Investigation of multi loop linear magnetic circuits of electromagnetic flow converters with ring channels

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Abstract. Mathematical models of single and multiloop magnetic circuits of electromagnetic flow transducers with an annular channel are designed taking into account the parameters of the magnetic circuit. The possibility of compensating the difference in magnetic resistances of coaxially located concentric ferromagnetic cores and the difference in their circumference by selecting the thickness of the cores is shown. It has been installed that the magnetic flux in ferromagnetic cores is distributed along the angular coordinate. The magnetic induction in the annular channel is unevenly distributed along the radial coordinates. It was found that with an increase in the value of the attenuation coefficient of the magnetic field in the magnetic circuit, the degree of magnetic induction in the annular channel increases along the angular coordinate, while it remains constant along the radial coordinate.

1 Introduction

When measuring liquid flow in process control systems, along with other converters, electromagnetic flow converters (EFC) are widely used. Along with the serially produced EFCs special EFCs are used to control and monitor some technological processes (to control and monitor the quantity and quality of dairy products, heat supply, the flow of liquid metals and various acids, etc.) [4,5,8].

Metrological characteristics of EFC mainly depend on the state of the magnetic field in the working channel of the transducer. Therefore, much attention is paid to the study of the magnetic fields of these converters. In this case, it is required to determine the law of change in the magnetic induction in the annular channel between the coaxial located by ferromagnetic cores depending on the coordinates α, ρ and z.

As is known electromagnetic processes are most fully described by the equations of the electromagnetic field, ie Maxwell's equations. But they are not very suitable for the study of magnetic fields of electrical measuring transducers, i.e. the solutions obtained are inconvenient for engineering calculations of the characteristics of converters. Therefore, the magnetic fields of measuring transducers are most often investigated in the form of circuits.

Magnetic circuits of electromagnetic flow transducers with annular channels belong to circuits with distributed parameters [7]. These parameters include linear values of magnetic

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resistances of annular coaxially located ferromagnetic cores ($Z_{\mu n}$) and magnetic capacitance (magnetic conductivity by classical analogy of electrical and magnetic circuits) of an annular air channel ($C_{\mu n}$) between them, per unit of angular coordinate α.

An analysis of publications devoted to the study of magnetic circuits of electrical and non-electric values showed [6] that in almost all cases they are single-loop magnetic circuits with distributed parameters and not accounting the distribution of parameters in displaced circuits significantly reduces the accuracy of their calculation. Therefore, the discovery of the article is devoted to the development of mathematical models of single-circuit and multi-circuit magnetic circuits by the method of partial electromagnetic flow transducer with an annular channel. A mathematical model has been designed and introduced into production [8].

2 Mathematical models of single-circuit EFC magnetic circuits with distributed parameters

The structural diagram of the magnetic circuit of the first EFC with an annular channel and the equivalent circuit of its elementary section dα are shown in Figure 1 a and b [8].

To simplify the analysis of magnetic circuits we will accept the following assumptions are made:

1) ring ferromagnetic rings and ferromagnetic rods connecting them to each other are made in a monolithic form from the same material;
2) 2), the magnetic fluxes at both ends of the annular ferromagnetic cores along the pipe axis are so small that they may not be taken into account;
3) the magnetic resistance of ferromagnetic cores does not depend on the value of the magnetic field induction in them, that is, the magnetic circuit operates in the linear part of the main magnetization curve (if we take into account the presence of a large air gap in the path of the working magnetic flux, this assumption is made quite acceptable);
4) due to the small value of the frequency of changes in the magnetic field over time, eddy currents in ferromagnetic cores have too small a value that they can be ignored.

These assumptions do not significantly affect the accuracy of magnetic circuit analysis, but they greatly simplify calculations.

Distinctive feature this magnetic chain is the fact that it created same conditions for circuit power lines magnetic across ring running clearance between coaxially located open ferromagnetic cores, united between various ends with ferromagnetic jumpers with magnetizing winding, i.e. magnetic resistance any way, for which connect power line magnetic flow the same. In this case, the difference between the values of the magnetic resistances of concentric ferromagnetic cores, which appears as follows: due to the difference in their lengths, it can be eliminated by choosing their thickness using the ratio $(h_2/h_1) = r_{2cp}/r_{1cp})$. Then we can assume that $Z_{\mu n1} = Z_{\mu n2} = Z_{\mu n}$.

To simplify calculations , we assume that the magnetic capacitances of non -working air gaps are $\delta_{s1}$and δ_{s2} are equal to each other, i.e.: .: $C_{\mu s1} = \mu_0\,(bh_1)/\delta_{s1} = C_{\mu s2} = \mu_0\,(bh_2)/\delta_{s2}$. From this equation it follows that $\delta_{s2} = \delta_{s1}(h_2/h_1)$. If this condition is met, we can assume that $Q_{\mu s1} = Q_{\mu s2} = Q_{\mu s}$.

