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Mathematical modeling of vertical axis wind turbines

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Abstract. The article talks about the growing demand for electricity in the Republic of Uzbekistan and about wind power devices that cover this need, as well as about some parameters that affect their efficiency. In addition, the NACA 4412 airfoil used in the manufacture of turbine blades and the proposed airfoil were analyzed. The parameters influencing the efficiency of vertical axis wind turbines have been studied. New device's a mathematical model is proposed.

1 Introduction

Frequent changes in wind speed in the region, shortening of the service life of wind turbines for consumers in desert areas (temperature +60 ^{0}C in summer, -20 ^{0}C in winter), dustiness of these territories (Due to the increase in mechanical losses in the supporting parts due to excess of normal 0,5 mg/m^3) there are problems with the supply of such consumers through wind turbines. Such problems do not allow the effective use of standard wind turbines in these areas. In addition, ensuring the excitation of turbines in low-speed wind flows remains an urgent problem [1]. Therefore, reducing the weight of turbines is an urgent issue. But solving these problems is not enough to introduce windmills in these areas. Therefore, research and development of vertical-axis wind turbines with improved technical parameters adapted to climatic conditions, acquire relevance. As you know, the preliminary determination of the real characteristics of ongoing projects in the field of mechanical engineering directly depends on the modeling of the object of study. Proper modeling of the designed object will save time, labor, raw materials and money. For this reason, when conducting research on the creation of a wind power device of an optimal design, the relevance of mathematical modeling of a wind power device with a vertical axis was initially determined.

2 Materials and methods

The torques that affect the rotation of a wind turbine are shown in Figure 1 below.

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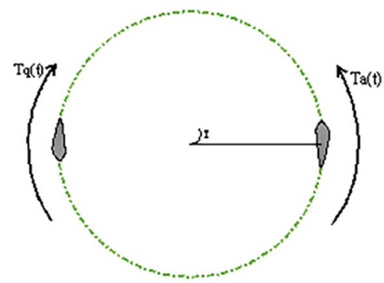

Fig. 1. Top (front) view of the turbine section.

$$T_a(t) - T_q(t) = J\theta(t) \tag{1}$$

Ta(t) - moment of aerodynamic driving force, Tq(t) - moment of drag force, J is the total inertial moment of the turbine, θ - the angle of installation of the blade relative to the turbine.

The result of the equality given in expression (1) above should reach its maximum. For this, the moment Ta(t) must arrive at its maximum value, and Tq(t) must get to its minimum value. In this case, the moment of inertia reaches its maximum, as a result of which the turbine has a proportional amount of kinetic energy. If we analyze the forces and moments acting on the turbine starting from the power, we take the equation 2.

We can evaluate the physical processes in the turbine through analytical analysis. As you know, the power of a wind power device is calculated as follows: [2, 3]

$$P = \frac{1}{2} \cdot \rho \cdot A \cdot V^3 \cdot C_p \tag{2}$$

Here: The wind turbine's mechanical power (3) is determined by the equation [4].

$$P = \omega \cdot T \tag{3}$$

But the power indicated in equation (2) is calculated relative to the Betz coefficient. According to Betz, this coefficient $C_p \leq \frac{16}{27}$ must satisfy equality. The Betz theory was developed on the example of a wind turbine which have a horizontal axis, according to [4-6] the use of this coefficient for a vertical axis wind turbine can cause inaccuracies in the calculations. There are many types of vertical axis turbines [7], so the power factor in such turbines should be determined based on experimental results [8]. In addition, the efficiency of the electric generator (η) also impact of the output power of the wind farm [9].

$$P = \frac{1}{2} \cdot \rho \cdot A \cdot V^3 \cdot C_p \cdot \eta \tag{4}$$

(2) and (4) are given in the equations A is the surface of the turbine and is calculated by expression (5) [10,11].

$$A = 2 \cdot R \cdot h \tag{5}$$

tip speed ratio (TSR) λ, power coefficient C_p is closely related to [12]. As shown in Equation (6) λ is determined by the ratio of the angular velocity to the wind speed [13,14].

