Vibration simulation of the spring element of the locomotive swing link

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Abstract. The article presents the simulation of vibrations of a cylindrical elastic rod bent along a helix for the swing link of a locomotive. As a result, methods for calculating the dynamic strength of helical springs for locomotives were developed.

1 Introduction

The development of the modern theory of oscillations of the bearing frame of locomotives and the system of their swing links is characterized by the wide use of theoretical research methods and numerical computer processing of the results and experimental data on the stress-strain state of structures and dynamic strength calculations.

The foundations of the theory and practical methods for studying the dynamics of vehicles were developed by N.P. Petrov, I.E. Zhukovsky, S.P. Timoshenko [1], and further by A.M. Goditsky - Tsvirko, M.V. Vinokurov. M.F. Verigo, S.V. Vershinsky, S.M. Kutsenko, V.A. Lazaryan, V.B. Medel, I.I. Chelnokov in their studies fully developed modern methods for the research of natural and forced oscillations of rail vehicles, their interaction with the superstructure of the track. A significant contribution to the solution of these problems was made by E.P. Blokhin, L.O. Grachev, V.P. Ivanov, I.P. Isaev, A.A. Kamaev, L.N. Nikolsky, E.N. Nikolsky, A.N. Savoskin, V.P. Koturanov, M.M. Sokolov, L.A. Shadur and others.

Research has been conducted and is being conducted on this topic by leading scientists worldwide such as S.A. Brebbia (Wessex Institute of Technology, UK), G.M. Carlomagno (University of Naples di Napoli, Italy), A. Varvani-Farahani (Ryerson University, Canada), S.K. Chakrabarti (USA), S. Hernandez (University of La Coruna, Spain), S.-H. Nishida (Saga University, Japan) [2-9]. Authoritative scientific schools and prominent scientists in the CIS countries from MIIT, PGUPS, MAI, VNIIZhT, JSC VNIKTI, JSC Russian Railways, etc. worked on these issues. A significant contribution to solving many complex problems and checking theoretical conclusions related to the study of the processes of oscillations of the spring suspension of the rolling stock was made by the Russian Research Institute of Railway Transport (CNII MPS) and the Russian Research Institute of Railcar Building (NIIV), where along with theoretical studies, a large number of experimental studies (bench and full-scale ones) were conducted [10-13]. In Uzbekistan, the academician of the Academy of Sciences of the Republic of Uzbekistan, Professor, Doctor of Technical Glushchenko A.D., Professors Fayzibaev Sciences Sh.S., Khromova G.A.,

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Shermukhamedov A.A., Mukhamedova Z.G. and their students studied the problems of optimizing the systems of spring suspension of rolling stock [14-21].

However, in the existing calculation methods, the curvilinearity of surfaces, impulse contact processes that occur during the operation of the spring suspension of ground vehicles, the complexity of the dynamic loading pattern, and the volumetric configuration of systems have not been taken into account so far.

2 Objects and methods of research

The objects of study are elastic curvilinear systems of a complex profile and various types of shock absorbers for locomotives, for example, helical springs for swing links of a rail vehicle.

The research is based on the use of standard methods of the strength of materials, the theory of vibrations and the theory of dynamic strength, operational calculus and model experimental studies. Numerical studies are based on the Boundary Element Technology. Scientific results obtained by the authors of the article were reported in 2004-2022, at 15 International Conferences (in the USA, Russia, Kazakhstan, Lithuania, Uzbekistan) and were published all over the world [14-21].

To derive the equations of spatial oscillations of a cylindrical elastic rod bent along a helix with a variable radius of curvature of the coils, we used the results obtained in [19-21] and the following assumptions.

1. The boundary element is taken as a single coil of a cylindrical elastic rod bent along a helix with a fixed radius of curvature (Fig. 1). N is the number of boundary elements (depending on the spatial configuration of the spring element); it is connected into a single dynamic system using boundary conditions. We used the calculation scheme shown in Fig.2 (for the axle box of the spring suspension for a rail vehicle, for example, for locomotives) [10].



Fig. 1. Boundary element in the form of a single coil of a cylindrical elastic rod bent along a helix with a fixed radius of curvature: a). one model coil; b). developed view.

2. One coil is described by a curvilinear coordinate system (Fig. 1), characterized by distance ℓ to fix the location of a particular section, measured along the length of a helical line bent along the radius R_s in plane $Y \ell$, passing through the centers of gravity of these sections.

