Wear resistance of gear teeth in the presence of slippage between gear teeth and during rolling

Hikmat Ishmuratov*

Tashkent State Technical University, Tashkent, Uzbekistan

Abstract. This article gives a brief overview of the fracture of contacting parts under the influence of circumferential force in meshing accompanied by rolling and slipping during plastic deformation.

1 Introduction

Profilograms obtained from the friction surfaces of gear teeth after the formation of equilibrium roughness show that their protrusions and depressions of irregularities are relatively close in height and have a fairly large radius of volume curvature than the technological roughness [2]. As a result of formation of equilibrium roughness on friction surfaces of open gearing teeth, the actual contact area of teeth is increased. Therefore, the friction surfaces of the gear teeth can operate without seizing at higher loads. According to the fatigue wear theory, the wear products from the friction surfaces of the gear teeth are separated after a certain amount of repeated deformation by rounded roughness protrusions.

2 Metods

As a result, the wear process of the gear teeth occurs when there is a slip between the rolling teeth, occurring on the head and foot of the teeth, and the wear of the teeth in pure rolling, occurring in the contact area of the initial circles. For these types of gear tooth contact the wear rate is determined.

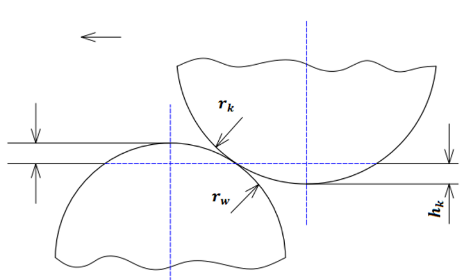

Fig. 1. Schematic diagram of roughness contact during friction

Wear resistance in the presence of slippage between gear teeth. Taking into account results of research given in [3, 4] and in order to simplify, calculation of wear rate, ledges of

*Corresponding author: x.ishmuratov@mail.ru

roughness of surfaces of a friction of the teeth participating in process of wear are modelled in the form of spheres (Fig. 1) with volumetric radius equal (r).

During contact, during wear of the gear teeth, roughness protrusions are embedded in the friction surfaces and this roughness's can be partially deformed by the radial component of the circumferential force transmitted by the gearing. When slippage occurs between the gear teeth, the embedded roughness protrusions plough a deepened raceway on the friction surfaces of the teeth [3].

In the process of force interaction between the teeth of meshing gears the volume of deformed material depends on the depth of penetration, radius of volume curvature of roughness projections, path of relative slippage of teeth, number of roughness projections that are along the length of contact area width and that participate in the process of friction surfaces deformation [3].

According to the research results obtained in [3-10], there is a connection between the diameter of the contact spot embedded in the friction surfaces of the gear teeth and the circumferential force acting on these protrusions (P), when the friction surfaces of the gear teeth are in contact:

$$P = \frac{\pi \cdot a^2_{ш,к} \cdot c \cdot M_{об} \cdot H_{ш,к}}{4} \cdot \text{Н} \tag{1}$$

where $a_{ш,к}$ - diameter of contact spot of embedded roughness projection with friction surfaces of teeth, m; s - coefficient depending on shape of projections and hardening of material [1]; $M_{об}$ - total number of roughness projections on working area of teeth contact; $H_{ш,к}$ - material hardness of driving (driven) gear, MPa.

To calculate the wear rate of the teeth, it was assumed that the dimensions of the roughness protrusions in height and volume radius of curvature are the same. Which are arranged consecutively along the length and height of the tooth. During friction, the roughness projections are partially embedded in the friction surfaces of the contacted tooth surfaces. According to the accepted conditions for the arrangement of the roughness projections of the friction surfaces, their number along the length of the gear teeth *L*, is equal:

$$M = \frac{L}{a_{ш,к}}$$

Then the load absorbed by a single roughness ledge:

$$\frac{P \cdot a_{ш,к}}{L} = \frac{\pi \cdot a^2_{ш,к} \cdot c \cdot H_{ш,к}}{4}$$

