

# Filtering errors in primary sensor signals

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**Abstract.** Sensors are used in the primary measurement of signals. When signals in the sensor are transmitted to the next device, there is an occurrence of errors due to various effects. Filters are used to eliminate the errors that have occurred. Depending on the frequency nature of the signals, there are high pass filters and low pass filters. We select the filters according to the frequency range of the signal. It is observed that the measurement result is different from the actual value if the signal is not filtered. This article analyzes the errors that occur when signals are processed. In time discretization, the state of the signal spectrum is considered. The cause of the appearance of errors has been studied.

## 1 Introduction

To determine the problems encountered in solving the filtering problem, consider the spectrum of the signal when it is temporally discretized. Temporal discretization meets the requirement of amplitude-pulse modulation of the viewed signal. Therefore, the signal after discretization can be determined by the spectral response, the signal before and after discretization. A distinction of amplitude-pulse modulation is the complex spectrum of the modulated signal. The spectrum of the sample pulse has harmonic components which broaden the spectrum of the signal after modulation. All valid signals have an infinite spectrum, and after sampling this is the basis for the so-called spectrum superposition error, where spectral components that were not in the original signal will appear in the signal after restoration.

## 2 Methods

Spectrum overlap error underlines the fact that discretization is not a filtering of the signal spectrum. If there are high-frequency components in the signal spectrum that are not needed for further use, the discretization rate cannot be selected without taking these components into account. This leads to significant errors in the final signal. The root-mean-square error of the superimposed spectra is estimated using the formula: [1].

$$\gamma = \frac{\delta_1 + \delta_2}{K} \quad (1)$$

where

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$$\delta_1 = \frac{1}{\pi} \int_0^{\frac{\omega_d}{2}} \left[ \sum_{\substack{l=-\infty \\ l \neq 0}}^{+\infty} A(\omega + \omega_d l) \right]^2 d\omega \quad (2)$$

$$K = \frac{1}{\pi} \int_0^{\infty} [A(\omega)]^2 d\omega \quad (3)$$

The component  $\delta_2$ - takes into account the overlapping error of the spectra from all spectral components of the signal. If limited to the first harmonic, the spectral superposition error can be estimated by the formula:

$$\gamma_n = 2 \frac{\int_{\omega_d}^{\infty} [A(\omega)]^2 d\omega}{\int_0^{\infty} [A(\omega)]^2 d\omega} \quad (4)$$

Spectrum overlap error can also occur in the case of a signal with a finite spectrum, where it is possible to indicate  $\omega_{gr}$ . An error appears when:  $\omega_d < 2\omega_{gr}$

Spectrum superposition error can be reduced by increasing the frequency of sampling and by limiting the signal spectrum. Filtering of the sensor signals in the analogue interface is done either to reduce the spectrum overlap error or to reduce the time sampling rate. But the filter setting is associated with introducing error into the signal conversion process. Analogue and digital filters are used in the sampling-reduction process. In order to reduce the magnitude, it is necessary to clearly limit the spectrum of the original signal. This can be done by placing an analogue filter upstream of the time sampling device. Therefore, filtering of the signal is often an integral part of the sampling process. The conditions from which the decision as to whether or not to apply a filter is derived are formed as follows. In the analogue interface, by solving a forward and inverse error allocation problem between some links, the total sampling-recapture error was allocated. Either a discretization frequency was obtained whose value is not satisfactory to the user or it was found impossible to perform the discretization with the raw data. The way out of this situation may be to apply a filter. But the filter introduces an error, which requires a redistribution of errors, which can only be produced by an error in the spectrum overlay. This in turn may lead to a new increase in the sampling frequency and raise the question of whether a filter should be applied. It is permissible to put a filter in place for other conditions. For example, if the data acquisition system operates in several modes with changing sampling rates, each mode modification must be accompanied by a switch of filters after the signal sources before the discretization device. Failure to do so may result in errors of hundreds of per cent and indicates that the discretization process is being attempted to filter the signal, which is unacceptable. Also of special interest is the evaluation of current analogue interfaces in terms of the largest parameters of the best analogue filters. General error distributions and their impact on parameters. Let us first consider the choice of sampling frequency in terms of methodological uncertainty. The error is determined by the type of approximating polynomial and its degree. Time sampling assumes that the signal recovery will use an algorithm whose operation is described by an approximating polynomial. Maximum values of absolute methodological error  $\Delta_{max}$  are of the form:

For stepwise extrapolation by power polynomial

$$|\Delta_{max}| \leq M'_{max} T \quad (5)$$

Where  $M'$  - is the maximum value of the first derivative of the signal to be sampled;  $T$  is the sampling period.

In linear extrapolation:

$$|\Delta_{max}| \leq M''_{max} \frac{T^2}{2} \quad (6)$$

For the case of linear interpolation by a first-order Lagrange polynomial:

$$|\Delta_{max}| \leq M''_{max} \frac{T^2}{8} \quad (7)$$

The degree of the approximating polynomial (n) rarely exceeds 2-4 in practice, but higher order polynomials can be used in mathematical modelling. The maximum values of the derivatives can be determined through the spectra of discrete signals. If the maximum frequency (boundary) spectral component in the signal spectrum can be specified, its value is written in the form:

$$A(\omega_{gr}) = A_{gr} \sin(\omega_{gr} t) \tag{8}$$

where  $A_{gr}$  - harmonic amplitude,  $\omega_{gr}$ - is the frequency of the boundary harmonic. It is not difficult to demonstrate by taking a series of derivatives of A ( $\omega$ ), that

$$|M'_{max}| = \left| \frac{dA(\omega_{gr})}{dt} \right|_{max} \leq A_{gr} \omega_{gr} \tag{9}$$

$$|M''_{max}| = \left| \frac{d^2 A(\omega_{gr})}{dt^2} \right|_{max} \leq A_{gr} \omega_{gr}^2 \tag{10}$$

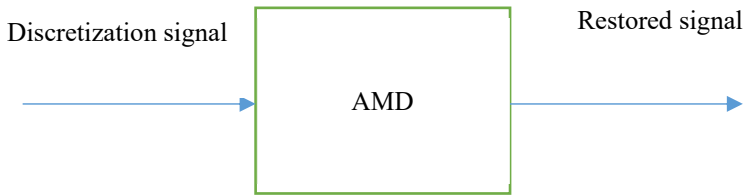
Analysis of expressions (9) (10) allows us to draw an important conclusion: approximation can be used only for recovery of signals, spectra of which are finite and it is possible to specify the boundary frequency. But, as noted earlier, real signals have infinite spectra and the description of the sampling-recovery process only by means of approximating polynomials does not allow taking into account all components of the error. Let us briefly settle on the instrumental error of information reconstruction, which depends on the methodological error and directly affects the choice of sampling frequency. Indeed, the choice of a polynomial may lead to a large instrumental error, which would make one reconsider the choice of the polynomial. Information recovery after sampling can be hardware, software, and mixed. Purely hardware-based recovery is characteristic of amplitude-pulse modulation, when the signal readout is analog in nature. Note that in this case, only extrapolating polynomials are actually applicable. The most commonly used recovery devices are low-pass filters of n-th order (LPF) and analog storage devices (ASD) [2,3,4].

### 3 Discussion

The RAM is used in a zero-order extrapolative polynomial recovery. The errors of the AMD integrated circuits in the temperature and supply voltage range are up to one per cent, so the effect of the reduction process is significant. AMD can be used directly (Figure 1.) or together with a RAM (Figure 2). The errors introduced by the filters, discussed earlier, are also present here. The software reconstruction can use both extrapolating and interpolating polynomials. The recovery task in this case is always mixed. This is due to the fact that the output signal after reconstruction must have an analog form, which can be obtained by means of a digital-to-analog converter (DAC). The register for storing the digital signal reference, which always precedes the DAC, plays the role of a zero-order extrapolator storing the signal values between the references. Therefore, software recovery really corresponds to the structural diagram of Figure 3. The input of the software interpolator or extrapolator is the digital signal counts after sampling at a frequency of  $\omega_d$ . The software recoverer assigns samples with frequency  $k \omega_d$  where  $k=2,3,4,\dots$ . The greater the value of k, the more accurately the output signal is reconstructed. At the same time the speed and volume of calculations grows. The algorithm of the software interpolator of the first order is shown in Figure 4. The output is k times the number of samples [5].