$$\lambda = \frac{\omega \cdot R}{V_{Sh}} \tag{6}$$

The area of motion of the wind turbine blade is equal to the height of the turbine (h) and radius (R) is characterized by the larger the radius of the turbine, the higher the generated torque, but the larger the radius of the turbine, the less stable the design. Turbine height (h), radius value (R), long compared to, the generated torque is small, requiring an increase in the turbine speed. The aspect ratio is calculated as follows:

$$AR = \frac{h}{2 \cdot R}; \Rightarrow h = 2 \cdot AR \cdot R \tag{7}$$

Solidity (σ) is an important quantity affecting the acting of wind turbines. Solidity of the total area of the turbine blade ($N \cdot c$) is determined by the ratio to the length of the wind turbine [15, 16].

$$\sigma = \frac{N \cdot c}{2 \cdot \pi \cdot R} \tag{8}$$

In the figure below, we analyze the velocity vectors and forces acting on the turbine blades made using the proposed aerodynamic surface, and consider their equations.

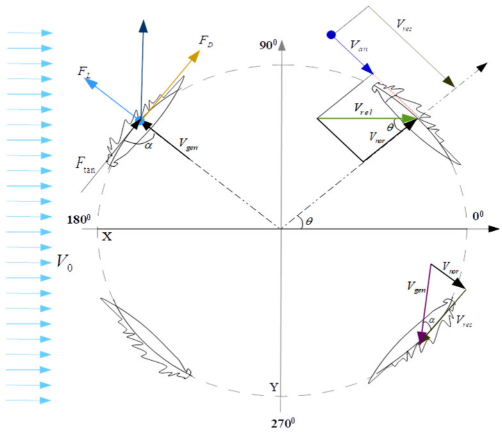

Fig. 2. Representation of parameters affecting the blade of a wind turbine, in vector values.

In this wind V_{an} - angular velocity; V_0 - to the turbine; V_{rel} - relative; V_{gen} - general; V_{nor} - normal; V_{res} - resulting speeds, F_L - lift and F_D - drag forces.

The angular velocity of the turbine is determined as follows:

$$V_{an} = \omega \cdot R = \frac{2\pi \cdot n \cdot R}{60} = \frac{\pi \cdot n \cdot R}{30} \tag{9}$$

In that n - turbine rotation speed, [rpm]

And the resulting speed:

$$V_{res} = V_{an} + V_{rel} \cdot \sin\theta = \omega \cdot R + V_{rel} \cdot \cos\theta = \lambda \cdot V_{rel} + V_{rel} \cdot \cos\theta = V_{rel}(\lambda + \cos\theta) \tag{10}$$

relative speed quantification $V_{rel} = (V_0 + V_2)/2$

In that V_2 - wind speed after (leaking) the turbine.

The normal speed acting on the wind turbine blades is given by equation (11).

$$V_{nor} = V_{rel} \cdot \sin\theta \tag{11}$$

The total speed acting on the wind turbine is:

$$V_{gen} = \sqrt{V_{nor}^2 + V_{res}^2} = \sqrt{(V_{rel} \cdot \sin\theta)^2 + (V_{rel}(\lambda + \cos\theta))^2} = V_{rel}\sqrt{1 + 2 \cdot \lambda \cdot \cos\theta + \lambda^2} \tag{12}$$

Wind angle α and (13) is defined by the expression: [8, 17, 18]

$$\alpha = \tan^{-1}(\frac{V_{nor}}{V_{res}}) = \tan^{-1}(\frac{V_{rel} \cdot \sin\theta}{V_{rel}(\lambda + \cos\theta)}) = \tan^{-1}\left(\frac{\sin\theta}{\lambda + \cos\theta}\right) \tag{13}$$