We assume that the circumferential force transmitted by an open gear working in dry friction is absorbed by the bending stresses on the tooth contact surface, then the circumferential force absorbed by all the bending stresses on the tooth contact surface is equal:

$$P = 2 \cdot k \cdot m \cdot \sigma_{из\, ш,к} \cdot M \cdot a_{ш,к} \tag{2}$$

where $\sigma_{изш,к}$ - maximum bending stress occurring on the plane of the concave circle of the leading (driven) gear tooth stem, MPa; k - tooth height coefficient relative to the dividing circle of the gears; m - meshing modulus, m.
For approximate calculations, equating expressions (1) and (2) and solving them with respect to the hardness of the material is obtained:

$$H_{ш,к} = \frac{8 \cdot k \cdot m \cdot \sigma_{изш,к}}{\pi \cdot a_{ш,к} \cdot c}.$$

From the resulting expression, the relationship between the hardness of the material and the bending stress in the gearing can be derived, with an optimum value of $H=_{ш,к}1.8\sigma$
Due to the fact that the contact area of the embedded roughness projection is approximately circular then the contact area of the friction surface of the teeth of the driving (driven) gears, is calculated by the expression [7].

$$a_{ш,к} = \frac{1.27 \cdot P}{L \cdot c \cdot H_{ш,к}} = \frac{2.54 \cdot m \cdot k \cdot \sigma_{изш,к}}{c \cdot H_{ш,к}} \text{ м.} \tag{3}$$

According to the scheme shown in Fig. the depth of penetration of roughness protrusions to the friction surfaces at the contact of gear teeth is 2determined and the dependence is obtained,

$$h_{ш,к} = \frac{0.0019 \cdot a_{шк}}{c \cdot H_{ш,к} \theta_{ш,к}} \tag{4}$$

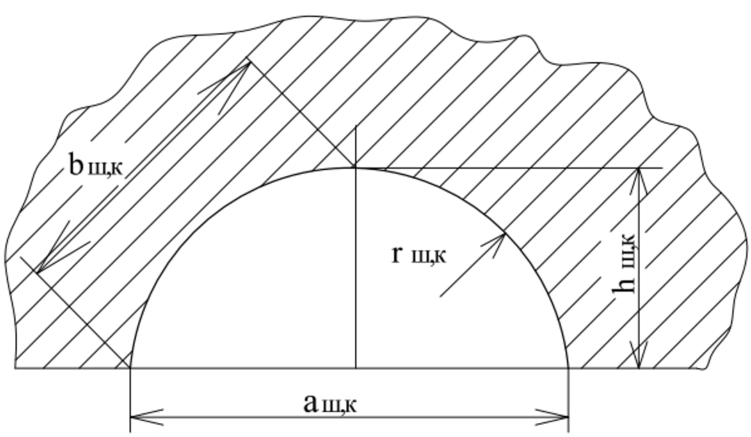

Fig. 2. Schematic diagram for determining the geometric parameters for introducing roughness *projections into* the friction surface of the gear teeth

There is a relationship between the volumetric radius of curvature $r_{ш,к}$ and the penetration depth of the gear tooth roughness $h_{ш,к}=8$:

$$r = \frac{h_{ш,к}}{3 \cdot c \cdot H_{ш,к} \theta_{ш,к}} \tag{5}$$

The chord length of the segment ($b_{ш,к}$) resulting from the intrusion of a rounded roughness is determined according to the scheme shown in Fig. 2. In this diagram, the calculated values of the contact diameter $a_{ш,к}$, the penetration depth $h_{ш,к}$ and the volumetric radius of curvature ($r_{ш,к}$) of the roughness protrusions are taken into account,

$$b_{w,k} = \sqrt{2 \cdot h_{ш,к} \cdot r_{ш,к}} = \frac{0.816 \cdot h_{ш,к}}{(c \cdot H_{ш,к} \cdot \theta_{ш,к})^{0,5}} \tag{6}$$