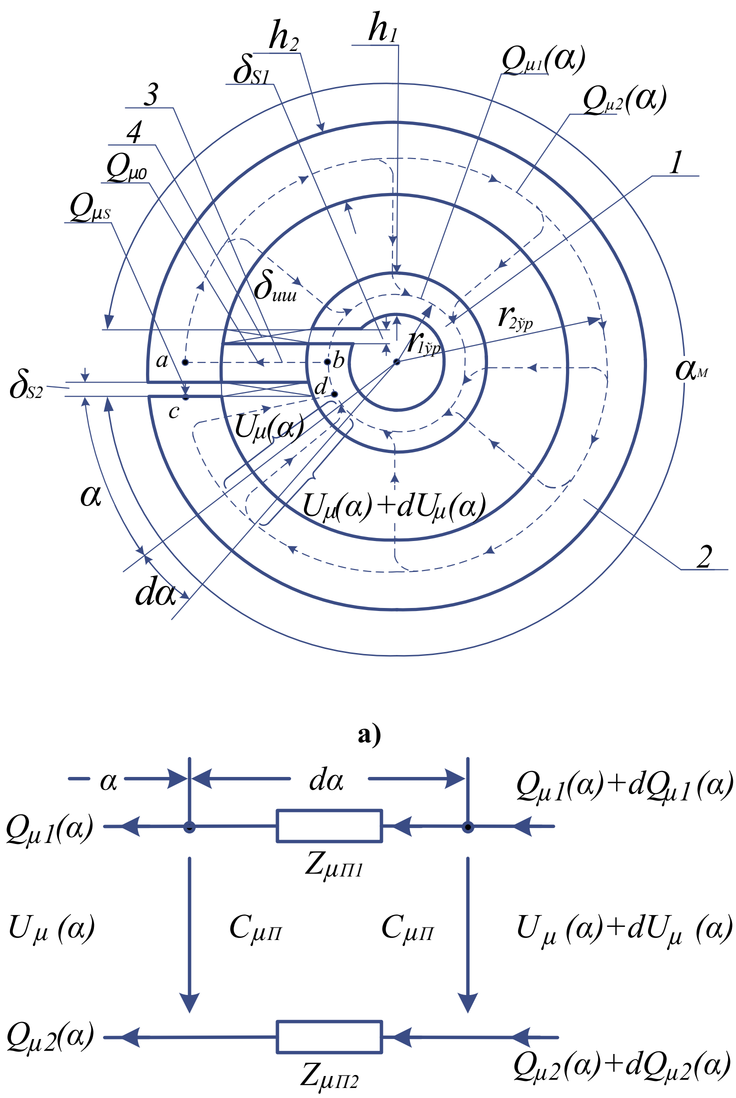

Fig.1. Structural diagram (a) and equivalent circuit of the elementary section (b) of a linear EFC magnetic circuit with an annular channel.

Differential equations based on the following laws Kirchhoff's method for the magnetic flux and magnetic voltage generated by MDS $F_в$ field windings, for an elementary section of a magnetic circuit they will look like this:

$$\frac{dQ_{\mu1}(\alpha)}{d\alpha} = U_\mu(\alpha)C_{\mu n}; \quad \frac{dQ_{\mu2}(\alpha)}{d\alpha} = -U_\mu(\alpha)C_{\mu n};$$
$$\frac{dU_\mu(\alpha)}{d\alpha} = Z_{\mu n}\left[Q_{\mu1}(\alpha) - Q_{\mu2}(\alpha)\right], \quad (1)$$

where: $Z_{\mu п1} = \frac{2\pi r_{1cp} - h_3 - \delta_{s1}}{\mu\mu_0 b h_1 \alpha_м}$, $Z_{\mu п2} = \frac{2\pi r_{2cp} - h_3 - \delta_{s2}}{\mu\mu_0 b h_2 \alpha_м}$; $C_{\mu п} = \mu_0 \frac{b\pi(r_{1cp} + r_{2cp}) - (\delta_{s1} + \delta_{s2} + h_3)}{\delta \alpha_м}$;

where h_3 - thickness of ferromagnetic jumpers, connecting interaction between each other concentric ferromagnetic materials cores. The remaining designations are shown in pic. 3, a.

After simple calculations, we obtain the following differential equation:

$$\frac{d^2U_\mu(\alpha)}{d\alpha^2} = 2Z_{\mu п}C_{\mu п}U_\mu(\alpha). \quad (2)$$

The general solution of this differential equation is as follows:

$$U_\mu(\alpha) = A_1 e^{\gamma\alpha} + A_2 e^{-\gamma\alpha}, \quad (3)$$

For the magnetic circuit under study, the following condition is met:

$$Q_{\mu 1}(\alpha) + Q_{\mu 2}(\alpha) = Q_{\mu 2}(\alpha_м) + Q_{\mu s} = Q_{\mu 1}(0) + Q_{\mu s}$$

here $Q_{\mu s}$ - magnetic flux closing through non -working air gaps δ_{s1} and δ_{s2}

From (4) finding it $Q_{\mu 2}(\alpha)$ and substituting it into the third equation (1) we determine the expression from the obtained equation $Q_{\mu 1}(\alpha)$ as follows:

$$Q_{\mu 1}(\alpha) = \frac{\gamma}{2Z_{\mu п}}(A_1 e^{\gamma\alpha} - A_2 e^{-\gamma\alpha}) + \frac{1}{2}\left[Q_{\mu 2}(\alpha_м) - Q_{\mu s}\right]. \quad (4)$$