Coefficient of tangential force when calculating the mechanical power of the turbine $C_{\tan}$ and the normal coefficient C_n, and to determine these values it is required C_L, C_D coefficients like these coefficients are determined by NACA special airfoil programs. For example, NACA 4412 for airfoil. $\alpha = 3^0$ V C_L, C_D The values are 0.5279, 0.04426 respectively, usually C_L, C_D chances are average 3^0 given regarding. We calculate the tangential and normal coefficients as follows:

$$C_{\tan} = C_L \cdot \sin\alpha - C_D \cdot \cos\alpha \quad (14)$$

$$C_n = C_L \cdot \sin\alpha + C_D \cdot \cos\alpha \quad (15)$$

Tangential force $F_{\tan}$ and normal force F_n is calculated according to the following expression:

$$F_{\tan} = \frac{1}{2} \cdot C_{\tan} \cdot \rho \cdot S_{blade} \cdot V_{gen}^2 \quad (16)$$

$$F_n = \frac{1}{2} \cdot C_n \cdot \rho \cdot S_{blade} \cdot V_{gen}^2 \quad (17)$$

In that S_{blade} - surface affecting the wind on the upper surface of the blade (Figure 3), in scientific research this value is approximately $S_{blade} = c \cdot h$ is set equal to c - chord of blade $c \cdot h$ - is not equal to the actual wind surface, to find the actual wind surface, the length of the curvature of the upper surface must be multiplied by the height of the wing: i.e. $S_{blade} = c^1 \cdot h$, in which c^1 - length of the upper surface, h -blade length. S_{blade} - a large amount according to expression (14). $F_{\tan}$ causing your power to increase. An growth in the tangential force causes an rise in the magnitude of the turbine torque, so it was concluded that an rise in the surface area of the turbine blade exposed to the wind would be effective.

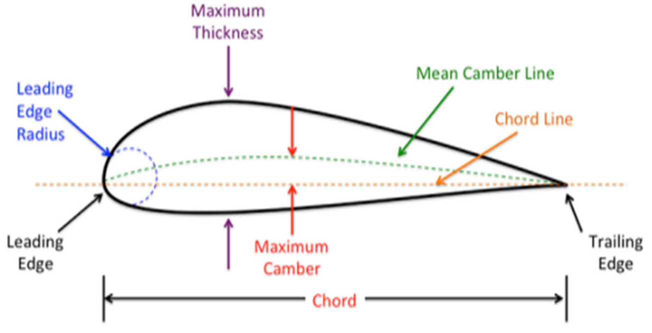

Fig. 3. Parts of the NACA 4412 airfoil.

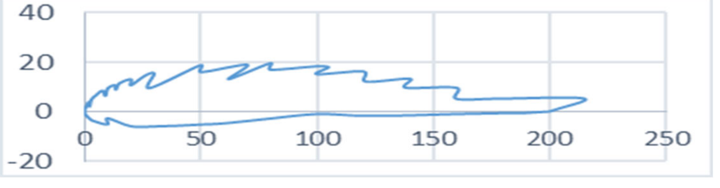

Fig. 4. Curved outside of the NACA 4412 airfoil.

The upper surface of the airfoil in Figure 4 is larger than the standard NACA 4412 airfoil, the surface of this shape is filled with special curves that work effectively with the wind flow. These curves lead, firstly, to an increase in the surface area, and secondly, to an improvement in the interaction of flows.

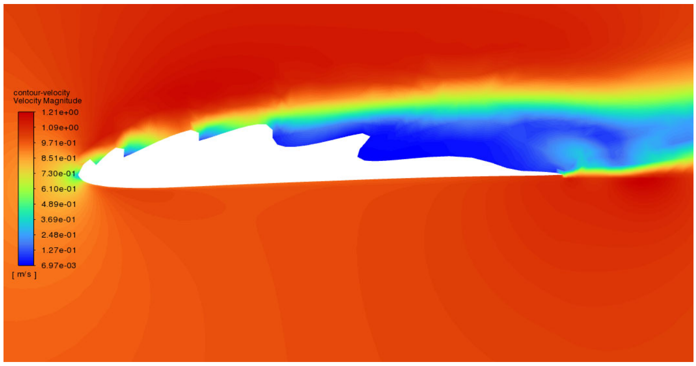

Fig. 5. Velocity characteristics of an uneven surface in the ANSYS program.