We calculate the cross-sectional area of the deformed metal volume resulting from the introduction of a single roughness projection located on the friction surface of the teeth of the driving (driven) pinion [7]:

$$F_{ш,к} = \frac{h_{шк} \cdot \left(6a_{ш,к} + 8b_{ш,к}\right)}{15} = \frac{2.448 \cdot k \cdot h_{ш,к} \cdot m \cdot \sigma_{из}}{c \cdot H_{ш,к}} \tag{7}$$

The number of roughness's located on the tooth length of the driving (driven) pinion [8] taking into account expression (3) we obtain:

$$M = \frac{0.39 \cdot L \cdot c \cdot H_{ш,к}}{k \cdot m \cdot \sigma_{из}} \quad 8)$$

Given the value of the cross-sectional area of the deformed metal volume as a result of the introduction of a single roughness $F_{ш,к}$ of (7), the path of the relative slip of the roughness projections located on the surface of the contacting teeth $s_{ш,к}$ [7] and the number of roughness's located on the gear tooth length M calculated by expression (8) after some simplification we get the expression for calculating the volume of deformation of the material of the drive (driven) gear, all the roughness projections, located on the contact surface in the presence of slippage

$$v_{ш,к} = F_{ш,к} s_{ш,к} M_{ш,к} = \frac{3}{z_{ш,к}} \cdot h_{ш,к} \cdot L \cdot m \cdot (i+1) \cdot \psi \tag{9}$$

Here the slip coefficient of the gear teeth in contact is denoted ψ by , which is defined through the reduced number of teeth:

$$\psi = \sqrt{z_{пр}^2 \sin^2 \alpha + 4k_{ш} \pm 4k^2} - z_{пр} \sin \alpha,$$

where z_{np} is the reduced number of teeth of the gears, $z_{np} = \dfrac{z_{ш} \cdot z_{к}}{z_{ш} + z_{к}}$; here $z_{ш}$, $z_{к}$ are the numbers of teeth of the driving and driven gears respectively.

In an expression, the plus sign is placed k^2 in front when the slip of the tooth head is calculated, the minus sign is used to calculate the slip of the tooth stem.

In the process of friction the roughness protrusions of the teeth of one gear for each cycle of loading come into contact with different protrusions of the roughness of the other gear. And the roughness of the teeth of these gears is different from each other by the density of the location on the tooth surface and the size. In addition, during contact the roughness projections themselves may partially deform and change their original shape. The same roughness ledge and the deformed surface of the counter body can meet each other after a certain number of load cycles of the gears, which is taken as the probability of repeated deformation, to calculate the value, its obtained dependence [7]:

$$\eta_{w,k} = \frac{1}{z_{ш,к} \cdot M} = \frac{k \cdot m \cdot \sigma_{из}}{0.39 \cdot z_{ш,к} \cdot L \cdot c \cdot H_{ш,к}} = \frac{2.56 \cdot k \cdot m \cdot \sigma_{из}}{z_{ш,к} \cdot L \cdot c \cdot H_{ш,к}} \qquad (10)$$

Fracture of the deformed gear tooth surfaces occurs according to fatigue wear theory, after a certain number of repetitive loadings.
The number of load cycles resulting in the failure of the deformed tooth surface of the driving (driven) pinion is equal:

$$n_{p(ш,к)} = \psi^{t}_{ш,к} \qquad (11)$$

where $\psi_{ш,к}$ - coefficient of relative elongation of pinion material; t - coefficient of friction fatigue of pinion material, for gears made of steel it is taken t=1,3 [1].