Similarly we find the expression $Q_{\mu 2}(\alpha)$:

$$Q_{\mu 2}(\alpha) = -\frac{\gamma}{2Z_{\mu п}}(A_1 e^{\gamma\alpha} - A_2 e^{-\gamma\alpha}) + \frac{1}{2}\left[Q_{\mu 2}(\alpha_м) - Q_{\mu s}\right]. \quad (5)$$

The integration constants A_{A1} and A_{A2} are found taking into account the following boundary conditions:

$$\left.\begin{aligned} Q_{\mu 1}(\alpha)|_{\alpha=0} = Q_{\mu 1}(0) = Q_{\mu 2}(\alpha_м), \\ Q_{\mu 2}(\alpha)|_{\alpha=\alpha_м} = Q_{\mu 2}(\alpha_м). \end{aligned}\right\} \quad (6)$$

Substituting in (7) the values of magnetic fluxes and magnetic stresses corresponding to the boundary conditions, and solving the resulting system of algebraic equations with respect to the unknowns, we have:

$$A_{A1} = -\frac{Z_{\mu n}\left[Q_{\mu 2}(\alpha_m) + Q_{Qs}\right]}{\mu s 2\gamma sh(\gamma\alpha_m)}(e^{-\gamma\alpha_m} + 1), \quad (7)$$

$$A_{A2} = -\frac{Z_{\mu n}\left[Q_{\mu 2}(\alpha_m) + Q_{Qs}\right]}{\mu s 2\gamma sh(\gamma\alpha_m)}(e^{\gamma\alpha_m} + 1). \quad (8)$$

Substituting the found values $A_{of\ A1}$ VA A_{A2} into equations (3), (5), and (6), we obtain the following expressions for the magnetic stresses between coaxial concentric ferromagnetic cores and for the magnetic fluxes in them:

$$U_\mu(\alpha) = -\frac{Z_\mu n\, Q_{\mu 0}}{\gamma sh(\gamma\alpha_m)}\{ch(\gamma\alpha) + ch[\gamma(\alpha_м - \alpha)]\}, \quad (9)$$

$$Q_{\mu 1}(\alpha) = -\frac{Q_{\mu 0}}{2sh(\gamma\alpha_м)}\{sh(\gamma\alpha) - sh[\gamma(\alpha_м - \alpha)]\} + \frac{1}{2}\left[Q_{\mu 2}(\alpha_м) - Q_{\mu s}\right], \quad (10)$$

$$Q_{\mu 2}(\alpha) = \frac{Q_{\mu 0}}{2sh(\gamma\alpha_м)}\{sh(\gamma\alpha) - sh[\gamma(\alpha_м - \alpha)]\} + \frac{1}{2}\left[Q_{\mu 2}(\alpha_м) - Q_{\mu s}\right]. \quad (11)$$

To determine the expressions $U_\mu(\alpha)$ and $Q_{\mu 1}(\alpha)$ BA $Q_{\mu 2}(\alpha)$, expressed in terms of the MDS F_{in} the field windings, it will be necessary to determine the values $of\ Q_{\mu 0}$ (total magnetic flux), $Q_{\mu s}$ and $Q_{\mu 2}(\alpha_m)$, expressed in terms of the MDS F_{in}.

To do this, we will make the following equations based on Kirchhoff's laws for the node "a", as well as for the closed contours "$aa'b'ba$" and "$baa'a''b''b$" of the magnetic circuit under study:

$$Q_{\mu 0} = Q_{\mu 2}(\alpha_м) + Q_{\mu s}, \quad (12)$$

$$Z_{\mu 0}Q_{\mu 0} + U_\mu(\alpha_m) + W_{\mu s}Q_{\mu s} = F_{in}, \quad (13)$$

$$Z_{\mu 0}Q_{\mu 0} + Z_{\mu n}\int_0^{\alpha_m} Q_{\mu 2}(\alpha)d\alpha + U_\mu(0) = F_{in}, \quad (14)$$

where $W_{\mu ss} = \frac{1}{C_{\mu s}}$ is the magneticaя stiffness of the magnetic circuit (according to the energy-information model of circuits of various physical nature [10]), and by the classical analogy of electric and magnetic circuits, this parameter is called the magnetic resistance of the circuit. Solving together equations (13)-(15) with respect $to\ q_{\mu 0}$, $Q_{\mu s}$, and $Q_{\mu 2}(\alpha_m)$, we obtain their following expressions:

$$Q_{\mu 0} = -F_{in}\frac{2bshb\,(W_{\mu s} + Z_{\mu}\, n\alpha_m)}{\Delta_1}, \quad (15)$$

$$Q_{\mu 2}(\alpha_m) = -F_{in}\frac{2bshb\ (W_{\mu s}+0.5\ Z_{\mu\ n}\alpha_m)}{\Delta_1}, \quad (16)$$

$$Q_{\mu s} = -F_{in}\frac{\beta sh\beta Z_{\mu n}\alpha_m}{\Delta_1}, \quad (17)$$

where $\Delta_1 = (Z_{\mu n}\alpha_m + W_{\mu s})\ (2Z_{\mu}n\ \alpha_m(1 + CHB) - 2Z_{\mu\ 0}\beta shb)\ - \beta Z_{\mu\ n}W_{\mu s}SHB.$

