An Updated Design Procedure for Tesla Turbines

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Abstract. Tesla turbine rotor, a special case of the flow between two corotating disks, has been studied in the past analytically and the performance is discussed both qualitatively and quantitatively. However, there is no systematic design criteria/process given to design the rotor of a Tesla expander in the peer-reviewed literature. Such design procedure, presented in this article, allows researchers and engineers to design and optimise the rotor for a given fluid and design condition (Power, flow and rotational speed). In this article, we present a 0-D design methodology to calculate rotor design parameters such as disk diameters, the gap between disks, the number of disks and the rotational speed of the expander, and efficiency and power estimation. This design procedure is based on the correlations and optimal ranges present in the literature. The 0-D model discussed in this article is a promising design approach to the preliminary design of the Tesla rotor and then further fine-tuning could be done based on the CFD simulations when coupled with the stator. A case study is presented with a 3-kW air bladeless expander prototype in which the rotor is designed using the 0-D model approach and compared with 2D Computational Fluid Dynamics results.

1 Introduction

Nikola Tesla developed the bladeless turbomachinery in 1913[1][2]. Based on the idea that the best performance will be reached when the changes in velocity and direction of the fluid's movement are as gradual as possible, Tesla claims in his patents a high efficiency because of the type of energy transfer, which occurs by viscous forces. Many researchers have performed experimental work on Tesla expanders and provided insight into the performance of the machine. The performance recorded is very low (i.e., total to static efficiency < 35%) [3]. The experimental data is available mostly for low-size expanders i.e., net power less than 1 kW. The performance of the Tesla expander at different sizes is not available for the same fluid/design conditions. The design methodology for most of the experimental prototypes is not clearly presented, which makes it even more difficult to analyse its performance. Analytical research has already been done on the Tesla Rotor, a special case of the flow between two corotating disks, and the performance is discussed both qualitatively and quantitatively. The rotor of a Tesla expander is not, however, given any systematic design

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criteria or method in the peer-reviewed literature. In this article, we present 0-D design technique for the Tesla rotor. Researchers and engineers can design and optimize the rotor for a specific fluid and design condition (Power, flow, and rotational speed) using the design technique described in this article.

2 Tesla rotor design parameters

The Tesla turbine rotor typically consists of multiple closely spaced flat disks mounted on a shaft and propelled by fluid flowing between them in spirals concentric with the shaft toward the output as shown in Fig. 1. The fluid enters from the outer perimeter in a tangential direction and leaves through the inner exit ports in an axial direction.

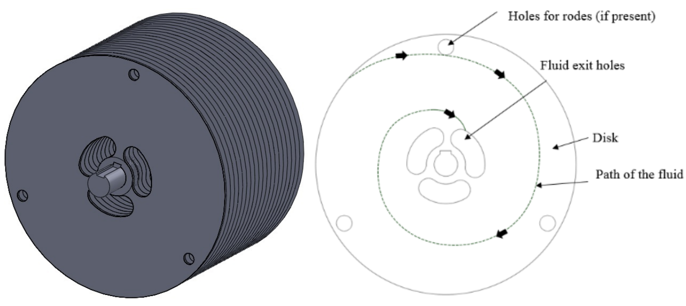


Fig. 1. Rotor of Tesla bladeless expander

The design of the rotor is done based on the following flow parameters.

2.1 Inlet flow angle (α_o) and inlet velocity ratio

The fluid should enter from the stator to the Tesla rotor in a nearly tangential direction for the best performance, that is a challenge. Hence, researchers in the past have used stator with positive flow angle (greater than 5°) for practical reasons. Fig. 2 shows the velocity triangle at inlet and exit of the Tesla rotor. The flow angles are measured with respect to tangential direction (along the disk tip velocity vector): the angle of absolute velocity vector is α_o, which is also the angle which stator makes with the rotor (inlet flow angle).

