

# On the problem of calculating steady state modes of electric power systems under conditions of interval data uncertainty

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**Abstract.** Currently, there are several approaches for dealing with uncertain (inaccurate) data. The main ones are methods of probabilistic analysis, fuzzy set theory and interval analysis. In this paper, we consider the problem of calculating the parameters of steady-state modes of electric power systems with interval uncertainty of the initial data. The main reasons and the relevance of the use of interval methods for calculating the parameters of steady-state modes of networks of electrical systems are described. First, an interval calculation model is formulated, and then some interval iterative methods for solving nonlinear nodal equations of electrical networks are studied. Algorithms for the interval methods of Gauss-Seidel and Newton-Raphson are proposed for solving nonlinear nodal equations of steady state electrical networks. To demonstrate the level of efficiency of the developed algorithms, several test calculations were carried out with interval parameters formed through the middle and radius of the interval. The results of numerical calculations using these methods show that the Newton-Raphson method is superior to the Gauss-Seidel method in terms of the number of iterations and optimality (with a smaller width) of interval solutions.

## 1 Introduction

The steady-state mode of the electrical network is understood as such a normal or post-emergency mode in which currents, voltages and powers in its elements are taken unchanged. The calculation of the steady state implies the determination of these currents, voltages and powers that characterize the mode of the electrical network are called mode parameters.

Calculations of steady-state conditions are the main task in the management and analysis of the functioning of electric power systems (EPS). Most of the existing calculation methods are based on a deterministic representation of the initial data [1-3]. Indeed, these methods do not describe the real situation, i.e. when using these methods, we will make certain assumptions. Most of the parameters for calculating the steady-state modes of the EPS have

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an uncertainty limited from below and from above, i.e. the values of these parameters fluctuate at a known amplitude, in other words, they are non-deterministic.

The problem of non-deterministic data occurs in solving a number of theoretical and practical problems. This problem is especially prominent when modeling electrical networks [4-9].

Recently, when solving problems with non-deterministic data, methods of interval analysis are increasingly used. These methods make it possible to naturally separate deterministic parameters from non-deterministic ones already at the stage of mathematical model synthesis. They operate with non-deterministic quantities, namely, interval quantities without any approximation within a certain structure and on the basis of a suitable calculus. As a consequence, the results themselves are obtained in the form of intervals, which, as a rule, has quite meaningful interpretations in the problems under study. In this regard, the problem of synthesizing mathematical models for the problem of calculating steady-state modes of EES, with a logically justified interval of a specific value, as well as the development and justification of the corresponding interval algorithms and software, is very relevant. Since the model should be resolved within the interval approach. In the mathematical modeling of complex systems and processes with non-deterministic parameters, probabilistic-statistical methods are traditionally used. Recently, due to the relative limitations and observed, in some cases, inadequacy of probabilistic-statistical models, researchers are turning to the methods of fuzzy set theory and methods of interval analysis, since these theories suggest the possibility of implementing a full cycle of computational experiments within the framework of the corresponding calculus. Of course, each of these approaches has its own advantages and disadvantages.

The advantages of interval methods include such facts as the relative similarity with the real case of the formalization of the verbally posed problem, the ability to save in some cases the prescriptions of the components of interval algorithms, as well as the applicability of software for solving purely real problems, which allows this approach to automatically take into account rounding errors. In general, interval algorithms allow, within the framework of a unified calculation of intervals, to take into account all kinds of errors and obtain solutions to problems in the form of interval objects that are guaranteed to contain the desired exact solutions.

## 2 Interval notation and Interval arithmetic

An informal draft of the international standard [10], developed by leading specialists in the field of interval analysis, was adopted to designate interval values. According to this project, in order to distinguish from point (non-interval) values, interval values are highlighted in the text in bold italic type.

An interval number (or sometimes called a real interval) is defined as

$$\mathbf{a} = [\underline{a}, \bar{a}] = \{x \in R \mid \underline{a} \leq x \leq \bar{a}\}, \quad (1)$$

and the set of interval numbers for classical interval arithmetic is denoted as  $IR$ . In (1) the real numbers  $\underline{a}$  and  $\bar{a}$  are respectively the lower and upper bounds of the interval  $\mathbf{a}$ .

The main characteristic functions of the real interval are defined as follows:

- The interval width is defined as  $\text{wid } \mathbf{a} = \bar{a} - \underline{a}$ ;
- The interval radius is calculated as  $\text{rad } \mathbf{a} = \frac{1}{2}(\bar{a} - \underline{a})$ ;
- The middle of the interval is defined as  $\text{mid } \mathbf{a} = \frac{1}{2}(\underline{a} + \bar{a})$ ;

- The absolute value (modulus or magnitude) of an interval number is defined as  $|a| = \max \{ |a| \mid a \in \mathbf{a} \} = \max \{ |\underline{a}|, |\bar{a}| \}$ .

