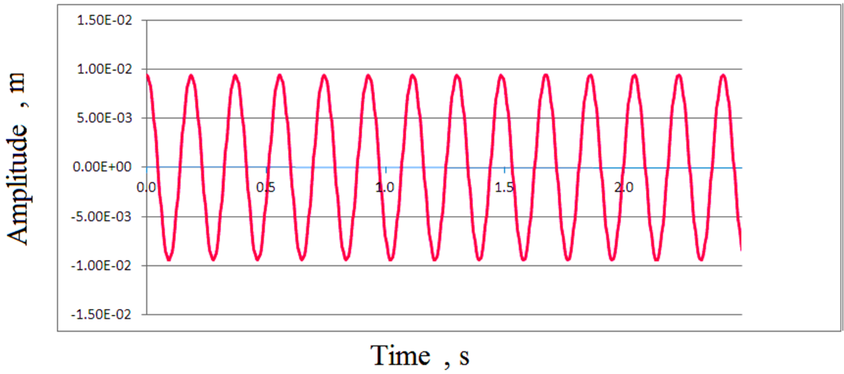


Fig. 3. Displacement-time diagram of concentrated mass, $R^* = 0$.

Let's compare the obtained solution with the analytical solution of the equation of motion of a concentrated mass with free undamped vibration (Fig. 4). It is known that the analytical solution of equations (3) and (4) with free vibrations has the following form:

$$Z(t) = Z_0 \cos(\omega t) + \frac{Z}{\omega}\sin(\omega t) \quad (17)$$

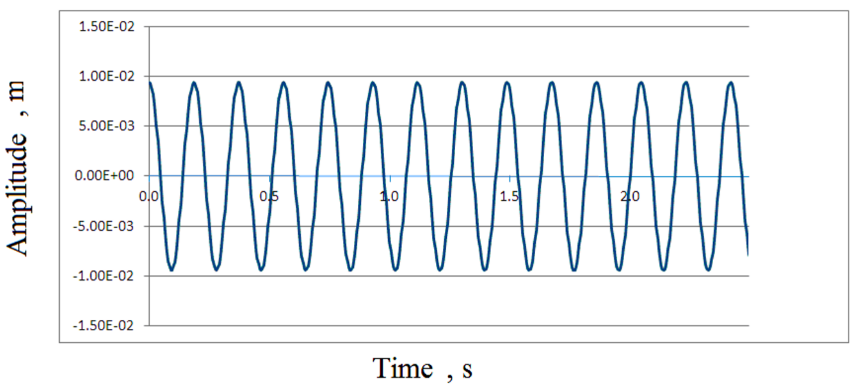


Fig.4. Displacement-time diagram of concentrated mass obtained by standard formula, γ=0

As a result of solving this problem, we can say that it coincides with the analytical solution. To study free damped vibrations, Table 2 is given, with the values of the coefficients of inelastic resistance and rheological parameters for various materials: steel, reinforced concrete, and wood. Figures 5, 6, and Figure 7 are presented diagrams of the displacement of concentrated mass in time, respectively, with the corresponding rheological characteristics. Figure 8 shows the solution for the standard equation of motion with a damping coefficient [13, 14].