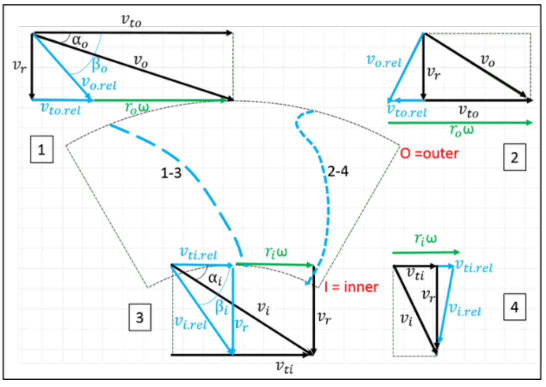


Fig. 2. Velocity diagram for Tesla rotor

The angle between relative velocity vector with tangential direction is β_o i.e., relative flow angle. There are two cases shown in the Fig. 2 based on the inlet velocity ratio (ratio between inlet tangential velocity and the disk tip velocity).

Case 1-3: This case is represented in Fig. 2 by #1 and #3 velocity triangles with the corresponding relative fluid path from outer to inner edge of the disk as 1-3. In this case, the inlet tangential fluid velocity, v_{to}, is higher than the disk tip speed (velocity ratio greater than 1). The relative velocity between fluid and disk, $v_{o.rel}$, is in the direction of the rotation of the disk. This positive relative velocity produces a shear force on the disks which in turn results into torque. Similarly, in the exit velocity triangle when the exit tangential velocity is higher than the disk tip velocity, it results in positive torque. However, energy is lost in the exit of the rotor due to the higher tangential velocity of the fluid without exchanging work with the disk. In this case, the inner radius of the disk must be carefully designed to recover maximum energy from the fluid. The typical relative fluid path is shown with dashed line 1-3.

Case 2-4: This case is represented in Fig. 2 by #2 and #4 velocity triangles with the corresponding relative fluid path from outer to inner edge of the disk as 2-4. The relative velocity between fluid and disk, $v_{o.rel}$, is in the opposite direction of the rotation of the disk: the flow reversal inside the disk is an inefficient energy transfer process which generates losses, and should be avoided in a Tesla rotor. Hence, for the best performance, inlet flow angle should be as small as possible, and the velocity ratio should be higher than 1.

2.2 Reynolds Number

Reynolds number defines the flow behaviour inside the gap between the disks i.e., laminar or turbulent. There are several definitions of the Reynolds number used depending upon the characteristic length and the velocity consideration. For the effective transfer of energy (momentum) of the fluid to the disks by acquiring the momentum of the fluid by the disk, the flow should be laminar [4].

Reynolds number based on relative velocity (difference between the fluid tangential velocity and disk velocity) and the gap between disk as the characteristic length:

$$Re_{b.rel} = \frac{\rho.b.(v_{to} - r_o.\omega)}{\mu} \tag{1}$$

Reynolds number based on radial velocity:

$$Re_{b.rel} = \frac{\rho.D_h.(v_r)}{\mu} \tag{2}$$

The characteristic length can be presented by hydraulic diameter:

$$D_h = \frac{4.S}{P} \tag{3}$$

where S is the flow cross-sectional area and P is the wetted perimeter.

$$D_h = \frac{4.(2.\pi.r.b)}{2.(2.\pi.r)} = 2b \tag{4}$$

Therefore, Eq. 4 becomes,

$$Re_{b.vr} = \frac{\rho.2.b.(v_r)}{\mu} \tag{5}$$

This Reynolds number is comparable with the Reynolds number with the flow in the pipe case. This will help to understand the flow behaviour inside the rotor of the Tesla turbine i.e., laminar flow if $Re_{b.vr} < 2000$, transition if $2000 < Re_{b.vr} < 4000$ and turbulence if $Re_{b.vr} > 4000$.

Rice [4] has performed the analytical investigation of the Tesla rotor using Reynolds number based on fictitious velocity (velocity represented as the product of the gap between disks and angular velocity, characteristic length represented by the gap between disks). Although this

Reynolds number does not give information qualitatively, the reasonable range for the higher performance of the rotor is recommended between 5 and 8.