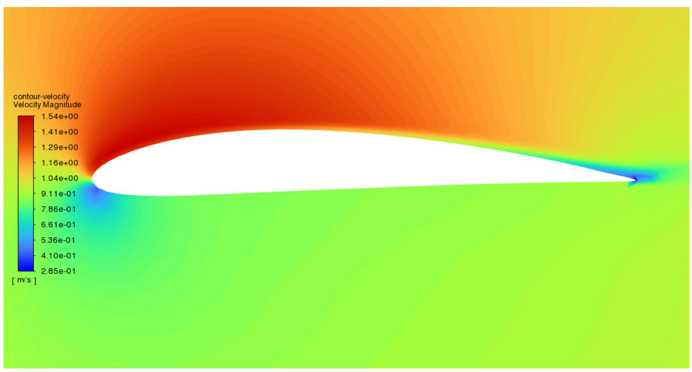

Fig. 6. Velocity characteristics of the NACA4412 surface in the ANSYS program.

In Figures 5 and 6, the proposed and standard airfoils are compared in ANSYS. The efficiency of vertical axis wind turbines is directly related to the width of the blue area.

3 Results and discussion

Continuing the above analysis, the value of the average tangential force $F_{\tan}$ ga and the angle of rotation of the turbine blade θ depends on the value as follows [19]:

$$F_{ta.av} = \frac{1}{2 \cdot \pi} \int_0^{2\pi} F_{\tan}(\theta) d\theta \tag{18}$$

Expanding expression (19), we obtain the following generalized equality

$$T = n \cdot F_{ta.av} \cdot R = \frac{2 \cdot \rho \cdot c^1 \cdot h \cdot V_{gen}^2 \cdot R}{\pi} \cdot \left(C_K \cdot \int_0^{2\pi} \sin arctg\,(\frac{\sin\theta}{\lambda + \cos\theta})(\theta) - C_T \cdot \int_0^{2\pi} \cos arctg\,(\frac{\sin\theta}{\lambda + \cos\theta})(\theta) \right) \tag{19}$$

Here c^1 - The outer surface of the NACA 4412 airfoil is filled with special curves. c^1 The value is determined by filling in the square. According to calculations, the length of the outer surface of the NACA 4412 airfoil is 0.1 m, and proposed surface is 0.109 m [20].

Summing up the obtained expressions and introducing them into the Matlab program, we obtain the following form of the model in Figure 7.

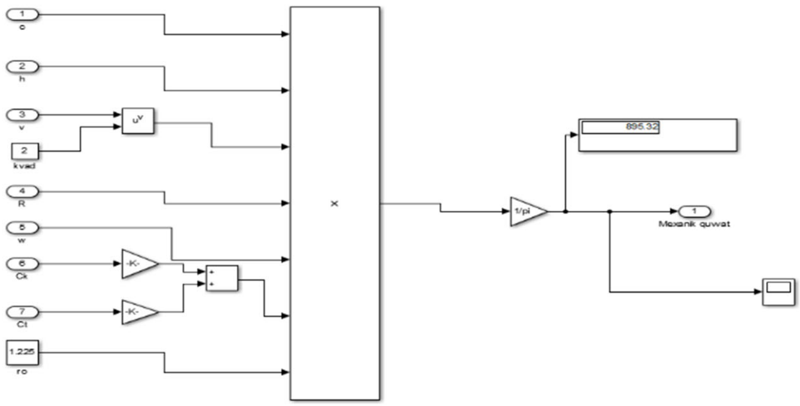

Fig. 7. Model of a wind power plant in the Matlab application package.

In Matlab, the actual dimensions of the turbine are taken into account as quantities when calculating power, for example $c = 30\ \ sm$, $h = 2\ \ m$, $R = 0{,}7\ \ m$ calculated for amounts such as.