Then, in general terms, the wear rate of the gear teeth of open gears, in the presence of slippage between the gear teeth is defined

$$\gamma_{\partial(ш,к)} = \frac{v_{ш,к} \cdot n_{ш,к} \cdot \eta_{ш,к}}{F \cdot n_{p(ш,к)}} \qquad (12)$$

where $n_{ш,к}$ is the speed of the driving (driven) gear; F is the contact area of the meshing pairs of gear teeth $F=BL$.
The contact widths of the gear teeth of the starting circle B area are calculated by the expression [7]:

$$B = \frac{3.04 \cdot \sqrt{P \cdot \rho_{np}} \cdot (1-\mu^2)}{\sqrt{L \cdot E_{np}}} \qquad (13)$$

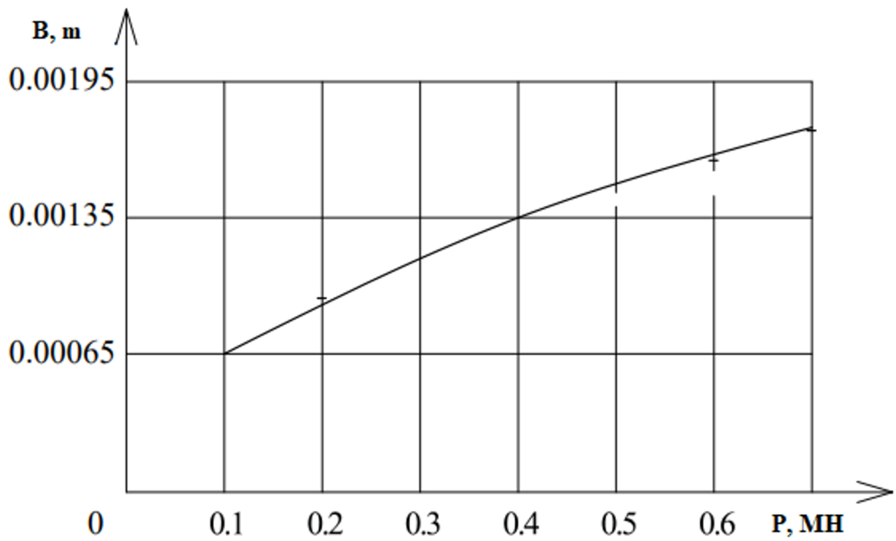

Fig. 3. Variation of tooth contact width as a function of the circumferential force in the gearing

Where μ is Poisson's ratio; ρ_{np} - reduced radius of curvature of the gear teeth of the rolling zone, $\rho_{np} = 0.5 \cdot m \cdot z_{np} \cdot \sin\alpha$.

The graph of change of width of contact of teeth, depending on circumferential force, presented on fig.3 is received by expression (12) at the following initial data: z_{np}=9.78; μ =0.03; $E=_{np}$ 215000 MPa shows, that increase of width of contact of gears teeth results in increase of circumferential force, transferred by meshing.

Substituting the values from $v_{ш,к}$ expression (9), $\eta_{w,k}$ from (10) and ($n_{p(ш,к)}$ 11) into (12) considering, and after some simplification, we obtain an expression for calculating the wear rate of the driving (driven) pinion, in the presence of slippage between the teeth,

$$\gamma_{\partial(w,k)} = \frac{910.8 \cdot h_{ш,к} \cdot m \cdot (i+1) \cdot \psi \cdot n_{ш,к} \cdot k \cdot \sigma_{из} \cdot P^{0.5}}{E_{np}^{0.5} \cdot \rho_{np} \cdot L \cdot (1-\mu^2) \cdot z_{ш,к}^2 \cdot c \cdot n_{рш,к} \cdot H_{ш,к}} \qquad (14)$$

Table 1 gives an example of the results of the calculation of the gear wear in the drive gears of a vertical cotton picker. As an example, the results of calculating the amount of wear of the pinion teeth of the drive pinion of a cotton harvester with vertical spindles, depending on the meshing module in the presence of slippage between the pinion teeth ψ=*1.035,* the tooth height factor from the initial circle of the drive pinion *k=1;* the rotation frequency of the drive pinion $n=_{ш}$ *r/s 2.92*; maximum bending tension on pinion teeth $\sigma=_{из}$ *153.7 MPa*; modulus of elasticity *E=215000MPa*$_{np}$; gear transmission ratio *i=0.125* (acceleration gearings); number of pinion teeth z=88$_{ш}$; strain factor c=3; hardness of pinion material *H=282MPa*$_{ш}$*;* number of deformation cycles, which cause destruction of deformed pinion surface at ψ =6% is *n=* $_{рш}$*10.273*. The results *of calculating* the service life of the pinion is shown in the table. For calculation, it is assumed that according to the recommendations proposed in the "Encyclopedia of Mechanical Engineering XXL" the ultimate wear of pinion teeth is 20% of the tooth thickness.