Substituting (29)-(31) in equations (11)-(13), respectively, we obtain the following final expressions for the magnetic voltage and magnetic fluxes:

$$U_{\mu}(\alpha) = F_{in}\frac{2(W_{\mu s}+Z_{\mu}n\ \alpha_m)}{\Delta_1}\{ch(\beta\alpha^*) + ch[\beta(1-\alpha^*)]\}. \quad (18)$$

$$Q_{\mu 1}(\alpha) = F_{in}\frac{\beta}{\Delta_1}\{(W_{\mu s} + Z_{\mu}n\ \alpha_m)\{sh(\beta\alpha^*) - sh[\beta(1-\alpha^*)]\} - W_{\mu s}sh\beta\}. \quad (19)$$

$$Q_{\mu 2}(\alpha) = -F_{in}\frac{\beta}{\Delta_1}\{(W_{\mu s} + Z_{\mu}n\ \alpha_m)\{sh(\beta\alpha^*) - sh[\beta(1-\alpha^*)]\} + W_{\mu s}sh\beta\}, \quad (20)$$

where $\alpha^* = \frac{\alpha}{\alpha_m}$.

The dependence of the magnetic induction of the magnetic field in the annular channel on the coordinate α is determined using the following expression:

$$B(\alpha) = \mu_0\frac{U_{\mu}(\alpha)}{\delta} = F_{\text{в}}\mu_0\frac{2(W_{\mu s}+Z_{\mu\text{п}}\alpha_{\text{м}})}{\delta\Delta_1}\{ch(\beta\alpha^*) + ch[\beta(1-\alpha^*)]\}. \quad (21)$$

The degree of uneven distribution of the magnetic field induction in the annular channel along the coordinate α is calculated by the formula:

$$\delta B(\alpha), \% = \left[\frac{B(0)-B(0{,}5\beta)}{B(0)}\right]\cdot 100\% = \left[1 - \frac{2ch(0{,}5\beta)}{1+ch\beta}\right]\cdot 100\%. \quad (22)$$

The law of variation of the magnetic field induction in an annular channel along the radial coordinate p is determined using the following expression:

$$B(\rho) = \left|\frac{Q_{\mu 2}(\alpha_m)}{b[2\pi\rho-(\delta_{s1}+\delta_{s2}+h_{\delta s1\ +\ \delta s2\ +\ H3})]}\right| = \left|F_{in}\frac{2\beta sh\beta(W_{\mu ss}+0.5\ Z_{\mu n}\alpha_m)}{b[2\pi\rho-(\delta_{s1}\delta s1\ +\ \delta s2\ +\delta_{H3)\ \Delta}+h_3)]\Delta_2}\right|. \quad (23)$$

If $W_{\mu s} \to \infty$ or $C_{\mu s} = 0$, expressions (29) - (31) and (32)-(35) will be equal to:

$$Q_{\mu 0} = Q_{\mu 2}(\alpha_m) = -F_{in}\frac{2bshb}{\Delta_2},\ Q_{\mu s} = 0, \quad (24)$$

$$U_{\mu}(\alpha) = F_{in}\frac{2Z_{\mu}n\ \alpha_m}{\Delta_2}\{ch(\beta\alpha^*) + ch[\beta(1-\alpha^*)]\}. \quad (25)$$

$$Q_{\mu 1}(\alpha) = F_{\text{в}}\frac{\beta}{\Delta_2}\{sh(\beta\alpha^*) - sh[\beta(1-\alpha^*)] - sh\beta\}. \quad (26)$$

$$Q_{\mu 2}(\alpha) = -F_{\text{в}}\frac{\beta}{\Delta_2}\{\{sh(\beta\alpha^*) - sh[\beta(1-\alpha^*)] + sh\beta\}\}, \quad (27)$$

$$B(\alpha) = F_{in}\mu_0\frac{2Z_{\mu}n\ \alpha_m}{\delta\Delta_2}\{ch(\beta\alpha^*) + ch[\beta(1-\alpha^*)]\}, \quad (28)$$

where $\Delta_2 = 2Z_{\mu}(1 + ch\beta) - 2Z_{\mu}0\ \beta sh\beta\ - Z_{\mu}\beta sh\beta; Z_{\mu} = Z_{\mu n}\alpha_m$.

Expressions (19) - (22) are mathematical models of the linear magnetic circuit of the EFC shown in Figure 2, taking into account the distribution of magnetic circuit parameters and scattering fluxes that close through non-working gaps. They can be used to determine the design parameters of magnetic circuits and study the static and dynamic characteristics of an EFC with an annular channel.