2.3 Gap between disks

The gap between two disks is an important design parameter and greatly depends on Reynolds number, boundary layer thickness, the kinematic viscosity of the fluid and the rotational speed of the disks. The gap between the disks is evaluated considering both the boundary layers i.e., tangential, and radial direction. There is an optimum gap that ensures effective energy transfer between fluid and disks (boundary layer in tangential direction) and to ensure positive flow through rotor with less viscous frictional loss (boundary layer in the radial direction). Boundary layer thickness for laminar flow [5] can be evaluated as follows,

$$\delta = 5.\sqrt{\frac{\nu . l}{U}} \tag{6}$$

where, l is the length of the channel and U is the free stream velocity outside boundary layer. In case of tangential direction, $l = 2\pi r_o$ and $U = v_{to}$. In case of radial direction, $l = r_o - r_i$ and $U = v_r$.

Breiter and Pohlhausen [6] used a similarity parameter that determines the shape of the radial and tangential flow profiles between the disks.

$$P = \frac{b}{2}.\sqrt{\frac{\omega}{\nu}} \tag{7}$$

A profile that is just deviating from a parabolic shape is considered to be optimum [7] i.e., flow conditions should be such that P value appears close to "π/2".

There is a similar parameter studied by some researchers, Ekman Number, which is crucial for velocity profile between the gap and the efficiency of the Tesla rotor. Ekman number is defined as the ratio of viscous forces to the Coriolis forces or the ratio of half gap to the boundary layer thickness. Ekman number is given by,

$$Ek = \frac{\frac{b}{2}}{\delta} = \frac{b}{2}.\sqrt{\frac{\omega}{\nu}} \tag{8}$$

Ekman number is identical to the Eq. 7 and the recommended range for high rotor efficiency is 1- 2.

From the above analysis, one can estimate, as a rule of thumb, gap between the disks is in the neighbourhood of double the boundary layer thickness, for Tesla expanders.

2.4 Flow parameter

Flow rate per disk gap is another important parameter to be evaluated for the maximum efficiency of the Tesla rotor. Rice [4] has performed analytical calculations for non-dimensional flow rate impact on efficiency with respect to velocity ratio and radius ratio.

Following are the recommended values for flow rate parameter, radius ratio and velocity ratio (inlet fluid tangential velocity to disk tip velocity) at disk tip.

Non dimensional flow rate parameter for optimum flow per disk gap:

$$q_f = \frac{Q}{\omega . r_o^3} \sim 0.00001 - 0.0001 \tag{9}$$

equivalent to:

$$U_0 = \frac{Q}{2.\pi . \omega . r_o^2 . b} \sim 0.1 - 0.25 \tag{10}$$

Velocity ratio:

$$VR = \frac{v_{to}}{\omega . r_o} = \sim 1.1 - 1.3 \quad (11)$$

Radius ratio:

$$RR = \frac{r_o}{r_i} = \sim 2 - 5 \quad (12)$$

Once the flow rate per gap is calculated, number of disks can be estimated based on the total flow available at the inlet of the turbine.

Torque per disk can be evaluated using Euler's equation for turbine,

$$T = m.(r_o v_{to} - r_i v_{ti}) \quad (13)$$

Power per disk is given by,

$$P = T.\omega \quad (14)$$

3 Tesla rotor design algorithm

Based on the parameters discussed in the above section, an algorithm to preliminary design the rotor can be established, obtaining the geometric and flow parameters for the turbine to be selected at design condition. The fine tuning of the geometry may be required mainly for the design case and feasibility with respect to manufacturing of the rotor.

3.1 Case study – 3 kW air expander rotor design

An air 3 kW expander, for which detailed numerical and experimental investigation is performed in Authors' previous work [8], is considered for the case study of rotor design. The generator has rated rotational speed of 40000 rpm with 3 kW power, which is considered a starting point for the analysis.