Table 1. Basic parameters for calculating the wear resistance of the teeth of an open pinion gear

Traction modulus m, m	Tooth length sesterni L, m	The width of the con-Toothcycle B, m	Tooth contact area $F_к$, m^2	Circumferential force in the coupling P, MN	Tooth base area $F_о$, m^2	Bending stress, at the tooth stem $\sigma_{из}$,MPa
1	2	3	4	5	6	7
0.001	0.025	0.00012	0.00000696	0.014	0.00009106	153.7
0.002	0.029	0.00024	0.00001392	0.028	0.00018212	153.7
0.004	0.037	0.00048	0.00002784	0.056	0.00036424	153.7
0.006	0.045	0.00072	0.00004176	0.084	0.00054636	153.7
0.008	0.053	0.00096	0.00005568	0.112	0.00072848	153.7
0.010	0.061	0.00120	0.00006960	0.140	0.00091060	153.7

Continuation of table № 2.

Traction modulus m, m	Depth of penetration of the roughening protrusion $h_ш$, m	Radius of curvature of pinion contact point $\rho_{тш}$,	Drive pinion tooth wear rate $\gamma_ш$, m/hour	Permissible gear tooth wear, m	Resource leading gears, hour
8	9	10	11	12	13
0.001	0.00014	0.00167	0.0000000662	0.000314	4743
0.002	0.00028	0.00334	0.0000001622	0.000628	3872
0.004	0.00056	0.00668	0.0000003596	0.001256	3492
0.006	0.00084	0.01002	0.0000005432	0.001884	3468
0.008	0.00112	0.01336	0.0000007100	0.002512	3538
0.010	0.00140	0.01670	0.0000008621	0.003140	3642

In the rolling area of the gear teeth, the wear process, as noted above, occurs as a result of deformation of local metal volumes of the friction surfaces. When between friction surfaces of gear teeth there is no slippage from introduction of roughness ledges in their contact zone crater-shaped wells are formed, herewith the wear products are formed after some number of repeated deformations of friction surface of gear teeth by ledges of roughness rounded shape. In this case the wear rate of gear teeth in the contact zone of initial circles, when rolling in general form is determined by the expression:

$$\gamma_{\partial(w,k)} = \frac{v_{1н(w,k)} \cdot M_{об} \cdot n_{w,k} \cdot \eta}{F_{nk} \cdot n_{p(w,k)}}, \ m/hour \tag{15}$$

where M_{ob} is the total number of roughness projections in the gear tooth contact area.
To calculate the deformed metal volume of the contact surfaces of gear teeth with a single roughness ledge, having a spherical shape, taking into account the diameter of the contact area a and $_{w,к}$ hardness of the gear material $H_{w,к}$, when the ledge is rounded tooth surface roughness, when rolling the initial circular area of contacting gear teeth received the dependence [7]:

$$v_{1H(wk)} = 5.75 \cdot \frac{\theta^2_{ш,к} \cdot k^3 \cdot m^3 \cdot \sigma^3_{изг}}{9 \cdot c \cdot H_{шк}} \qquad (16)$$

In the contact area of the starting circles of the meshing gears - the value of the gear tooth height factor in the starting circle area k can be represented by the ratio of the tooth contact width to the meshing modulus,

$$k = \frac{B}{m}$$

then the expression (16) has the form,

$$v_{1H(wk)} = 0.639 \cdot \frac{\theta^2_{ш,к} \cdot B^3 \cdot \sigma^3_{изг}}{c \cdot H_{шк}}$$