3 Mathematical models of multi-circuit EFC magnetic circuits with distributed parameters

The design diagram of the magnetic circuit of the EFC with ring channels developed with the participation of the authors of the article, and the scheme of replacing its elementary part are shown in Figure 2 [9]. The magnetic system consists of external and 1 internal 2 closed cylindrical pole pieces, two open hollow cylindrical magnetic cores 3 and 4, and mutually coaxially positioned mirror and are alternately connected to the pole pieces 1 and 2 using ferromagnetic jumpers 5 and 6, the source MDS, made in the form of an electromagnet connecting the inner ends of the pole pieces 2 and unconfined cylindrical magnetic circuit 4, the side surfaces of which the insulating plates 8 and 9 are mounted flat electrodes 10 and 11. Pole tips 1 and 2, open cylindrical magnetic lines 3 and 4, ferromagnetic jumpers 5 and 6, and a MDS source 7 form a multi-circuit magnetic circuit with distributed parameters.

We will start developing mathematical models of a multi-loop magnetic circuit by drawing up a substitution scheme for its elementary section of length dx (Figure 3).

Differential equations based on Kirchhoff's laws for an elementary section of a multi-loop magnetic circuit with distributed parameters have the following form:

$$\frac{dQ_{\mu 1}}{dx} = U_{\mu 12} C_{\mu n 12}, \quad (29)$$

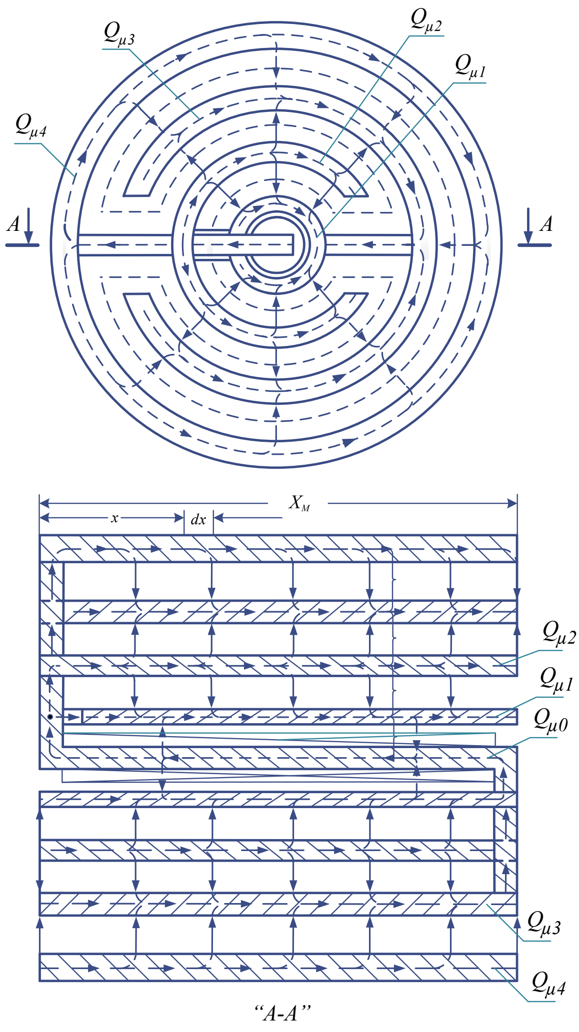

Fig. 2. Structural diagram of an EFC multiloop magnetic system with annular channels.

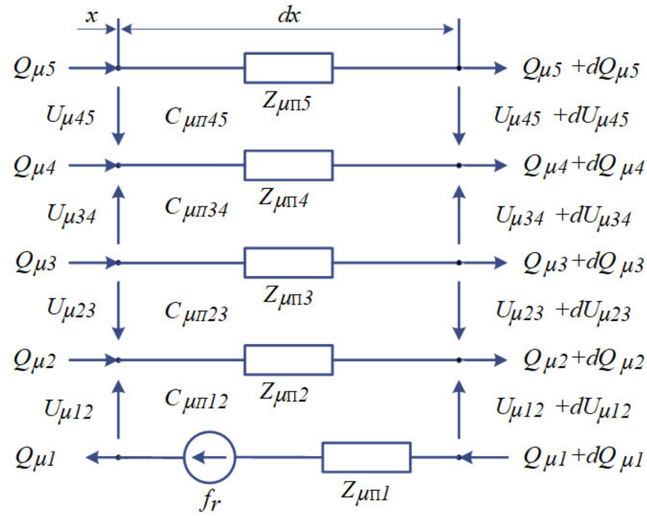

Fig. 3. Equivalent circuit of an elementary section of a multiloop magnetic circuit with distributed parameters

$$\frac{dU_{\mu12}}{dx} = -f_{\text{b}} + Z_{\mu п2}Q_{\mu2} + Z_{\mu\mu п1}Q_{\mu1}, \tag{30}$$

$$\frac{dQ_{\mu2}}{dx} = U_{\mu23}C_{\mu п23} + U_{\mu12}C_{\mu п12}, \tag{31}$$

$$\frac{dU_{\mu23}}{dx} = -\left[Z_{\mu п3}Q_{\mu3} - Z_{\mu п2}Q_{\mu2}\right], \tag{32}$$

$$\frac{dQ_{\mu3}}{dx} = -\left[U_{\mu34}C_{\mu п34} + U_{\mu23}C_{\mu п23}\right], \tag{33}$$

$$\frac{dU_{\mu34}}{dx} = Z_{\mu п4}Q_{\mu4} - Z_{\mu п3}Q_{\mu3}, \tag{34}$$

$$\frac{dQ_{\mu4}}{dx} = U_{\mu45}C_{\mu п45} + U_{\mu34}C_{\mu п34}, \tag{35}$$

$$\frac{dU_{\mu45}}{dx} = -\left[Z_{\mu п5}Q_{\mu5} - Z_{\mu п4}Q_{\mu4}\right], \tag{36}$$

$$\frac{dQ_{\mu5}}{dx} = -U_{\mu45}C_{\mu п45}, \tag{37}$$

where f_{in} is the linear value $of\ F_{in}$.