We can represent any interval as  $\mathbf{a} = \text{mid } \mathbf{a} + [-1, 1] \cdot \text{rad } \mathbf{a}$ , which is equivalent to

$$\mathbf{a} = \{x \in R \mid |x - \text{mid } \mathbf{a}| \leq \text{rad } \mathbf{a}\}. \quad (2)$$

It is known that in the calculations of the steady-state modes of the EPS, the design parameters, in the general case, take on complex values. In interval analysis, rectangular and circular complex intervals are most often used as complex intervals [11, 12]. The corresponding sets are denoted by  $IC_{rect}$  and  $IC_{circ}$ . Further, we will consider intervals only from  $IC_{rect}$ , since for brevity we denote this set simply by  $IC$ :

$$\mathbf{a} = \mathbf{a}_1 + i \mathbf{a}_2 = \{a = a_1 + ia_2 \in C \mid a_1 \in \mathbf{a}_1, a_2 \in \mathbf{a}_2\}$$

for real intervals  $\mathbf{a}_1, \mathbf{a}_2 \in IR$ .

Similarly to the real case, we introduce the corresponding arithmetic operations and functions for complex intervals:

$$\begin{cases} (\mathbf{a}_1 + i \mathbf{b}_1) + (\mathbf{a}_2 + i \mathbf{b}_2) = (\mathbf{a}_1 + \mathbf{a}_2) + i(\mathbf{b}_1 + \mathbf{b}_2) \\ (\mathbf{a}_1 + i \mathbf{b}_1) - (\mathbf{a}_2 + i \mathbf{b}_2) = (\mathbf{a}_1 - \mathbf{a}_2) + i(\mathbf{b}_1 - \mathbf{b}_2) \\ (\mathbf{a}_1 + i \mathbf{b}_1) \cdot (\mathbf{a}_2 + i \mathbf{b}_2) = (\mathbf{a}_1 \mathbf{a}_2 - \mathbf{b}_1 \mathbf{b}_2) + i(\mathbf{a}_1 \mathbf{b}_2 + \mathbf{a}_2 \mathbf{b}_1) \\ (\mathbf{a}_1 + i \mathbf{b}_1) / (\mathbf{a}_2 + i \mathbf{b}_2) = \frac{1}{\mathbf{a}_2^2 + \mathbf{b}_2^2} ((\mathbf{a}_1 \mathbf{a}_2 + \mathbf{b}_1 \mathbf{b}_2) + i(\mathbf{a}_2 \mathbf{b}_1 - \mathbf{a}_1 \mathbf{b}_2)), \quad 0 \notin (\mathbf{a}_2 + i \mathbf{b}_2) \end{cases}$$

Definition 1. Let  $\mathbf{a} = \mathbf{a}_1 + i \mathbf{a}_2 \in IC$ . Then the quantity  $|\mathbf{a}| = \sqrt{|\mathbf{a}_1|^2 + |\mathbf{a}_2|^2}$  is called the absolute value or the modulus of the complex interval  $\mathbf{a}$ .

Definition 2. Let  $\mathbf{a} = \mathbf{a}_1 + i \mathbf{a}_2 \in IC$ . Then the radius of the complex interval  $\mathbf{a}$  is the quantity  $\text{rad } \mathbf{a} = \text{rad } \mathbf{a}_1 + \text{rad } \mathbf{a}_2$ .

Definition 3. Let  $\mathbf{a} = \mathbf{a}_1 + i \mathbf{a}_2 \in IC$ . Then the width of the complex interval  $\mathbf{a}$  is the value  $\text{wid } \mathbf{a} = \text{wid } \mathbf{a}_1 + \text{wid } \mathbf{a}_2$ .

Let us introduce the Hausdorff metric [5] in the space  $IC^n$ .

Definition 4. Let  $\mathbf{a} = [a_1, a_2], \mathbf{b} = [b_1, b_2] \in IC$ . Then the distance between elements  $\mathbf{a}$  and  $\mathbf{b}$  is entered as follows:

$$\text{dist}(\mathbf{a}, \mathbf{b}) := \max \{ |a_1 - b_1|, |a_2 - b_2| \}.$$

Definition 5. Let  $\mathbf{x} = [x_1, x_2], \mathbf{y} = [y_1, y_2] \in IR^n$ . Then the metric on the multidimensional interval space for the vectors  $\mathbf{x}$  and  $\mathbf{y}$  is defined as:

$$\text{dist}(\mathbf{x}, \mathbf{y}) := \max \{ \|x_1 - y_1\|, \|x_2 - y_2\| \},$$

where  $\|\cdot\|$  is the absolute vector norm in  $R^n$ .