The contact area of the friction surfaces of the gear teeth is equal:

$$F_{пк} = L \cdot B \qquad (17)$$

The number of roughness projections determines the M_b amount of deformation of the friction surfaces, a relationship is obtained to calculate the number of roughness projections located across the contact width of the gear teeth:

$$M_b = \frac{1.69 \cdot \sqrt{\rho_{шк}} \cdot (1-\mu^2) \cdot c \cdot H_{ш,к}}{\sqrt{E \cdot B \cdot \sigma_{из}}} \qquad (18)$$

The number of successive roughnesses per tooth length of the driving (driven) gear is M determined by expression (8).
According to [7] in the contact area of the starting circles of the meshing gears only rolling, without tooth slippage occurs. For this case:
radius of curvature of the pinion tooth profile,

$$\rho_ш = 0.5 \cdot m \cdot z_ш \cdot \sin\alpha$$

radius of curvature of the idler tooth profile,

$$\rho_k = 0.5 \cdot m \cdot z_ш \cdot i \cdot \sin\alpha$$

The total number of roughness projections on the tooth contact area, taking into account expressions (8) and (18), is equal:

$$M_{об} = M_b \cdot M = \frac{0.34 \cdot \sqrt{\rho_{шк}} \cdot (1-\mu^2) \cdot \theta \cdot L \cdot c^2 \cdot H^3_{ш,к}}{\sqrt{E \cdot (B \cdot \sigma_{из})^3}} \qquad (19)$$

The calculated value of the re-deformation $\eta_{ш,к}$ probability, by a roughness protrusion of the same deformed surface is determined by the dependence (10) [8]:
Substituting the values of ηfrom (10), $n_{p(ш,к)}$ from (11), $v_{1н(w,k)}$ from (12), $F_{пк}$ from (16), $M_{об(ш,к)}$ from (19), into (14), we finally obtain

$$\gamma_{\partial(w,k)} = \frac{5950 \cdot \theta^2_{ш,к} \cdot B^{5/2} \cdot \sigma^{5/2}_{из} \cdot n_{ш,к}}{i \cdot L^{1/2} \cdot \psi^t_{ш,к} \cdot P^{1/2}} \tag{20}$$

The obtained expression shows that the wear rate of gear teeth in the contact zone of initial circles of meshing gears, rolling zone depends on the tooth length, gear ratio and friction fatigue material, the contact width of gear teeth, bending stress arising at the foot.

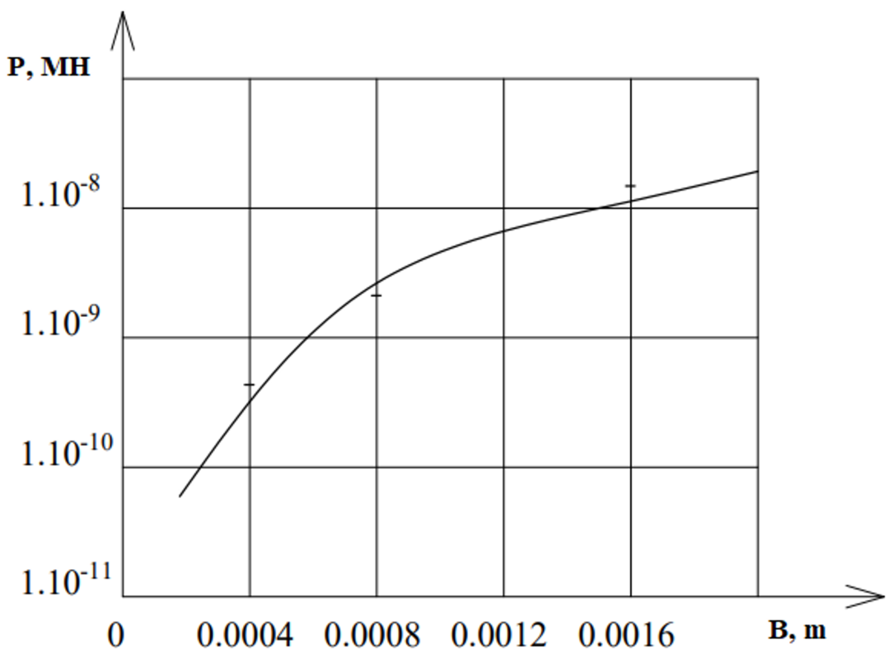

Fig. 4. Variation of pinion tooth wear rate as a function of pinion tooth contact width