The parameters of the sections of the spring coil (Fig. 1) are taken into account according to

- cross-sectional area
$$F_s = \frac{\pi d_s^a}{4}$$
;

- mass intensity $M_1 = \frac{\pi d_s^2 \cdot \rho}{4}$ and i_1 mass moment of inertia, where ρ is the density of the spring coil material;

- equatorial $I_x = I_y = \frac{\pi d_s^4}{64}$ and polar $I_0 = I_x + I_y = -\frac{\pi d_s^4}{32}$ moments of inertia of the cross-sectional area of the spring coil,

- modulus of elasticity of the first E and the second G kinds of the coil material.

3. We introduce generalized coordinates that take into account:

- elastic bending deformations $x_s(t, \ell)$, $y_s(t, \ell)$ in two planes - tangent to the helix and parallel to the axis of the cylinder of radius R_s of the winding of the coil, and perpendicular to the first plane;

- elastic deformations under torsion $Q_s(t, \ell)$ and compression $U_s(t, \ell)$ relative to the longitudinal axis of the helix of the spring.

4. To accept the model, considering [1, 10, 19], we use the following functions:

- moments of elastic deformations, bent in the $X\ell$ plane [19, 20].

$$M_{y}(\ell) = EI_{y}\left(\frac{\partial^{2}Y_{s}}{\partial\ell^{2}} + \frac{Y_{s}}{R_{s}^{2}}\right)$$
(1)

taking into account the curvature $\frac{1}{R_s}$ of the coils in plane Y ℓ ,

$$M_{x}(\ell) = EI_{x} \frac{\partial^{2} x_{s}}{\partial \ell^{2}}$$
(2)

- torsional moments about the longitudinal axis of the helix [19, 20].

$$M_{xy}(\ell) = GI_o\left(\frac{\partial Q_s}{\partial \ell} + \frac{1}{R_s}\frac{\partial x_s}{\partial \ell}\right)$$
(3)

where the second component takes into account the value of the additional angle of rotation of the section under bending in the $X \ell$ plane and the increasing value of moment M_{xy} , reduced to the arc of radius R_s

- compression by axial force relative to the longitudinal axis 0ℓ

$$P_{s}(\ell) = EF \frac{\partial U_{s}}{\partial \ell}$$
(4)

- moments of external forces (for bending sections) in planes $X \ell$ and $Y \ell$

$$M_{yef}(t,\ell) = P_s R_s \, \sin\lambda \, \left(1 - \cos\frac{2\pi\ell}{\ell}\right) \tag{5}$$

$$M_{xef}(t,\ell) = P_s R_s \, \cos \lambda \, \left(1 - \cos \frac{2\pi\ell}{\ell_s}\right) \tag{6}$$

where $P_{DYN}(t)$ is the dynamic load on the coil, containing the static component P_s and the dynamic component with amplitude of P_{ADYN}

$$P_{DYN}(t) = P_s + \sum_{n=0,1,\dots}^{\infty} \left\{ \sum_{k=0,1,2,\dots}^{\infty} P_{ADYN} \cdot \left[\cos(n\omega_a t) \cdot \left[\sigma_o(t - kt_I) - \sigma_o(t - kt_I - t_{IP}) \right] \right\} \right\}$$
(7)

where $\sigma_o(t)$ are impulsive functions (Heaviside functions [1]), λ - is the helix angle; k=0, 1, 2,... - is the number of impulses of dynamic load; n=0, 1, 2,... is the number of harmonics;

 $\ell_s = 2\pi R_s$ is the estimated length of the helix of one coil of the spring, therefore

$$2\pi \frac{\ell}{\ell_s} = \frac{\ell}{R_s}$$

- the twisting moment of external forces is

$$M_{Tef}(t,\ell) = P_s R_s \cos \lambda \left(1 - \cos \frac{2\pi\ell}{\ell_s}\right)$$
(8)

- the intensity of external forces bending the sections of one coil of the spring is

$$q_{yef}(t,\ell) = \frac{4\pi P_s R_s \sin \lambda}{\ell_s^2 \cos \frac{\ell}{R_s}}$$
(9)

$$q_{xef}(t,\ell) = \frac{4\pi P_s R_s \cos \lambda}{\ell_s^2 \cos \frac{\ell}{R_s}}$$
(10)

$$Q_{Tef}(t,\ell) = \frac{2\pi P_s R_s \cos \lambda}{\ell_s^2 \sin \frac{\ell}{R_s}} P_{DYN}(t)$$
(11)



Fig. 2. Calculation scheme for simulation of vibrations of cylindrical elastic rod bent along a helix (for swing link of a rail vehicle (a locomotive)).