Definition 6. Let  $\mathbf{x} = \mathbf{x}_1 + i \mathbf{x}_2, \mathbf{y} = \mathbf{y}_1 + i \mathbf{y}_2 \in IC^n$ . Then the metric on the space  $IC^n$  for the vectors  $\mathbf{x}$  and  $\mathbf{y}$  is defined by the relation:

$$\text{dist}(\mathbf{x}, \mathbf{y}) := \text{dist}(x_1, y_1) + \text{dist}(x_2, y_2).$$

To get acquainted with the basics of interval analysis, you can refer to the site [13] called "Interval analysis and its applications", developed by the professor of Novosibirsk State University S.P. Shary. This website contains extensive resources and hyperlinks on various aspects of interval analysis and its applications.

### 3 Mathematical models for calculating steady-state modes of EPS

#### 3.1 Real model

The problem of steady state modes of electric power systems is reduced to solving a system of nonlinear algebraic equations with complex coefficients [1, 2]. They are usually called the equation of state in the form of a balance of currents connecting currents and voltages in the nodes of an electrical network:

$$Y_{ij}\dot{U}_i = Y_{i0}\dot{U}_0 - \dot{I}_i, \quad i = 1, \dots, n, \quad (3)$$

where  $Y_{ij}$  is a matrix of mutual conductivities,  $\dot{U}_i$  are voltages and  $\dot{I}_i$  are currents of nodes,  $Y_{i0}$  is a matrix-column of conductivities of communication branches of a balancing node with other nodes, and  $\dot{U}_0$  is a voltage of a balancing node.

However, in real calculations of modes in EPS nodes, instead of currents, the corresponding powers  $\hat{S}_i$  of generators and loads are used, determined with active  $P_i$  and reactive powers  $Q_i$ :

$$\hat{S}_i = P_i + jQ_i, \quad i = 1, \dots, n.$$

According to formula  $\hat{S} = \dot{U}\hat{I}$ , we have

$$\hat{I}_i = \frac{\hat{S}_i}{\hat{U}_i}, \quad i = 1, \dots, n, \quad (4)$$

where  $\hat{I}$ ,  $\hat{S}$ ,  $\hat{U}$  are complex conjugate numbers for complex-valued parameters  $I$ ,  $S$ ,  $U$ , respectively. Then equation (3) has the following form:

$$Y_{ij}\dot{U}_i = Y_{i0}\dot{U}_0 - \frac{\hat{S}_i}{\hat{U}_i}, \quad i = 1, \dots, n. \quad (5)$$

System (5) is a direct form for describing nonlinear nodal equations; there is also an inverted form, i.e. the nonlinear system of nodal equations are described using the matrix of nodal resistances, which is inverse to the matrix of nodal conductivities  $Z = Y^{-1}$ :

$$\dot{U}_i = A_{i0}\dot{U}_0 - Z_{ij}\frac{\hat{S}_i}{\hat{U}_i}, \quad i = 1, \dots, n. \quad (6)$$

The main difference between these two forms of writing (5) and (6) is related to the features of the matrices  $Y$  and  $Z$ , i.e. the  $Y$  matrix is sparse, it has a large number of zero elements, and in the  $Z$  matrix, almost all elements can be non-zero. The advantages of form (5) are explained by the fact that this case makes it possible to save computer memory and it is possible to develop the most efficient calculation algorithms that take into account the sparseness of the matrix, since this leads to a reduction in the time complexity of calculations. In case (6), the inverse matrix  $Z$  has a larger number of non-zero elements, which requires a large amount of memory, in addition, the calculation in this case has an oscillatory nature of convergence, sometimes the iterative process diverges. These shortcomings limit the possibilities of using form (6) for calculating the steady state modes of the EPS main network.

The calculation according to the inverted form (6) is effective when the calculation is repeated for the same system many times, as well as in the calculation of short-circuit currents.

Due to the nonlinearity of the steady state equations, they are usually solved only by approximations, i.e. iterative methods. This raises the question of the convergence of the

iterative process and the existence of a solution. Also, given the large dimension of the problem, there are certain difficulties in choosing iterative procedures.