Differentiating equations (31), (33), (35) VA (37) by the $x\ coordinate$ and substituting in them (30), (32), (34), (36) in (38), we obtain the following system of second-order differential equations with constant coefficients:

$$\begin{cases} \frac{d^2U_{\mu12}}{dx^2} = \left(Z_{\mu п1} + Z_{\mu п2}\right)C_{\mu п12}U_{\mu12} + Z_{\mu п3}C_{\mu п23}U_{\mu23}, \\ \frac{d^2U_{\mu23}}{dx^2} = Z_{\mu п2}C_{\mu п12}U_{\mu12} + \left(Z_{\mu п2} + Z_{\mu п3}\right)C_{\mu п23}U_{\mu23} + Z_{\mu п3}C_{\mu п34}U_{\mu34}, \\ \frac{d^2U_{\mu34}}{dx^2} = Z_{\mu п3}C_{\mu п23}U_{\mu23} + \left(Z_{\mu п3} + Z_{\mu п4}\right)C_{\mu п34}U_{\mu34} + Z_{\mu п4}C_{\mu п45}U_{\mu45}, \\ \frac{d^2U_{\mu45}}{dx^2} = Z_{\mu п4}C_{\mu п34}U_{\mu34} + \left(Z_{\mu п4} + Z_{\mu п5}\right)C_{\mu п45}U_{\mu45}. \end{cases} \tag{38}$$

The system of equations (39) is a mathematical model of the investigated multi-loop magnetic circuit with distributed parameters in the form of differential equations and it can be solved using standard computer programs provided in reference books on higher mathematics.

In this article, we will limit ourselves to solving a system of differential equations for a two-circuit magnetic circuit with distributed parameters. For this chain, the system of equations (39) takes the following form:

$$\begin{cases} \frac{d^2U_{\mu12}}{dx^2} = (Z_{\mu п1} + Z_{\mu п2})C_{\mu п12}U_{\mu12} + Z_{\mu п3}C_{\mu п23}U_{\mu23}, \\ \frac{d^2U_{\mu23}}{dx^2} = Z_{\mu п2}C_{\mu п12}U_{\mu12} + (Z_{\mu п2} + Z_{\mu п3})C_{\mu п23}U_{\mu23}. \end{cases} \quad (39)$$

The characteristic equation of system (40) has the following form:

$$\begin{vmatrix} (a_{11} - k^2) & a_{12} \\ a_{21} & (a_{22} - k^2) \end{vmatrix} = 0, \quad (40)$$

here $a_{A11} = (Z_{mkn11} + Z_{mkn2})C_{\mu\text{mkn}12}$; $a_{A12} = Z_{\text{mkn}3}C_{mkn23}$; $a_{21}A21 = Z_{\text{mkn}2}C_{mkn12}$; $a_{A22} = (Z_{mkn2} + Z_{mkn3})C_{\mu\text{mkn}23}$.

Roots of the characteristic equation (41):

$$k_{1,2} = \pm\sqrt{\frac{(a_{11}+a_{22})+\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}} = \pm\gamma_1,$$

$$k_{3,4} = \pm\sqrt{\frac{(a_{11}+a_{22})-\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}} = \pm\gamma_2.$$

The General solution to the system of differential equations (40) is written as:

$$U_{\mu12} = A_1m^{(1)}e^{\gamma_1x} + A_2m^{(2)}e^{-\gamma_1x} + A_3m^{(3)}e^{\gamma_2x} + A_4m^{(4)}e^{-\gamma_2x}, \quad (41)$$

$$U_{\mu23} = A_1n^{(1)}e^{\gamma_1x} + A_2n^{(2)}e^{-\gamma_1x} + A_3n^{(3)}e^{\gamma_2x} + A_4n^{(4)}e^{-\gamma_2x}, \quad (42)$$

here $A_1 \div A_4$ – A1 ÷ A4 are integrationconstants; $m^{(1)} \div m^{(4)}$ BA $n^{(1)} \div n^{(4)\,are\ constant}$ coefficients corresponding to the roots of the characteristic equation ($K1_1 \div k_4 \div K4$).