4 Conclusion

The expressions above, in some cases, i.e. the angle of the wind flow α depends on It is given in expressions (14 and 15). C_L And C_D lift and drag coefficients α varies depending on These coefficients can be determined using the Xfoil and Ansys software for the NACA4412 airfoil. In addition, the angle of wind arrival through the base of the instrument airfoils α , moment coefficient C_m And C_L , C_D we can get graphical relationships between the values Using these graphs, the wind flow for different angles C_m And C_L , C_D We determine the coefficient at, and according to these graphs, the sum of the Reynolds number C_L true C_D and conclude that it is inversely proportional to .

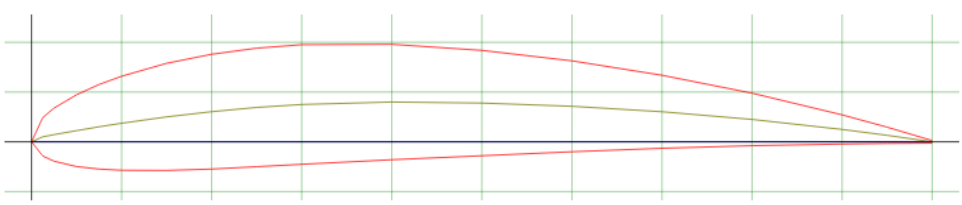

Fig. 8. Overview of the NACA4412 airfoil.

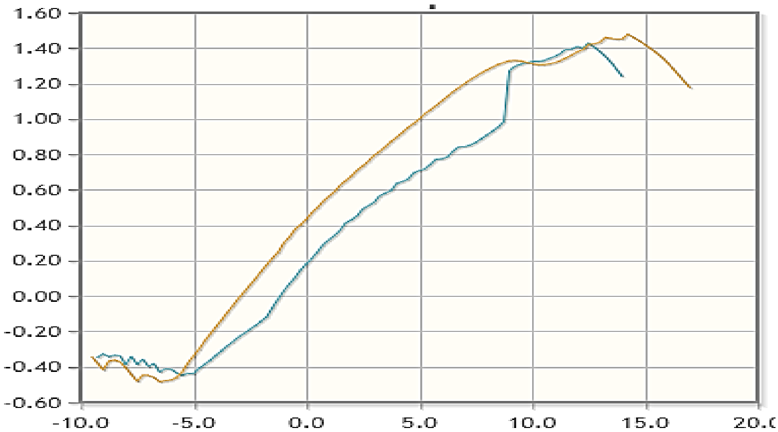

Fig. 9. Relation between C_L and α .

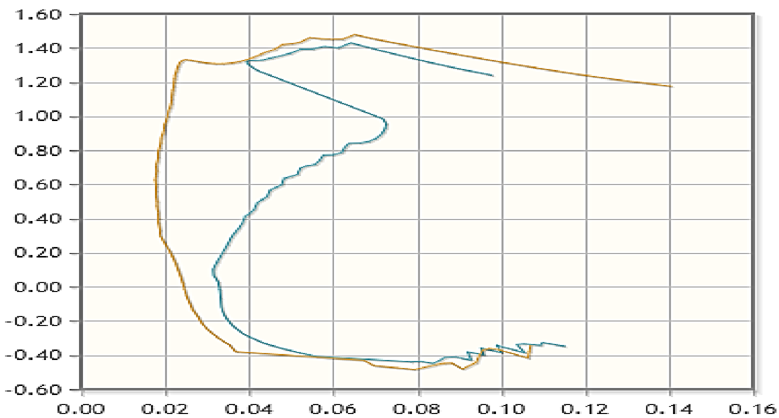

Fig. 10. Relation between C_L and C_D .