a. The outer diameter for the rotor is selected to limit the Mach number to 1 or less. Peripheral velocity of 250-300 m/s is taken as a starting point. The outer diameter of 120 mm is calculated for tip speed of ~ 250 m/s.
b. Total absolute pressure at the rotor inlet, P_{t0} , is considered 2 times the dynamic pressure due to fluid velocity at rotor periphery. At this total pressure at rotor inlet, density of 2 kg/m^3 is calculated for the temperature of 300K.
c. Reynolds number, $Re_{b,b}$, of 4 is selected for the calculation of gap between disks. Using the fluid thermodynamic properties, gap between disks comes out to be around 0.1 mm.
d. As a check for P, similarity parameter according to Eq. (7) is evaluated which is ~1. This is close to the recommended value of π/2.
e. The fluid being low density, inner diameter is calculated using diameter ratio 2. The diameter ratio of 2 is chosen considering the outlet area blockage due to shaft and discrete exhaust holes on the disks, as shown in Fig. 2. The calculated outlet diameter for the rotor is 60 mm.
f. The inlet tangential velocity is calculated using optimum velocity ratio of 1.2. The calculated inlet tangential velocity is 300 m/s.
g. Radial velocity is calculated by setting inlet flow angle (a_o) of 1-2 degree for near tangential flow at the rotor. In this case, for the inlet flow angle of 1.36 degree, we get radial velocity of 7 m/s.
h. Using the radial velocity, mass flow and flow rate at the periphery of the disks is calculated which is 0.45 g/s and 0.000283 m^3/s per gap respectively.
i. Flow rate is checked with respect to flow rate parameter, Eq. (9): the calculated flow rate parameter in this case is 0.0003 which is in the acceptable range.

j. Torque per disk is calculated using Euler's Eq. (13), which is 0.006487 Nm, and power per disk, calculated by multiplying torque with angular velocity, is 27 W.
k. Number of disks of 120 is selected to have a total power higher than 3 kW (being upper limit power, in practice power will be less than 3 kW). The power and mass flow for 120 disks is 3.26 kW and 54 g/s.
l. The efficiency calculated using the ratio of output power and inlet isentropic power is 85.3%, that is the expected rotor-only efficiency.

The design of rotor is performed for various inlet flow angles (α_o) i.e., with different radial velocities which leads to different mass flow per gap. The performance of 0-D design is compared with 2D CFD analysis in the following section.

3.2 2-D Rotor of 3 kW Tesla expander – 2D CFD

In this section 2-D CFD of the rotor for the 3-kW expander with air is performed. Details about the CFD model set-up may be found in [9].

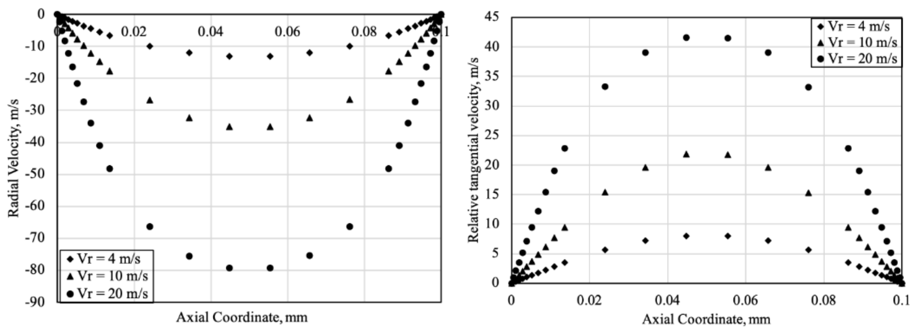


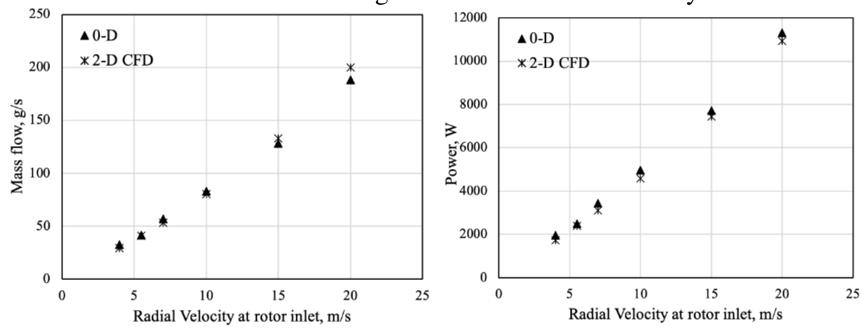
Fig. 3. Radial velocity and relative tangential velocity profile at r = 35 mm for different inlet radial velocity i.e. mass flow