For the two-circuit magnetic circuit under study, the following condition is met:

$$Q_{\mu1}(x) = Q_{\mu2}(x) + Q_{\mu3}(x). \quad (43)$$

From (44) find $Q_{\mu3}(x)$, substitute it in (33) and together with (31) we obtain the following system of equations:

$$\begin{cases} Z_{\mu п1}Q_{\mu1} + Z_{\mu п2}Q_{\mu2} = \frac{dU_{\mu12}}{dx} + f_в, \\ Z_{\mu п3}Q_{\mu1} - (Z_{\mu п2}+Z_{\mu п3})Q_{\mu2} = -\frac{dU_{\mu23}}{dx}. \end{cases} \quad (44)$$

Solving the system of algebraic equations (45), we find the values $of\ Q_{\mu1\,and}\ Q_{\mu2}$:

$$Q_{\mu1} = -\frac{(Z_{\mu n2}+Z_{\mu n3})}{\Delta_2}\frac{dU_{\mu12}}{dx}\mu12\ DX + \frac{Z_{\mu n2}}{\Delta_2}\frac{dU_{\mu23}}{dx}\mu23\ DX - \frac{(Z_{\mu n2}+Z_{\mu n3\,\Delta\,2\,f})}{\Delta_2}f_{\text{in}}. \quad (45)$$

$$Q_{\mu2} = -\frac{Z_{\mu n3}}{\Delta_2}\frac{dU_{\mu12}}{dx} - \frac{Z_{\mu n1}}{\Delta_2}\frac{dU_{\mu23}}{dx} - \frac{Z_{\mu n3}}{\Delta_2}f_{\text{in}}, \quad (46)$$

here $\Delta_2 = -(Z_{\mu n11}Z_{\mu n2} + Z_{\mu n1}Z_{\mu n3} + Z_{\mu n2}Z_{\mu n3})$.

$The\ q\ values_{of\ \mu3}$ are based on (44):

$$Q_{\mu3} = Q_{\mu1} - Q_{\mu2} = -\frac{Z_{\mu n2}}{\Delta_2}\frac{dU_{\mu12}}{dx}\mu12\ DX + \frac{(Z_{\mu n1}+Z_{\mu n2})}{\Delta_2}\frac{dU_{\mu23}}{dx}\mu23\ DX - \frac{Z_{\mu n2\,\Delta\,2\,f}}{\Delta_2}f_{\text{in}}. \quad (47)$$

Differentiating (42) and (43) by x and substituting them in (46), (47) and (48), we obtain:

$$Q_{\mu1} = \left[\frac{(Z_{\mu п2}n^{(1)}-(Z_{\mu п2}+Z_{\mu п3})m^{(1)}}{\Delta_2}\right]A_1\gamma_1e^{\gamma_1x} + \left[\frac{((Z_{\mu п2}+Z_{\mu п3})m^{(2)}-Z_{\mu п2}n^{(2)}}{\Delta_2}\right]A_2\gamma_1e^{-\gamma_1x} + \\ + \left[\frac{(Z_{\mu п2}n^{(3)}-(Z_{\mu п2}+Z_{\mu п3})m^{(3)}}{\Delta_2}\right]A_3\gamma_2e^{\gamma_2x} + \left[\frac{((Z_{\mu п2}+Z_{\mu п3})m^{(4)}-Z_{\mu п2}n^{(4)}}{\Delta_2}\right]A_4\gamma_2e^{-\gamma_2x} - \\ \frac{(Z_{\mu п2}+Z_{\mu п3})}{\Delta_2}f_в. \quad (48)$$

$$Q_{\mu2} = -\left[\frac{Z_{\mu п1}n^{(1)} + Z_{\mu п3}m^{(1)}}{\Delta_2}\right]A_1\gamma_1e^{\gamma_1x} + \left[\frac{Z_{\mu п1}n^{(2)} + Z_{\mu п3}m^{(2)}}{\Delta_2}\right]A_2\gamma_1e^{-\gamma_1x} - \\ -\left[\frac{Z_{\mu п1}n^{(3)}+Z_{\mu п3}m^{(3)}}{\Delta_2}\right]A_3\gamma_2e^{\gamma_2x} + \left[\frac{Z_{\mu п1}n^{(4)}+Z_{\mu п3}m^{(4)}}{\Delta_2}\right]A_4\gamma_2e^{-\gamma_2x} - \frac{Z_{\mu п3}}{\Delta_2}f_в. \quad (49)$$

$$Q_{\mu3} = \left[\frac{((Z_{\mu п1}+Z_{\mu п2})n^{(1)}-Z_{\mu п2}m^{(1)}}{\Delta_2}\right]A_1\gamma_1 e^{\gamma_1 x} + \left[\frac{(Z_{\mu п2}m^{(2)}-(Z_{\mu п1}+Z_{\mu п2})n^{(2)}}{\Delta_2}\right]A_2\gamma_1 e^{-\gamma_1 x} +$$
$$+\left[\frac{((Z_{\mu п1}+Z_{\mu п2})n^{(3)}-Z_{\mu п2}m^{(3)}}{\Delta_2}\right]A_3\gamma_2 e^{\gamma_2 x} + \left[\frac{(Z_{\mu п2}m^{(4)}-(Z_{\mu п1}+Z_{\mu п2})n^{(4)}}{\Delta_2}\right]A_4\gamma_2 e^{-\gamma_2 x} - \frac{Z_{\mu п2}}{\Delta_2}f_{в}. \quad (50)$$

In order to simplify the analysis of the magnetic circuit under consideration, we accept the following conditions:

$$Z_{mkn1} = Z_{mkn2} = Z_{mkn3} = Z_{MKN};\ C_{mkn12п12} = C_{mkn23п23} = C_{\mu MKN}. \quad (51)$$

should be noted that for most EFCs with a ring channel, conditions (52) are satisfied.