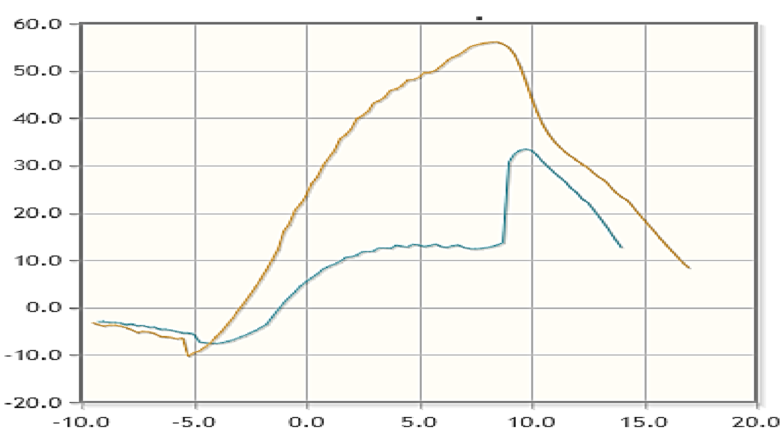

Fig. 11. Relation between C_L / C_D and α .

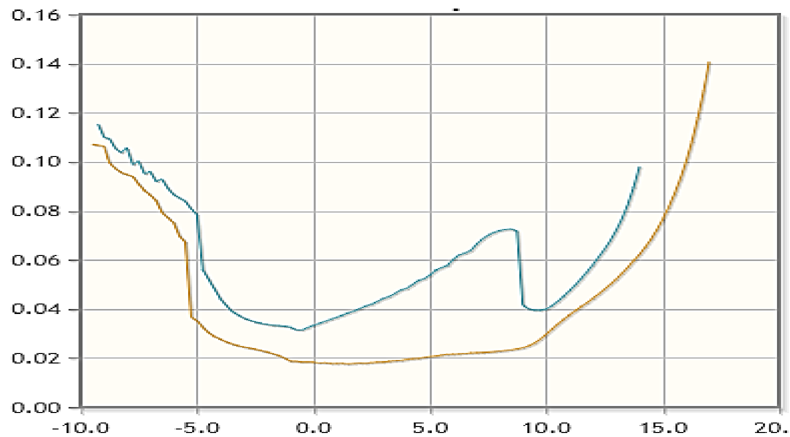

Fig. 12. Relationship between C_D and α .

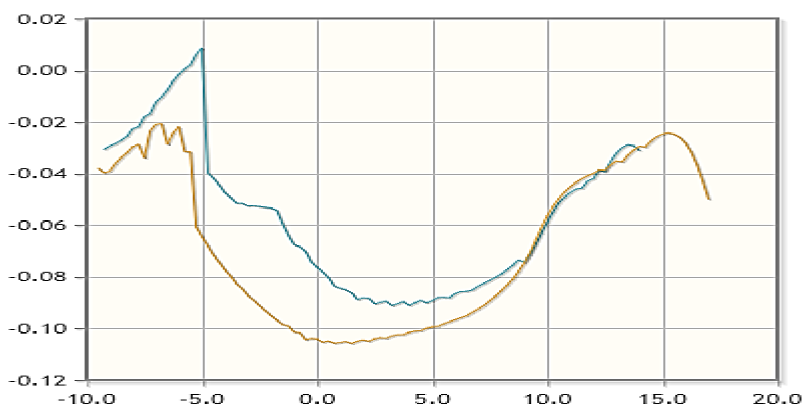

Fig. 13. Relationship between C_m and α .

The blue line in the graphs in Figure 8 is the change in the Reynolds number. $R_e = 0{,}5 \cdot 10^6$, and the yellow line $R_e = 10^6$ obtained for values equal to The reason for the unusual shape of these graphs is that when the wing moves, it constantly changes its stereometric position, the angle of the flow acting on it continuously changes, as a result of which the values of lift and drag coefficients change dramatically, so the characteristics are such as mentioned above [21]. For NACA 4412 $R_e = 0{,}5 \cdot 10^6$ and $R_e = 10^6$ the characteristics associated with can be said to be relatively parallel to each other. From Figures 9 10, 11, 12, 13 we can conclude that the Reynolds number is directly proportional to the lift coefficient, and from figures d, e we can conclude that it is inversely proportional to the drag and moment coefficients.

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