The main stages in the calculation of the steady state mode of the EPS are:

- 1) creating a matrix of nodal conductivities (to create this matrix, you can use the technique from [14]);
- 2) setting the initial data for the required variables, i.e. node voltages (in this case, you can take the nominal values of the node voltage);
- 3) calculation of power imbalance values;
- 4) creation of the Jacobian matrix;
- 5) solution of the formed system of linear equations;
- 6) determination of new values for voltage and phase angles of voltage;
- 7) checking the conditions for terminating the calculation, if a solution with the required accuracy is not found, then return to step 3.

### 3.2 Interval model

The main reason for creating mathematical models for calculating the steady state modes of EPS within the framework of interval analysis is the emerging limited uncertainties in the system parameters. For example, parameters that can change due to seasonal, climatic and other reasons, leads us to consider interval models of this problem [6]. Since when designing or studying the functioning of an EPS, various uncertainties appear, caused by the following main reasons:

- approximate mathematical models are used to describe EPS processes;
- initial data in mathematical models are either unknown or approximate;
- approximate methods are used to solve the problem of EPS calculation;
- in the process of calculation on a computer, a rounding error occurs.

A convenient and simpler way to formalize such uncertainties is to use interval calculation methods.

So, let's write equation (5) in interval form:

$$Y_{ij} \hat{U}_i = Y_{i0} \hat{U}_0 - \frac{\hat{S}_i}{\hat{U}_i}, \quad i = 1, 2, \dots, n. \quad (7)$$

In the interval equation of steady states, the parameters are given by intervals, and the operations are replaced by operations of interval arithmetic.

## 4 Interval iterative methods for solving the nodal equations of EPS

Classical (deterministic) calculation methods do not make it possible to take into account the uncertainties of the parameters. So, with a deterministic approach, inaccuracies in setting certain parameters are practically not taken into account, or, taking into account certain assumptions or assumptions, uncertain parameters are replaced by expert estimates or average values.

Currently, there are a large number of point methods for solving nonlinear systems of nodal equations of EPS, which can be classified as Newton, Seidel, optimization type methods, as well as methods using expansions in a Taylor series. They cannot be used directly for interval data due to the nature of interval operations. In interval methods, we can vary not only the choice of a point from a given interval, which the expansion is carried out in the neighborhood of this point, but also the form of the interval expansion of derivatives and the

method of external estimation of the set of solutions of intermediate interval linear systems [15], to which the solution bar is estimated.

#### 4.1 Algorithm for solving interval nonlinear systems of nodal equations by the Gauss-Seidel method

One of the most popular and effective algorithms for solving interval non-linear systems of nodal equations in the form of current balance (7) is the interval Gauss-Seidel method (IGS method) [8]. According to this method, the determination of the interval values of the voltages of the nodes  $\dot{U}_i$  sequentially from each  $i$ -th equation (7) using the methods of interval arithmetic leads to the iterative formula

$$\dot{U}_i^{(k+1)} = \dot{U}_i^{(k)} \cap \frac{1}{Y_{ii}} \left( Y_{i0} \dot{U}_0 + \sum_{j=1}^{i-1} Y_{ij} \dot{U}_j^{(k+1)} + \sum_{j=i+1}^n Y_{ij} \dot{U}_j^{(k)} - \frac{\hat{S}_i}{\hat{U}_i^{(k)}} \right), \quad i = 1, 2, \dots, n, \quad (8)$$

where  $\dot{U}_i^{(k)}$  and  $\dot{U}_i^{(k+1)}$  are the voltage values of the nodes, respectively, at the  $k$ -th and  $k+1$ -th iterations.

The iterative process continues until the condition  $\Delta U_i^{(k+1)} = \text{dist}(U_i^{(k)}, U_i^{(k+1)}) < \varepsilon$  is met, where  $\varepsilon$  is the required accuracy.

In the algorithm that we present for the Gauss-Seidel interval method, we assume that the intervals of the calculation parameters are specified through the center and radius of the interval in percentages:  $a = [\text{mid } a - \text{rad } a, \text{mid } a + \text{rad } a]$ . For example, a point value of  $U_0 = 220$ , then 5% interval width would be equal to

$$U_0 = [220 - 0.05 * 220, 220 + 0.05 * 220] = [219.94, 220.06].$$

Thus, we formulate the algorithm of the interval Gauss-Seidel method in the form of pseudocodes.