5. With the introduced assumptions, using the Ostrogradsky-Hamilton method [1], we

compose the equations of oscillations for one coil of the spring along each generalized coordinate of elastic deformations. Then, using the Euler equation for elastic systems, we obtain, as a result, a system for describing the *bending (in two planes), longitudinal and torsional vibrations of a cylindrical rod bent along a helix, which generally characterizes the spatial vibrations of a coil of a helical spring*

$$\frac{\partial^{4} y_{s}}{\partial \ell^{4}} + \frac{\partial^{2} y_{s}}{\partial \ell^{2}} \cdot \frac{1}{E I_{y}} \left(\frac{2}{R_{s}^{2}} + P_{s} \sin \lambda \right) + \frac{y_{s}}{R_{s}^{4}} + \frac{1}{E I_{y}} \left(M_{1} \frac{\partial^{2} y_{s}}{\partial t^{2}} - i_{1} \frac{\partial^{4} y_{s}}{\partial t^{2} \partial \ell^{2}} \right) + \\ + \frac{1}{R_{s}} \cdot \left(\frac{1}{R_{s}^{2}} \frac{\partial U_{s}}{\partial \ell} + \frac{\partial^{3} U_{s}}{\partial t^{2}} \right) = \frac{2\pi \sin \lambda}{\ell_{s} E I_{y}} \cdot P_{s}(t) \cos \left(\frac{\ell}{R_{s}} \right), \qquad (12)$$

$$\frac{\partial^{4} x_{s}}{\partial \ell^{4}} + \frac{\partial^{2} x_{s}}{\partial \ell^{2}} \cdot \frac{1}{E I_{x}} \left(P_{s} \sin \lambda - \frac{G I_{0}}{R_{s}^{2}} \right) + \frac{1}{E I_{x}} \left(M_{1} \frac{\partial^{2} x_{s}}{\partial t^{2}} - i_{1} \frac{\partial^{4} x_{s}}{\partial t^{2} \partial \ell^{2}} \right) + \\ + \frac{\partial^{2} Q_{s}}{\partial \ell^{2}} \cdot \frac{G I_{0}}{R_{s} E I_{x}} = \frac{2\pi \cos \lambda}{\ell_{s} E I_{x}} \cdot P_{s}(t) \cos \left(\frac{\ell}{R_{s}} \right), \qquad (13)$$

$$\frac{\partial^{3} y_{s}}{\partial \ell^{3}} + \frac{1}{R_{s}^{2}} \cdot \frac{\partial y_{s}}{\partial \ell} + \frac{F_{s} R_{s}^{2}}{I_{y}} \cdot \frac{\partial^{2} U_{s}}{\partial \ell^{2}} - \frac{M_{1} R_{s}}{E I_{y}} \cdot \frac{\partial^{2} U_{s}}{\partial t^{2}} = 0 \qquad (14)$$

$$\frac{1}{R_{s}} \cdot \frac{\partial^{2} x_{s}}{\partial \ell^{2}} + \frac{\partial^{2} Q_{s}}{\partial \ell^{2}} - \frac{i_{1}}{G I_{0}} \cdot \frac{\partial^{2} Q_{s}}{\partial t^{2}} = -\frac{1}{G I_{0}} \cos \lambda \cdot \sin \left(\frac{\ell}{R_{s}} \right) \cdot P_{s}(t) \qquad (15)$$

The resulting system of differential equations allows approximate solutions for the cases when $P_s \sin \lambda$ and $P_s \cos \lambda$ are constants. These solutions include functions of static $x_s(\ell), y_s(\ell), U_s(\ell), Q_s(\ell)$ and dynamic $x_a(t, \ell), y_a(t, \ell), U_a(t, \ell), Q_a(t, \ell)$ components.