Table 2. Rheological parameters and inelastic resistance coefficients

ε	α	β	γ
0.14	0.23	0.05	0.022
0.11	0.5	0.05	0.013
0.1	0.3	0.05	0.0422

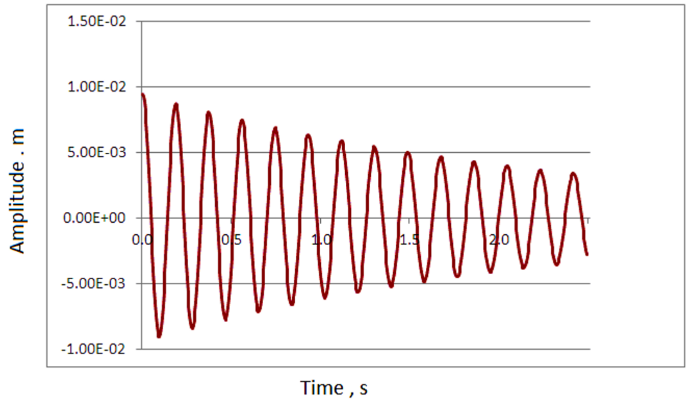


Fig.5. Displacement of concentrated mass on the steel support

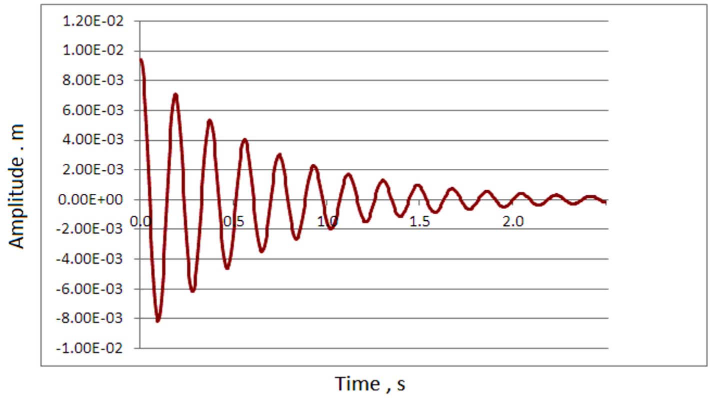


Fig. 6. Displacement of concentrated mass on the reinforced concrete support

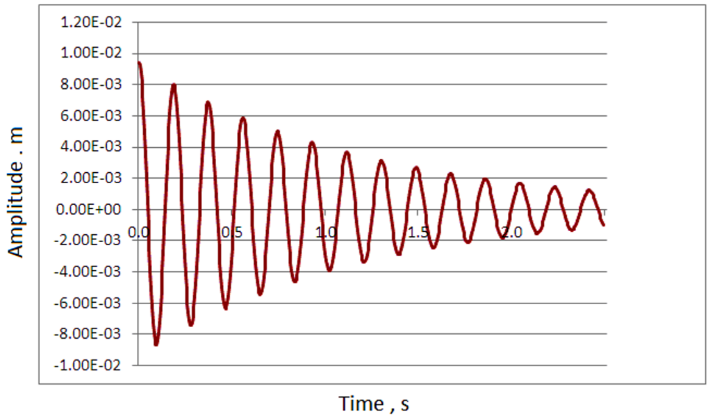


Fig. 7. Displacement of concentrated mass on the wood support

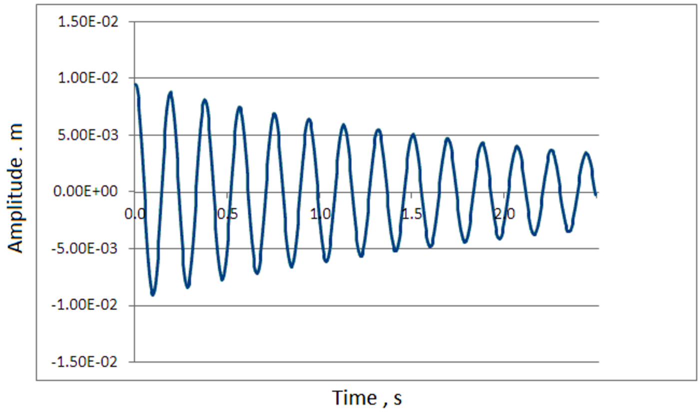


Fig.8. Displacement of concentrated mass on the steel support, standard solution

The vibration is damped faster at a higher damping coefficient. For example, in a reinforced concrete support, complete damping of vibration occurs already by 4-5 period natural (self) vibration, while in a steel one with the same dynamic characteristic complete damping of oscillation will occur by 11-12 period. Fig. 9 is shown the diagram of displacement at forced F(t)=m$\ddot{\Delta}(t)\sin\theta t$ damped vibration on the harmonic load at a nonresonant situation in rheological parameters: β = 0.05, α = 0.23, ε = 0.11, θ = 42 rad/s.