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##### Algorithm 1 (IGS method)

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- 1: Input of initial data:
    - a) circuit data:  $R_{ij}, X_{ij}, Bc_{ij}, Kt_{ij}$ ;
    - b) mode data:  $P_j, Q_j, U_n, U_0$ ;
  - 2: Formation of the design equivalent scheme, taking into account the spread of the values of the design parameters;
  - 3: Entering the required precision and maximum iteration value:  $\varepsilon, i_{\max}$ ;
  - 4:  $i = 1$ ;
  - 5: Determining the values of voltage increments  $\Delta U_i^{(k+1)}$  at the nodes of the circuit using the ISLAE solution:  $Y_{ij} U_i = Y_{i0} U_0 - I_i$ ;
  - 6: Calculation of new voltage values in circuit nodes:  
 $U_i^{(k+1)} = U_i^{(k)} \cap (U_i^{(k)} + \Delta U_i^{(k+1)})$ ;
  - 7: if  $i > i_{\max}$  then stop;
  - 8: else if  $\text{dist}(U_i^{(k)}, U_i^{(k+1)}) < \varepsilon$  then  
 output of calculation results:  $U_i, P_{ij}, Q_{ij}, P_{ji}, Q_{ji}, \Delta P_{ij}, \Delta Q_{ij}$ ;
  - 9: else  $i = i + 1$ ; goto 5;
  - 10: end if.
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In *Algorithm 1*, active  $R_{ij}$  and reactive  $X_{ij}$  resistances, conductivity  $Bc_{ij}$ , transformation ratio  $Kt_{ij}$  for network sections are used as circuit data. Also, mode data are entered for nodes, such as active  $P_j$  and reactive  $Q_j$  powers of load nodes, the value of nominal  $U_n$  and balancing voltage  $U_0$  of the node.

### 4.2 Interval Newton-Raphson (INR) method for solving nonlinear nodal equations of EPS

The most universal and suitable for solving a large class of nonlinear equations is the Newton–Raphson method.

According to this method, the determination of unknown parameters (voltages at the nodes) consists in solving nonlinear systems of current balance equations

$$W(I'_i) = g_{ii}U'_i - b_{ii}U''_i - \sum_{j=0, j \neq i}^n (g_{ij}U'_j - b_{ij}U''_j) + (P_iU'_i + Q_iU''_i) / U_i^2, \quad (9)$$

$$W(I''_i) = b_{ii}U'_i + g_{ii}U''_i - \sum_{j=0, j \neq i}^n (b_{ij}U'_j + g_{ij}U''_j) + (P_iU'_i - Q_iU''_i) / U_i^2, \quad (10)$$

Then for the iterative process we have

$$\begin{pmatrix} \frac{\partial W(I')}{\partial U'} & \frac{\partial W(I')}{\partial U''} \\ \frac{\partial W(I'')}{\partial U'} & \frac{\partial W(I'')}{\partial U''} \end{pmatrix}^{(k)} \cdot \begin{pmatrix} \Delta U' \\ \Delta U'' \end{pmatrix}^{(k+1)} = - \begin{pmatrix} W(I') \\ W(I'') \end{pmatrix}^{(k)}, \quad (11)$$

where the Jacobian elements are calculated by the expressions:

$$\frac{\partial W(I'_i)}{\partial U'_j} = \begin{cases} -g_{ij}, & j \neq i; \\ g_{ii} - G_i, & j = i; \end{cases} \quad \frac{\partial W(I'_i)}{\partial U''_j} = \begin{cases} b_{ij}, & j \neq i; \\ -b_{ii} + B_i, & j = i; \end{cases} \quad (12)$$

$$\frac{\partial W(I''_i)}{\partial U'_j} = \begin{cases} -b_{ij}, & j \neq i; \\ b_{ii} + B_i, & j = i; \end{cases} \quad \frac{\partial W(I''_i)}{\partial U''_j} = \begin{cases} -g_{ij}, & j \neq i; \\ g_{ii} + G_i, & j = i. \end{cases} \quad (13)$$

In this case, the voltage increments at the iteration step are determined by solving the linear system of equations (11). In expressions (12)-(13), the calculation of  $G_i$  and  $B_i$  is usually calculated according to the formulas [1]:

$$G_i = \frac{P_i(U_i'^2 - U_i''^2) + 2Q_iU'_iU''_i}{U_i^4}, \quad B_i = \frac{Q_i(U_i'^2 - U_i''^2) - 2P_iU'_iU''_i}{U_i^4}. \quad (14)$$

The main advantage of the Newton-Raphson method is its fast convergence, when even the first approximation can give a satisfactory result. The application of this method to other forms of nonlinear nodal equations is described in detail in the book [14].

The algorithm for iterative solution of interval systems of nonlinear nodal equations is practically similar to the algorithm for iterative solution of a nonlinear system of equations with deterministic coefficients, except that all calculations must be carried out in accordance with the operations of interval arithmetic.