For numerical calculation in the *MATHCAD 15* programming environment, we accept the following initial data and assumptions:

1. We accept a model of a cylindrical helical spring, characterized by a wire diameter d_s , an average coil diameter D_s , a pitch between turns S_s in an unloaded state, a number of turns i_s , a static load P_s , a lead angle in a loaded state λ .

2. We present the initial model as a single coil with a vertical axis of symmetry and length passing through the center of gravity of the sections of the helix ℓ_s (Fig. 1, a).

3. For the accepted model of a single coil, the upper section for $\ell = 0$ is considered cantilever (free end), loaded with concentrated static load P_s , and the lower section for $\ell = \ell_s$ - is considered clamped.

Boundary conditions for one coil (Fig. 1, a) are given in the following form: For $\ell = 0$

$$\frac{\partial^2 y_s(\ell=0)}{\partial \ell^2} = 0 \ ; \ \frac{\partial^2 x_s(\ell=0)}{\partial \ell^2} = 0 \ ; \ \frac{\partial^3 y_s(\ell=0)}{\partial \ell^3} = 0 \ ; \ \frac{\partial^3 x_s(\ell=0)}{\partial \ell^3} = 0 \ ; \ \frac{\partial Q_s(\ell=0)}{\partial \ell} = 0 \ ; \tag{16}$$

for $\ell = \ell_s$

$$y_s(\ell = \ell_s) = 0 \; ; \; x_s(\ell = \ell_s) = 0 \; ; \; \frac{\partial y_s(\ell = \ell_s)}{\partial \ell} = 0 \; ; \; \frac{\partial x_s(\ell = \ell_s)}{\partial \ell} = 0 \; ; \; Q_s(\ell = \ell_s) = 0.$$
(17)

4. The upper section of the coil is loaded with a vertical static load P_s , which is decomposed into two components

- perpendicular to the axis of the helix and equal to $P_x = P_s \cos \lambda$,

- directed along the helix and equal to $P_y = P_s \sin \lambda$, perpendicular to vector P_x .

5. Ignoring the effect of static longitudinal strains $U_s(\ell)$ on bending $x_s(\ell), y_s(\ell)$, we obtain a system of equations for calculating the *static strains of the sections of a helical spring* in a form similar to the ones obtained in [1-3].

$$\frac{\partial^4 y_s}{\partial \ell^4} + \frac{\partial^2 y_s}{\partial \ell^2} \cdot \frac{1}{E I_y} \left(\frac{2}{R_s^2} + P_s \sin \lambda \right) + \frac{y_s}{R_s^4} = \frac{2\pi \sin \lambda}{\ell_s E I_y} \cdot P_s \cos \left(\frac{\ell}{R_s} \right), \tag{18}$$

$$\frac{\partial^4 x_s}{\partial \ell^4} + \frac{\partial^2 x_s}{\partial \ell^2} \cdot \frac{1}{E I_x} \left(P_s \sin \lambda - \frac{G I_0}{R_s^2} \right) + \frac{\partial^2 Q_s}{\partial \ell^2} \cdot \frac{G I_0}{R_s E I_x} = \frac{2\pi \cos \lambda}{\ell_s E I_x} \cdot P_s \cos \left(\frac{\ell}{R_s} \right), \quad (19)$$

$$\frac{\partial^3 y_s}{\partial \ell^3} + \frac{1}{R_s} \cdot \frac{\partial y_s}{\partial \ell} + \frac{F_s R_s^2}{I_y} \cdot \frac{\partial^2 U_s}{\partial \ell^2} = 0 , \qquad (20)$$

$$\frac{1}{R_s} \cdot \frac{\partial^2 x_s}{\partial \ell^2} + \frac{\partial^2 Q_s}{\partial \ell^2} = -\frac{1}{GI_0} \cos \lambda \cdot \sin\left(\frac{\ell}{R_s}\right) \cdot P_s \ . \tag{21}$$

We combine solutions to equations (18) and (20), and equations (19) and (21). Taking into account the introduced assumptions, after transformation, for (18) and (20) we obtain the following

$$\frac{\partial^4 y_s}{\partial \ell^4} + \beta_y^2 \cdot \frac{\partial^2 y_s}{\partial \ell^2} + \alpha_y^2 y_s = P_{sy} \cos\left(\frac{\ell}{R_s}\right), \qquad (22)$$
$$\frac{\partial^2 U_s}{\partial \ell^2} = -\beta_u^2 \frac{\partial^3 y_s}{\partial \ell^3} - \alpha_u^2 \frac{\partial y_s}{\partial \ell}, \qquad (23)$$