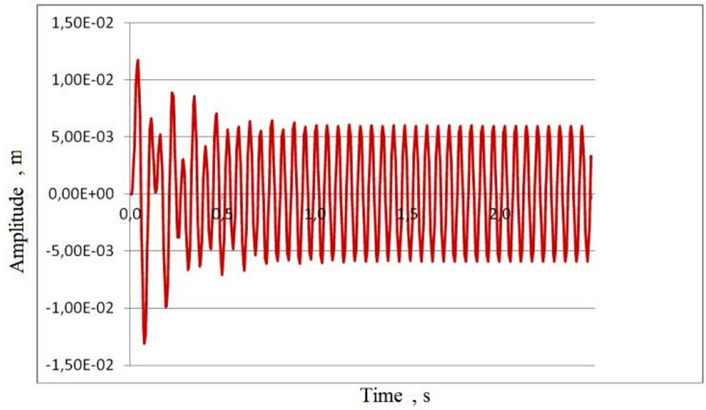


Fig. 9. Forced vibration with account of damping in the steel support(nonresonance,Θ> ω)for R*≠0

It can be seen that over time natural free vibration is damped out, and the vibration is restored with a frequency equal to the forced frequency θ. At resonance (Fig. 10), that is, with an increase in the total amplitude, the same thing happens; forced vibration with a frequency θ is established [11, 12].

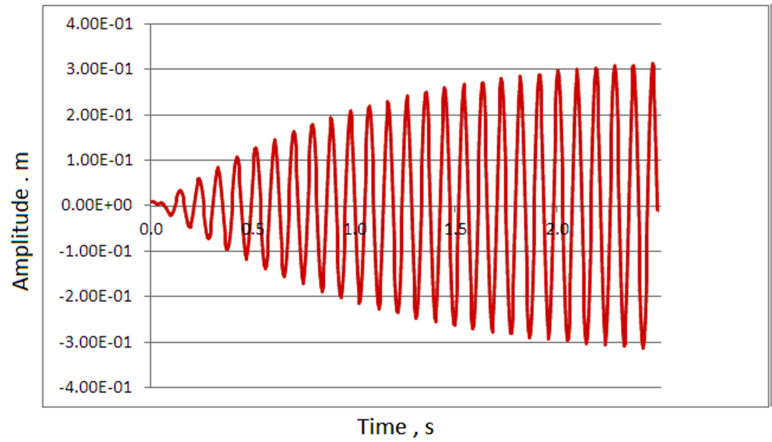


Fig. 10. Forced vibration with account of damping in the steel support(resonance,Θ=ω)for R* ≠ 0

When comparing Figures 9 and 10, it can be seen that at resonance. During the time from the origin (t=0) to the moment of stationary vibration (t=2s), the nonstationary vibration occurs, the so-called transient process. Let's consider damped vibrations from a nonharmonic dynamic suddenly acting load F(t)=const with rheological parameters: R*≠0, β=0.05, α=0.5, ε=0.11. This result shows that under the action of a constant external load, the vibration of the hereditarily deformable element occurs around the creep function curve and damps out over time along this curve (Fig. 11).

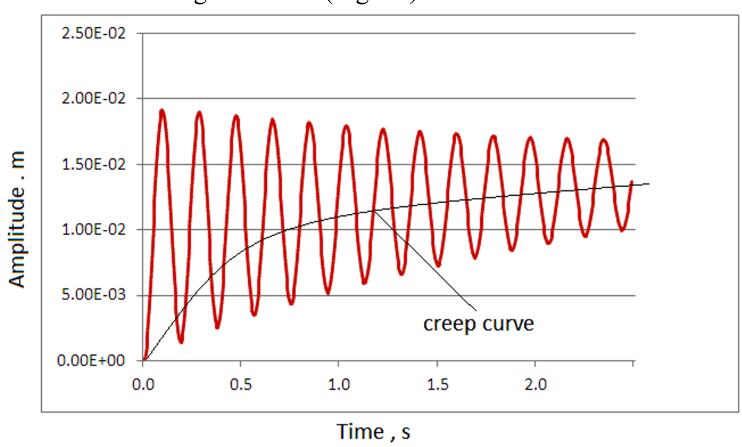


Fig. 11. Forced vibration with account of damping in the steel support (F(t)=const) for R * ≠ 0

4 Conclusion

The proposed method for implementing a numerical method for solving integro-differential equations of motion of concentrated masses with the account of the viscoelasticity of deformation of materials according to the hereditary theory is satisfactory. It can be applied in practice to solve engineering problems. A new opportunity can optimize the damping properties of materials of vibrating structures, which is important in solving the problem of introducing new materials. In mechanics, a new approach was proposed that knowing the rheological parameters of material can directly find the damping coefficients without resorting to experiment.

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