Verification of convergence by the interval method of Newton-Raphson is carried out by the condition:

$$\begin{cases} \text{mid}(W(\mathbf{P}_i, \mathbf{U}))^{(k+1)} \leq \varepsilon, \\ \text{mid}(W(\mathbf{Q}_i, \mathbf{U}))^{(k+1)} \leq \varepsilon, \end{cases} \quad (15)$$

where  $\text{mid}(W(\mathbf{P}_i, \mathbf{U}))$  and  $\text{mid}(W(\mathbf{Q}_i, \mathbf{U}))$  are the average values of the interval values of the power imbalance equations; and  $\varepsilon$  is the required calculation accuracy. We also formulate the algorithm of the Newton-Raphson interval method in the form of pseudocodes.

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**Algorithm 2 (INR method)**

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- 1: Input of initial data:
    - a) circuit data:  $R_{ij}, X_{ij}, B_{Cij}, Kt_{ij}$ ;
    - b) mode data:  $P_j, Q_j, U_n, U_0$ ;
  - 2: Formation of the design equivalent scheme, taking into account the spread of the values of the design parameters;
  - 3: Entering the required precision and maximum iteration value:  $\varepsilon, i_{\max}$ ;
  - 4:  $i = 1$ ;
  - 5: Calculation of interval values:  $W(\mathbf{P}, \mathbf{U}), W(\mathbf{Q}, \mathbf{U})$  and  $J(\mathbf{U})$ ;
  - 6: if  $\begin{cases} \text{mid}(W(\mathbf{P}_i, \mathbf{U}))^{(k+1)} \leq \varepsilon, \\ \text{mid}(W(\mathbf{Q}_i, \mathbf{U}))^{(k+1)} \leq \varepsilon, \end{cases}$  then
  - 7: Output of calculation results:  $U_i, P_{ij}, Q_{ij}, P_{ji}, Q_{ji}, \Delta P_{ij}, \Delta Q_{ij}$ ;
  - 8: Determining the values of voltage increments  $\Delta U_i^{(k+1)}$  at the nodes of the circuit using the ISLAE solution:  $Y_{ij}U_i = Y_{i0}U_0 - I_i$ ;
  - 9: Calculation of new voltage values in circuit nodes:  
 $U_i^{(k+1)} = U_i^{(k)} \cap (U_i^{(k)} + \Delta U_i^{(k+1)})$ ;
  - 10: else if  $i < i_{\max}$  then goto 5;
  - 9: else stop;
  - 10: end if;
  - 11: end if.
- 

As it can be seen from the algorithms of both methods, at each iteration step, they involve the solution of interval linear systems of equations. They can be solved, for example, using the interval methods of Gauss, Krawczyk and Hansen-Blik-Rohn. [12, 17].

To identify the most acceptable of them, a series of calculations were carried out using each of the methods. As an example of calculations using these methods, we present the results for various intervals for a 4-nodal scheme.

**Table 1.** Estimated calculations of interval methods for solving ISLAE for small values of intervals.

Nodes Voltage	Gauss Method		Krawczyk Method		Hansen-Blik-Rohn Method	
	$\underline{U}$	$\overline{U}$	$\underline{U}$	$\overline{U}$	$\underline{U}$	$\overline{U}$
U1re	0	0	0	0	0	0
U1im	0	0	0	0	0	0
U2re	-9.4004	7.2846	-1.3510	-0.9167	-1.3527	-0.9062
U2im	-5.8626	8.7387	0.6383	0.9277	0.6303	0.9300
U3re	-0.0907	0.1175	-0.0147	0.0118	-0.0149	0.0121



U3im	-2.5101	9.3175	3.3618	3.7961	3.3513	3.7978
U4re	-2.4835	7.3577	0.6136	0.9030	0.6056	0.9053
U4im	0.0502	0.2006	0.0883	0.1144	0.0876	0.1145