Taking into account (52), the roots of the characteristic equation (41) take the following form:

$$k_{1,2} = \pm\gamma_1 = \sqrt{3Z_{\mu n}C_{\mu nп}};\quad k_{3,4} = \pm\gamma_2 = \sqrt{Z_{\mu n}C_{\mu nп}}. \quad (52)$$

The values of the coefficients $m^{(1)} \div m^{(4)}$ and $n^{(1)} \div n^{(4)\ are}$ determined by sequentially substituting the roots $Ki_{(}i = 1 \div 4)$ into the following system of equations and solving it with respect to mi , Ni$m^{(i)}$, $n^{(i)}$:

$$\begin{cases}(a_{11} - k_i^2)m^{(i)} + a_{12}n^{(i)} = 0,\\ a_{21}m^{(i)} + (a_{22} - k_i^2)n^{(i)} = 0,\end{cases} \quad (53)$$

$$m^{(1)} = 1;\quad m^{(2)} = 1;\quad m^{(3)} = 1;\quad m^{(4)} = 1; \quad (54)$$

$$n^{(1)} = 1;\quad n^{(2)} = 1;\quad n^{(3)} = -1;\quad n^{(4)} = -1. \quad (55)$$

Substituting (55) and (56) in (42) and (43), respectively, we obtain the following expressions for the magnetic stresses:

$$U_{\mu12} = A_1e^{\gamma_1 x} + A_2e^{-\gamma_1 x} + A_3e^{\gamma_2 x} + A_4e^{-\gamma_2 x}, \quad (56)$$

$$U_{\mu23} = A_1e^{\gamma_1 x} + A_2e^{-\gamma_1 x} - A_3e^{\gamma_2 x} - A_4e^{-\gamma_2 x}. \quad (57)$$

Taking into account (52), (55) and (56) equations (49), (50) and (51) takes the following form:

$$Q_{\mu1} = \frac{\gamma_1}{3Z_{\mu n}}A_1A1\ e^{\gamma_1 x} - \frac{\gamma_1}{3Z_{\mu n}}A_{A2}e^{-\gamma_1 x} + \frac{\gamma_2}{Z_{\mu п}}A_{\mu n\ A3}e^{\gamma_2 x} - \frac{\gamma_2}{Z_{\mu n}}A_{A4}e^{-\gamma_2 x} + \frac{2}{3Z_{\mu n}}f_{\text{in}}, \quad (58)$$

$$Q_{\mu2} = \frac{2\gamma_1}{3Z_{\mu n}}A_1A1\ e^{\gamma_1 x} - \frac{2\gamma_1}{3Z_{\mu n}}A_{A2}e^{-\gamma_1 x} + \frac{1}{3Z_{\mu n}}f_{\text{in}}, \quad (59)$$

$$Q_{\mu3} = -\frac{\gamma_1}{3Z_{\mu n}}A_1A1\ e^{\gamma_1 x} + \frac{\gamma_1}{3Z_{\mu n}}A_{A2}e^{-\gamma_1 x} + \frac{\gamma_2}{Z_{\mu п}}A_{\mu n\ A3}e^{\gamma_2 x} - \frac{\gamma_2}{Z_{\mu n}}A_{A4}e^{-\gamma_2 x} + \frac{1}{3Z_{\mu n}}f_{\text{in}}. \quad (60)$$

The integration constants $A_{A1} \div A_{A4}$ are determined using the following boundary conditions: (Figure 4):

$$U_{\mu23}(x)|_{x=0} = Q_{\mu s}W_{\mu s} - Q_{\mu 3т}Z_{\mu de};\qquad Q_{\mu3}(x)|_{x=x_m} = 0; \quad (61)$$

$$U_{\mu12}(x)|_{x=0} = Q_{\mu s}W_{\mu s} + Q_{\mu 1т}Z_{\mu cd};\ U_{\mu12}(x)|_{x=X_м} = -Q_{\mu1}(x)|_{x=X_м}Z_{\mu ab}; \quad (62)$$

here $Q_{\mu 1т} = Q_{\mu1}(x)|_{x=X_m}$; $Q_{\mu s} = Q_{\mu2}(x)|_{x=0}$; $Q_{\mu 3т} = Q_{\mu3}(x)|_{x=0}$; $Z_{\mu ab}$, $Z_{\mu cd}$ and $Z_{\mu de}$ – magnetic resistance of the magnetic circuit sections “ab”, “cd” and “de”, respectively. If condition (52) is met we can assume $Z_{\mu cd} = Z_{\mu de} = Z_{\mu t}$.

4 Conclusion

Expressions are mathematical models of a two-circuit EFC magnetic circuit with ring channels, taking into account the distribution of parameters in both circuits of the circuit.

Thus, the paper develops mathematical models of single-and multi-circuit EFC magnetic circuits with distributed parameters. They can be used to determine the design parameters of magnetic circuits and study the static and dynamic characteristics of EFC with ring channels.

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