Figure 3 shows the radial and relative tangential velocity profiles for different mass flows (represented in terms of inlet radial velocity) between the gap of disks at a radius of 35 mm. It can be seen that the velocity profile doesn't show any inflection or reverse flow. The parabolic profile both in the radial and tangential directions ensures the good design of the rotor. The comparison between 0-D and 2-D models are shown in the Fig. 4 and Fig. 5. The graphs are plotted with respect to inlet radial velocity as this is the fixed parameter in both the simulations. The mass flow from 0-D calculation agrees well with the 2D CFD results at lower inlet radial velocities. The difference at higher inlet radial velocities is due to the approximate inlet density of the fluid. The correction of a density involves the good prediction of inlet total pressure which is the most difficult parameter to calculate by 0-D as it depends on several other factors. The high error in the prediction of pressure at higher inlet radial velocity conditions can be seen in the Fig. 5(a). There is a good agreement between turbine power with slight overprediction in case of 0-D model as it does not take into account the losses of energy transfer between fluid and disks. The 0-D model efficiency curve follows the similar trend as that of 2-D CFD but with overestimating the efficiency at higher inlet radial velocity.

4 Conclusion

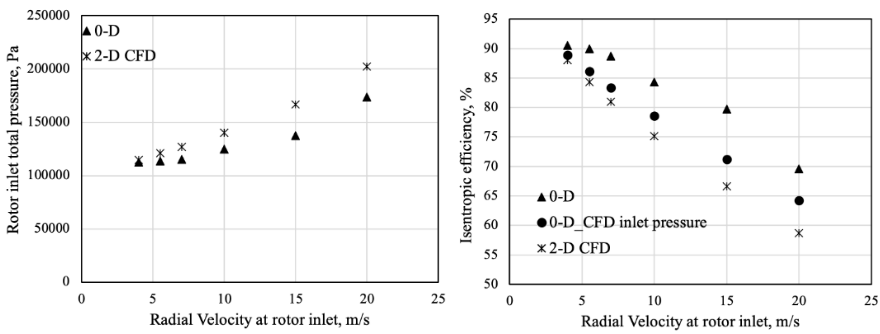
A 0-D design technique for Tesla rotor design is presented in this article. This algorithm is used in the initial design phase of Tesla expanders. The geometry and thermodynamic parameters evaluated from the algorithm are compared with 2-D CFD results. The geometry parameters indeed produce a highly efficient rotor design as verified with 2-D CFD analysis. Moreover, the algorithm predicts the power and efficiency in an acceptable range.

The 0-D model discussed above is a promising design approach to preliminary design the Tesla rotor and then further fine-tuning could be done based on very few CFD simulations.



(a) (b)

Fig. 4. Comparison between 0-D and 2-D CFD analysis. (a) mass flow versus radial velocity at the rotor inlet; (b) turbine power versus radial velocity at the rotor inlet for 120 disks



(a) (b)

Fig. 5. Comparison between 0-D and 2-D CFD analysis. (a) rotor inlet total pressure versus radial velocity at the rotor inlet; (b) isentropic efficiency versus radial velocity at rotor inlet for 120 disks

Nomenclature

b	gap between disks	[mm]
d	diameter	[mm]
$\dot{m}$	mass flow rate	[g/s]
p	pressure	[Pa]
r	radius	[mm]
v	velocity	[m/s]
C	specific heat	[J/kgK]
D	hydraulic diameter	[m]
L	length	[m]
N	rotational speed	[rpm]
P	power	[W]
Po	Poiseuille's number	[-]
Q	volume flow rate	[m^3/s]
Re	Reynolds number	[-]
α_o	inlet flow angle (absolute)	[°]
β_o	Inlet flow angle (relative)	[°]
ε	expansion ratio	[-]

η	efficiency	[-]
μ	dynamic viscosity	[Pa·s]
ρ	density of fluid	[kg/m^3]
τ	torque	[N·m]
ω	angular velocity	[rad/s]
Ψ	Degree of reaction (DoR)	[-]

Subscripts

amb	ambient
avg	average
i	inner edge of disk
o	outer edge of disk
p	pressure
p.o	pressure at outer edge of disks
r	radial
rel	relative

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