**Table 2.** Estimated calculations of interval methods for solving ISLAE for large values of intervals.

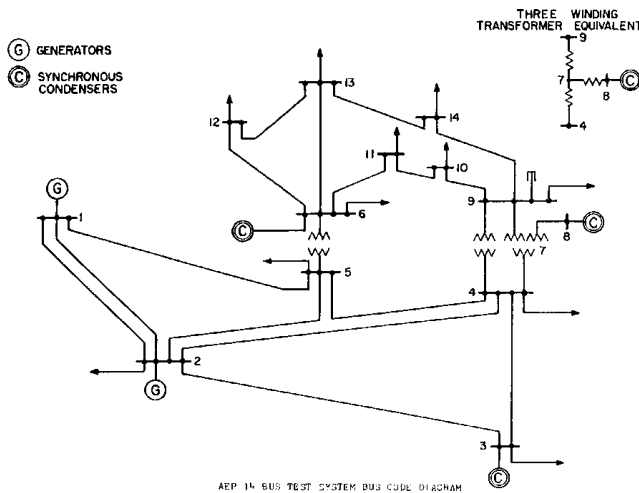
Nodes Voltage	Gauss Method		Krawczyk Method		Hansen-Bliek-Rohn Method	
	$\underline{U}$	$\overline{U}$	$\underline{U}$	$\overline{U}$	$\underline{U}$	$\overline{U}$
U1re	0	0	0	0	0	0
U1im	0	0	0	0	0	0
U2re	-45.4586	41.9069	-1.6926	-0.6106	-Inf	Inf
U2im	-31.8765	43.6184	0.4426	1.1469	-Inf	Inf
U3re	-0.4452	0.5999	-0.0348	0.0319	-Inf	Inf
U3im	-23.4152	24.0702	3.0551	4.1371	-Inf	Inf
U4re	-5.7805	34.6850	0.4173	1.1216	-Inf	Inf
U4im	0.0165	0.5271	0.0704	0.1344	-Inf	Inf

The calculation results showed that the Krawczyk method turned out to be the most suitable method. The results obtained by the Gauss method are much broader than other methods. For large intervals of initial data, the Hansen-Bliek-Rohn method does not give a solution.

### 5 Numerical results of calculations of steady-state modes of control circuits

Using the algorithms proposed above in the GNU Octave software environment [18, 19], a program was developed for calculating the steady-state modes of the main EPS networks, taking into account the interval uncertainty of the initial parameters.

As an example, we present the calculations of the 14-node IEEE test circuit [20, 21] (Figure 1).



**Fig. 1.** IEEE 14 bus test system.

Interval initial data are formed through the radius of the interval (in percent (%), relative to the center of the interval), for example, as indicated in Table 3. The initial data for the scheme are presented in Tables 4 and 5.

**Table 3.** Formation of interval parameters in percentage terms.

Schematic Parameter Interval Width			Mode parameter interval width		
rad( $R$ )	rad( $X$ )	rad( $Bc$ )	rad( $U$ )	rad( $P$ )	rad( $Q$ )
0 %	0 %	0 %	5 %	5 %	5 %

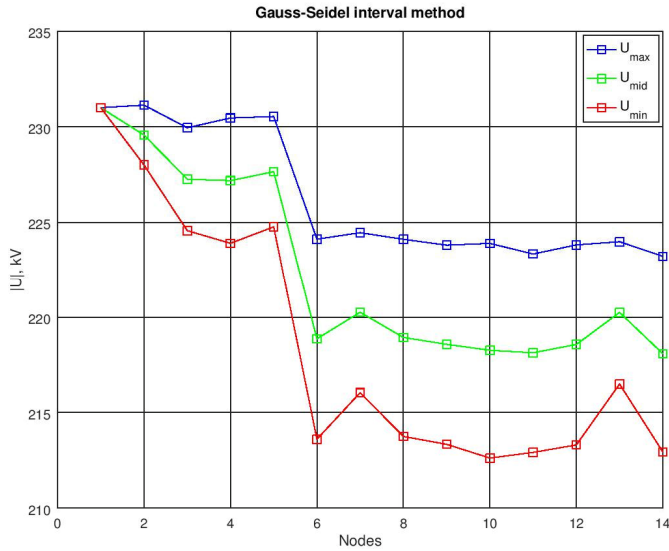
**Table 4.** Initial data on the branches of the 14 nodal test circuit.

Branch number	Begin	End	$R$ (Ohm)	$X$ (Ohm)	$Gk \cdot 10^{-3}$ (Sm)	$Bc \cdot 10^{-3}$ (Sm)	$Kt'$	$Kt''$
1	1	2	1.93	5.92	0	0.528	1	0
2	1	5	5.403	22.3	0	0.492	1	0
3	2	3	4.699	19.8	0	0.438	1	0
4	2	4	5.811	17.63	0	0.374	1	0
5	2	5	5.695	17.39	0	0.34	1	0
6	3	4	6.701	17.1	0	0.346	1	0
7	4	5	1.335	4.21	0	0.128	1	0
8	4	7	0	20.91	0	0	1.02	0
9	4	9	0	55.62	0	0	1.02	0
10	5	6	0	25.2	0	0	1.02	0
11	6	11	9.498	19.89	0	0	1	0
12	6	12	12.291	25.58	0	0	1	0
13	6	13	6.615	13.03	0	0	1	0
14	7	8	0	17.61	0	0	1	0
15	7	9	0	11	0	0	1	0
16	9	10	3.181	8.45	0	0	1	0
17	9	14	12.711	27.04	0	0	1	0
18	10	11	8.205	19.21	0	0	1	0
19	12	13	22.092	19.99	0	0	1	0
20	13	14	17.093	34.8	0	0	1	0