Where

$$\beta_y^2 = \frac{1}{E \, I_y} \left(\frac{2}{R_s^2} + P_s \sin \lambda \right); \, \alpha_y^2 = \frac{1}{R_s^4}; \, \beta_u^2 = \frac{I_y}{F_s R_s^2}; \, \alpha_u^2 = \frac{I_y}{F_s R_s^3}; \, P_{sy} = \frac{2\pi \sin \lambda}{\ell_s E \, I_y} \cdot P_s$$

The solution to equation (22) for static deflections $y_{sstat}(\ell)$ is represented as the sum of the solution to homogeneous equation (22) and a partial solution [1]

$$y_{sstat}(\ell) = y_s(\ell) + \overline{y_s}(\ell) = (C_{1y}sh(\omega_{ky}\ell) + C_{2y}ch(\omega_{ky}\ell) + C_{3y}sin(\omega_{by}\ell) + C_{3y}cos(\omega_{by}\ell) + \frac{1}{y_{sa}}(\ell),$$
(24)

where coefficients C_{1y} , C_{2y} , C_{3y} , C_{4y} are calculated with the boundary conditions according to the frequency equation by the iteration method using the *MATHCAD 15* programming environment, and eigenfrequencies for bending vibrations of one coil of the spring $y_{sstat}(\ell)$ are:

$$\omega_{ky} = \sqrt{-\frac{\beta_y^2}{2} + \sqrt{\left(\frac{\beta_y^2}{2}\right)^2 - \alpha_y^2}}; \ \omega_{ky} = \sqrt{\frac{\beta_y^2}{2} + \sqrt{\left(\frac{\beta_y^2}{2}\right)^2 - \alpha_y^2}},$$
(25)

partial solution to equation (22) $\overline{y}_s(\ell)$ is sought in the following form

$$\overline{y}_{s}(\ell) = \overline{y_{sa}}(\ell) \cdot \cos\left(\frac{\ell}{R_{s}}\right), \qquad (26)$$

where $\overline{y_{sa}}(\ell) = \frac{P_{sy}R_s^4 \cdot \cos\left(\frac{\ell}{R_s}\right)}{1 - R_s^2\beta_y^2 - \alpha_y^2R_s^4}$.

The solution to equation (23) for the longitudinal oscillations of one coil of the spring $U_{sstat}(\ell)$ is obtained using the Simson procedure

$$U_{sstat}(\ell) = \int_0^{\ell_s} \left[\int_0^{\ell_s} \left(-\beta_u^2 \frac{\partial^3 y_s}{\partial \ell^3} - \alpha_u^2 \frac{\partial y_s}{\partial \ell} \right) d\ell \right] d\ell$$
(27)

in the following form

$$U_{sstat}(\ell) = A_{1u}ch(\omega_{ky}\ell) + A_{2u}sh(\omega_{ky}\ell) + A_{3u}\cos(\omega_{by}\ell) + A_{4u}\sin(\omega_{by}\ell) + A_{5u}\cos\left(\frac{\ell}{R_s}\right) + A_{6u}\sin\left(\frac{\ell}{R_s}\right) + A_{7u}\ell + A_{8u}$$
(28)

where the following notation is introduced

$$\begin{split} A_{1u} &= -\beta_{u}^{2}C_{1y}\omega_{ky} - \alpha_{u}^{2}\frac{c_{1y}}{\omega_{ky}}; A_{2u} = -\beta_{u}^{2}C_{2y}\omega_{ky} - \alpha_{u}^{2}\frac{c_{2y}}{\omega_{ky}}; A_{3u} = -\beta_{u}^{2}C_{3y}\omega_{by} - \alpha_{u}^{2}\frac{c_{3y}}{\omega_{by}}; \\ A_{4u} &= -\beta_{u}^{2}C_{4y}\omega_{by} - \alpha_{u}^{2}\frac{c_{4y}}{\omega_{by}}; A_{5u} = -\beta_{u}^{2}\frac{\overline{y_{sa}(\ell)}}{R_{s}}; A_{6u} = \alpha_{u}^{2}R_{s}; \\ A_{7u} &= \beta_{u}^{2} \cdot \left(C_{2y}\omega_{ky}^{2} - C_{4y}\omega_{by}^{2}\right) + \alpha_{u}^{2} \cdot \left(C_{2y} + C_{4y} + \overline{y_{sa}(\ell)}\right); \\ A_{8u} &= \beta_{u}^{2} \cdot \left(C_{1y}\omega_{ky} + C_{3y}\omega_{by} - \frac{\overline{y_{sa}(\ell)}}{R_{s}}\right) + \alpha_{u}^{2} \cdot \left(\frac{C_{1y}}{\omega_{ky}} - \frac{C_{3y}}{\omega_{by}}\right). \end{split}$$