Dependence of change in the wear rate of the rolling area of the teeth of the pinion shown in Fig. 4 is obtained from expression 20 with the following initial data:θ = 4.23*10 -6 1/MPa; *n*= $_ш$2,92r/s*;* *p* = 0.14MN*;* *i*=0,125 ;*L = m0,058*; $\sigma_{из}$ = *1 53,7MPa;* $\psi_ш$ *= %6; i = 2.*

4 Conclusions

1. The wear rate of the gear teeth in the presence of slippage between the gear teeth increases with increasing modulus of engagement, speed of rotation of the driving (driven) gear, decreases with increasing number of teeth of the driving (driven) gear and frictional fatigue of the material of the driving (driven) gear.
2. In the zone of contact of initial circles of driving and driven gears, increase of width of contact of teeth, bending stress and rotation frequency of driving (driven) gears, rotation frequency of driving (driven) gears, decrease, increase of gear ratio, length of gear tooth,

friction fatigue and circumferential force transmitted by gear meshing lead to increase of wear rate.

3. By increasing the contact width up to 0,0008 m the wear rate of the teeth in the rolling area of the initial circles of the gears involved in the meshing grows more intensively, a further increase in the contact width of the teeth up to 0,0020 m results in a less intensive increase in the wear rate of the teeth.

4. A relationship has been established between the bending stress that occurs on the tooth shaft and the hardness of the gear wheel material, which increases the wear resistance of the gear teeth most effectively when the ratio of hardness to the bending stress that occurs on the tooth shaft is 1.88 once.

References

1. Park, J. Y., & Salmeron, M. Fundamental aspects of energy dissipation in friction. Chemical reviews, 114(1), 677-711. (2014).
2. Ishmuratov H. K. Theoretical justification of gear life of cotton harvesting machines according to wear criterion. Tashkent (2019).
3. Ishmuratov H.K. Wear resistance of gear teeth, while rolling without participation of abrasive particles in wear process. In International Scientific and Practical Conference "Automobile and Tractor Engineering. Minsk, pp. 16-20 (2019).
4. Ishmuratov, K. K., & Irgashev, B. A. Assessment of the wear resistance for gearwheel teeth in an open toothed gear under the conditions of a high level of dust. Journal of Friction and Wear, 41, 85-90. (2020).
5. Vinogradov, V. M., & Cherepakhin, A. A. Influence of a method of gear treatment on durability on a bend of teeth of cylindrical wheels of automobiles and tractors. Izvestiya MGTU MAMI, 3(2), 136-141. (2009).
6. Gorlenko O. A. A., Makarov G. N., Shnyrikov I. O. Increasing the contact endurance of teeth of spur gear drives. Friction and Lubrication in Machines and Mechanisms. № 6, pp.25-27. (2014).
7. Düzcükoğlu, H. PA 66 spur gear durability improvement with tooth width modification. Materials & Design, 30(4), 1060-1067. (2009).
8. Irgashev A. Methodological bases of increasing the wear resistance of gears of heavy loaded toothed gears of machine units. Tashkent (2005).
9. Al-quraan, T. M., Mikosyanchik, O. O., & Mnatsakanov, R. G. The Effect of the Slippage Degree at Rolling with Slipping on the Wear Resistance of Contact Surfaces. Mechanical Engineering Research, 6(2), 48-61. (2016).
10. Irgashev, A., Shaabidov, S. A., Irgashev, B. A., & Egamberdieva, N. A. Roughness of meshing gear teeth. In IOP Conference Series: Earth and Environmental Science (Vol. 1112, No. 1, p. 012003). IOP Publishing. (2022).