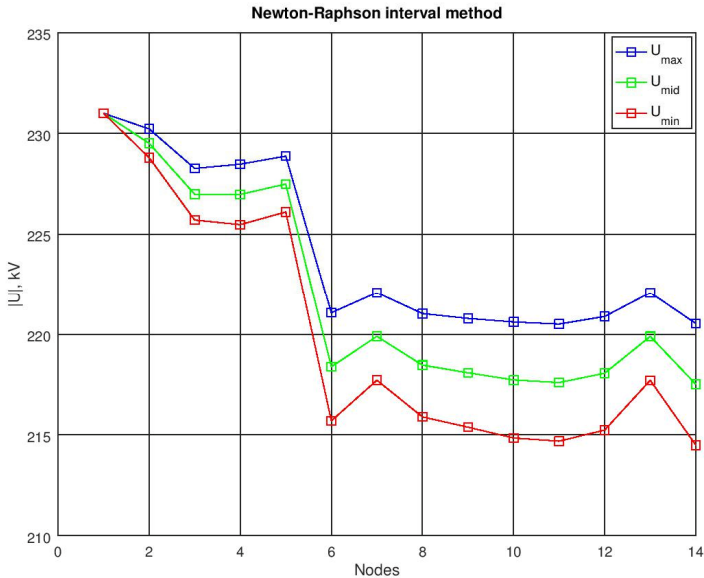
**Table 5.** Initial data on nodes 14 nodal test circuit.

Nodes		Generation + Load		$U_n$ - Nominal voltage
№	Type	$P$	$Q$	
1	3	0	0	220
2	2	-18.3	12.7	220
3	2	94.2	19	220
4	1	47.8	3.9	220
5	1	7.6	1.6	220
6	2	30	30	220
7	1	0	0	220
8	2	29.5	35.6	220
9	1	9	5.8	220
10	1	6.1	1.6	220
11	1	13.5	5.8	220
12	1	3.5	1.8	220
13	1	0	0	220
14	1	14.9	0	220

For these schemes, a series of calculations were carried out for different values of the initial data intervals. As an illustration of the results of calculations for both methods, we present at 5% interval width (Figures 2-3). In this case, the numerical results were obtained in the Gauss-Seidel interval method in 7 iterations, and in the Newton-Raphson interval method in 1 iteration.



**Fig. 2.** The result of applying the Gauss-Seidel method at 5% interval width.



**Fig. 3.** The result of applying the Newton-Raphson method at 5% interval width.

According to the results of calculations, it can be concluded that the voltage values at the nodes distant from the balancing node are the most dependent on the change in the load in the circuit, and the nodes located near are the least.

The boundaries of the interval values of voltages in the nodes obtained by the Newton-Raphson method are better than the values obtained by the Gauss-Seidel method. This is due to the small number of iterations of calculations, as well as the calculation of the values of the power imbalance equations and the values of the elements of the Jacobi matrix using the average values of the interval voltages at the nodes, which is necessary to exclude division by zero containing the interval.

## 6 Conclusion

The available initial information for performing calculations of steady state modes of electric power systems is always non-deterministic, i.e. is incomplete or limitedly reliable.

Interval models using external interval estimation adequately approximate the possible areas of change in the voltage modules of electrical networks for given intervals of variation of the initial data. On the basis of interval modeling, important information for designers can be obtained about the ranges of possible changes in operating parameters; in particular, in the calculation example, the values of voltage deviations that go beyond the permissible limits are obtained. Also, by means of interval simulation of loads, it is possible to obtain an integral assessment of power quality indicators based on voltage deviations and unbalance. This does not require the implementation of multi-stage procedures of statistical and simulation modeling. Accounting for the uncertainty of the initial data in the calculation of emergency modes of electric power systems can be performed based on the methods of interval analysis.

In this paper, two of the most popular iterative methods for calculating the steady state modes of electric power systems are investigated. An analysis of the numerical results shows that the voltage intervals found using the Newton–Raphson method have a smaller width than the resulting intervals of the Gauss–Seidel method. This difference is due to a small number of iterations, the calculation of the values of the power imbalance equations and the values of the elements of the Jacobi matrix using the average values of the interval voltages at the nodes at the iteration step.

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