Now, as a result of combined solution to equations (19) and (21), we obtain solutions for static elastic deformations of bending $x_{sstat}(\ell)$ and elastic deformations of torsion $Q_{sstat}(\ell)$ with assumption ($\frac{\partial^4 x_s}{\partial \ell^4} \rightarrow 0$; $\frac{\partial^2 Q_s}{\partial \ell^2} \rightarrow 0$), and with boundary conditions (17). The solution was obtained by the method of operational Laplace transformation [1] by analogy with equations (18) and (20).

 $4\pi^2 R_s^2 \gg \ell_s$ holds for coil springs, so the solution of $x_{sstat}(\ell)$ is simplified

$$x_{sstat}(\ell) \approx R_s ctg\lambda \cdot \left(1 - ch\frac{\ell}{R_x}\right) + x_{0s}ch\frac{\ell}{R_x} + R_s \dot{x}_{0s}sh\frac{\ell}{R_x}, \qquad (29)$$

$$x_{0s} \approx R_s ctg\lambda \cdot \left(1 - ch\frac{\ell_s}{R_x}\right); \qquad x_{0s} \approx -\frac{R_s ctg\lambda}{R_x} \cdot sh\frac{\ell_s}{R_x}.$$
(30)

Then, integrating equation (21), with boundary conditions (17), we obtain solution for $Q_{sstat}(\ell)$ using the Simpson procedure

$$Q_{sstat}(\ell) = \frac{P_s R_s \ell \cos \lambda}{G I_p} - \frac{P_s R_s \ell \cos \lambda}{G I_p} \cdot \frac{\ell_s}{2\pi} \cdot \sin \frac{2\pi \ell}{\ell_s} + A_1 R_x ch \frac{\ell}{R_x} - A_2 \frac{\ell_s}{2\pi} \cos \frac{2\pi \ell}{\ell_s} + A_3 R_x sh \frac{\ell}{R_x} + Q_0 , \qquad (31)$$

where

$$A_{1} = \frac{x_{0s}}{R_{x}} - \frac{4\pi^{2}R_{0}R_{x}}{R_{s}}; A_{2} = \frac{2\pi\ell_{s}R_{0}}{R_{s}}; A_{3} = x_{0s}^{\cdot};$$

$$R_{0} = \frac{R_{s}P_{s}\cos\lambda}{EI_{x}} \cdot \frac{R_{x}^{2}}{\ell_{s}^{2} + 4\pi^{2}R_{x}}; Q_{0} = -\frac{R_{s}P_{s}\ell_{s}\cos\lambda}{GI_{p}} - A_{1}R_{x}ch\frac{\ell_{s}}{R_{x}} + A_{2}\frac{\ell_{s}}{2\pi} - A_{3}R_{x}sh\frac{\ell_{s}}{R_{x}}$$

The projection of the elastic deformation of the center of gravity of the internal section of a single coil model onto the vertical axis of symmetry of the spring is

$$x_V = Q_0 R_s = x_{0s} + \frac{R_s P_s \ell_s \cos \lambda}{G I_p} \,. \tag{32}$$

For helical springs wound from wire with a diameter of d_s into the last formula (32) we introduce the value of $I_p = \frac{\pi d_s^4}{32}$ and obtain a formula for calculating the deflection of one coil of a helical spring

$$x_V = x_{0s} + \frac{64P_s R_s^3}{Gd_s^4}.$$
 (33)

The resulting formula differs by the term X_0 from the known formula according to

studies given in [1, 10]. This indicates the reliability of the adopted model of a single coil, which, unlike the well-known ones, takes into account the bending stresses of coils in two mutually perpendicular directions, and the shear stresses arising from the torsion of the sections.

When using a PC to calculate stresses in sections of a model of a helical spring with a tubular section, it is appropriate to simultaneously control the reduced stresses σ_i at 8 characteristic points of each section in comparison with the allowable stress for the material of this spring. The solution to such a problem, considering the data in Table 1, is related to a variation in diameter d_2 and pressure in the inner hole.

3 Theoretical and experimental results

A method for calculating stresses in a coil spring of a tubular section was proposed as a result of the analytical and numerical studies. The results of the numerical calculation of the stress state of helical springs with swing links for the mainline electric locomotive VL-80s are summarized in Table 1.

Table 1. Selection of rational parameters based on the stress state of a helical spring for sp	oring
suspension of the mainline electric locomotive VL-80s	
(varying the inner diameter d_2)	

		Diameter d_2 , in mm			According to [19]
№	Spring vibration parameters	12	16	20	for a spring wound from a bar d = 42 mm
1.	Eigenfrequencies (5 modes of vibration)				
1.1. 1.2. 1.3 1.4	$p_x \\ p_y \\ p_Q \\ p_1 \approx \\ \approx p_3$	1.945 0.989 1.042 5.52 5.05	2.006 1.03 1.041 5.71 5.24	2.08 1.09 1.04 5.95 5.47	- - - - -
2	Total maximum static stresses σ_{stat} , MPa	636.66	684.7	730.11	546 (not considering the bending)
3	Total maximum dynamic bending stresses, σ_{DYN}^{bend} , MPa	114.24	154.24	214.84	-
4	Total maximum dynamic torsion stresses, τ_{DYN} , MPa	-43.97	-94.56	-145.38	-
5	Total maximum dynamic stresses, σ_{DYN} , MPa	122.41	180.91	259.66	148.6
6	Total maximum stresses σ_{max} , MPa	759.07	865.61	989.77	694.6
7	Tensile strength (endurance limit of spring steels of type 65C228A or 60C2XA), [σ], MPa		1000		923
8	Safety factor $K_{dirability} = \frac{[\sigma]}{\sigma_{max}}$	1.32	1.155	1.01	1.33

According to Table 1, a spring was selected for cradle suspension of the VL-80s main electric locomotive with the following parameters: D=198 mm, H=648 mm, $d_1=40$ mm, $d_2=12$ mm, N=6.

The total stresses in the most loaded section of the spring at $\ell = 0.33 \ell_s$ are in our calculation:

- static at $P_{stat} = 53.50$ kH $\sigma_{stat} = 636.66$ MPa;
- dynamic (under bending) for $P_{aDYN} = 1.4 \text{ kH} \sigma_{DYN}^{bend} = 114.24 \text{ MPa}$;
- dynamic (under torsion) $P_0 = 1.0 \text{ kH}$ $\tau_{DYN} = -43.97 \text{ MPa}$;
- total dynamic stresses $\sigma_{DYN} = 122.41$ MPa;
- total maximum stresses $\sigma_{max} = 759.07$ MPa;
- safety factor (for $[\sigma] = 1000$ MPa)

$$K_{dirability} = \frac{[\sigma]}{\sigma_{max}} = 1.32.$$

4 Conclusions

1. The proposed numerical-analytical applied method and refined methods of dynamic strength calculation (using the method of boundary elements - Boundary Element Technology) for curvilinear elements of the rolling stock of railways of a complex profile (springs, vibration dampers, cradle suspension units, spring suspension) are planned to be used in the design , operation and modernization with the extension of the useful life of locomotives.

2. The proposed methods are relevant for the Republic of Uzbekistan, and for the CIS countries, as they allow us to obtain better dynamic characteristics of ground vehicles, which determine their reliability and key performance indices.

References

- 1. Timoshenko S.P., Young DH, Weaver W. Fluctuations in engineering. Translated from English. L.G. Korneichuk; ed. E.I. Grigolyuk. M.: Mashinostroenie, p. 472 (1985).
- Branislav Titurus, Jonathan du Bois, Nick Lieven, Robert Hansford. A method for the identification of hydraulic damper characteristics from steady velocity inputs. Mechanical Systems and Signal Processing, Vol. 24, (8), pp. 2868–2887. (2010).
- 3. Kim D. H., Park J. W., Lee G. S., Lee K. I. Active impact control system design with a hydraulic damper, Journal of Sound and Vibration, 250, (3), pp. 485–501. (2002).
- Anderson R., Elkins J. & Brickle B. Rail Vehicle Dynamics for the 21st Century. eds. Aref H. & Phillips J. Mechanics for a New Millennium. Netherlands: Kluwer Academic Publishers, pp. 113–126. (2001).
- Wang W. L., Zhou Z. R., Yu D. S., Qin Q. H., Iwnicki S. Rail vehicle dynamic response to a nonlinear physical 'in-service' model of its secondary suspension hydraulic dampers. Mechanical Systems and Signal Processing, (95), pp. 138-157. (2017).
- 6. Jehle G., Fidlin A. Hydrodynamic optimized vibration damper. Journal of Sound and Vibration, 440, pp. 100-112. (2019).
- 7. Spiryagin, M., Cole, C., Sun, Y.Q., McClanachan, M., Spiryagin, V., & McSweeney, T. Design and Simulation of Rail Vehicles (1st ed.). CRC Press. (2014).
- 8. Popp K. and Schiehlen W., System Dynamics and Long-Term Behavior of Railway Vehicles, Track and Subgrade, *Springer Science and Business Media*, (2013).
- 9. Wang, K., Huang, C., Zhai, W., Liu, P., & Wang, S. Progress on wheel-rail dynamic

performance of railway curve negotiation. Journal of traffic and transportation engineering (English edition), 1(3), 209-220. (2014).

- 10. Biryukov I. S. and Savoskin A. N., *Mechanical part of the rolling stock*. Moscow, Transport, p. 440 (1991).
- 11. A.N. Savoskin. Durability and reliability of railway rolling stock: Mashinostroenie, (1990).
- 12. Ibragimov M.A., Kiselev V. I., Ramlov V. A. and Scalin A. V., *Dynamics of locomotives* (RGOTUPS, 2005).
- 13. Oganyan E. S., Volokhov G. M. Calculations and strength tests of load-bearing structures of locomotives: textbook. manual. M.: FGBOU "Educational and methodological center for education in railway transport", p. 326. (2013).
- 14. Khromova G., Makhamadalieva M. and Khromov S. Generalized dynamic model of hydrodynamic vibration dampener subject to viscous damping. *E3S Web of Conferences, EDP Sciences* 264 (2021).
- 15. Khromova G., Kamalov I. and Makhamadalieva M. Development of a methodology for solving the equations of bending vibrations of the hydro friction damper of the electric train of disk type. *AIP Conference Proceedings*, 2656(1) (2022). https://doi.org/10.1063/5.0108814
- 16. Mukhamedova Z. Diagnostics of Asynchronous electric motors on the basis of spectral analysis of amplitude-modulated stator current. // *International Scientific Journal «Europen Sciences review»*, 2018, № 7-8 (July-August), ISSN 2310-5577, Section 13. Transport, pp. 281-285.
- Khromova G. A., Makhamadalieva M. A. Calculation diagram of the hydrodynamic friction support of the flexible shaft of the hydro-friction vibration damper used in railway transport. «Universum: tekhnicheskie nauki» (Scientific journal: «Universum: Engineering Sciences»), Moscow, 7 (76), pp 77–80. (2020).
- Khromova G. A., Makhamadalieva M. A. Development of a method for extending the service life of the spring suspension of the afrosiyob high-speed electric train «Universum: tekhnicheskie nauki» (Scientific journal: «Universum: Engineering Sciences»), Moscow, 2 (95), pp 66–70. (2022). DOI -10.32743/UniTech.2022.95.2.13017
- G. A. Khromova, Z. G. Mukhamedova, and I. S. Yutkina, *Optimization of dynamic characteristics of emergency recovery rail service cars*. Monograph. ISBN 978-9943-975-96-6. Tashkent: "Fan va tekhnologiya", 2016. 253 p.
- Khromova G.A., Vershkov S.A., Babadzhanov A.A. Development of a new numericalanalytical method for predicting the resource of elastic elements of complex configuration Proc. of the 5th International Conference "Train traffic safety", Moscow: 2004, pp.107-108.
- 21. Mukhamedova Z.G. Forecasting residual resource and prolongation of service life of electrical installations of railway service cars and ray motor trolleys. // «American Scientific Journal of Research», USA, Michigan, 2018, № 9-10, pp. 122-127.