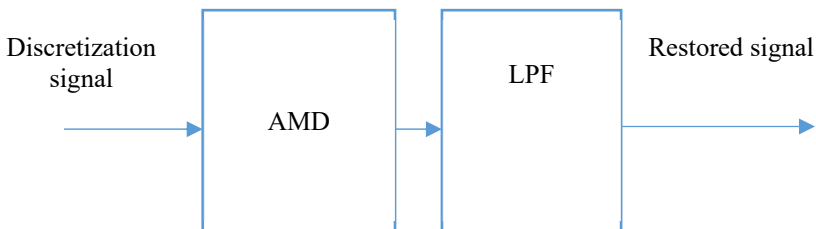
Consideration of the algorithm of Figure 4. confirms the conclusion that it is practically impossible to implement interpolation algorithms with analog form of reference representation on analog devices due to difficulties to implement division and multiplication operations in analog form with acceptable errors. If the software recovery device is implemented on a controller that uses fixed-point numbers, then the implementation of

algorithms like Figure 4 that have division and multiplication operations will lead to rounding errors that must be accounted for in the instrumental recovery error.



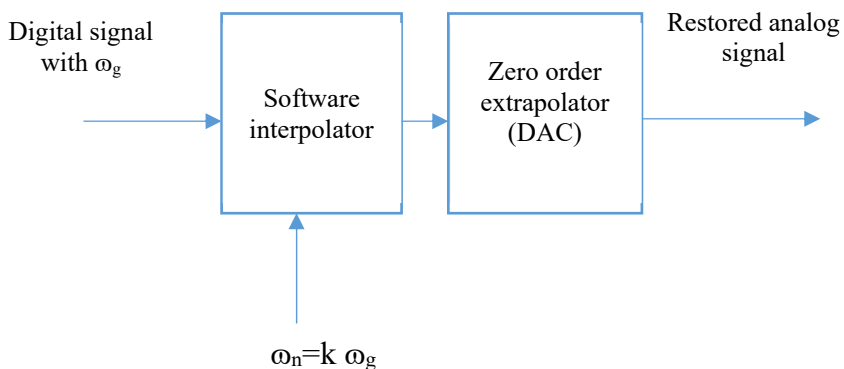
**Fig. 1.** Analog memory device.

Let us consider the choice of sampling frequency from the point of view of spectrum overlap error. In various papers, there is a practice of ignoring the spectrum superposition error or creating conditions where its influence could be neglected. It is known that all real signals have infinite spectra, which always determines the presence of spectrum superposition error [6].



**Fig. 2.** Analogue memory device and low-pass filter.

Sometimes this error is really negligible, but more often its influence is significant. When approximating the spectral response of a signal with a Butterworth filter function, it is necessary to ensure the coverage of the approximated function (Figure 4). This results in redundancy, usually evaluated in units of percent. The approximation problem usually consists in choosing the order of the Butterworth filter. Optimal results can be obtained by using special computer programs which, by brute force, give the required minimum value of the Butterworth filter order [7,8].



**Fig. 3.** Signal processing with software interpolator and digital to analogue converter.

Let us also focus on the use of the sample rate selection theorem. The frequency If a continuous time function  $f(t)$  does not contain spectral components with a frequency higher than it is well defined by discrete values counted with a frequency of  $2 \omega_{gr}$ . This unambiguous and simple approach to the selection of the sampling frequency differs from the approaches discussed above. Indeed, in the case of using a power series or a Lagrange polynomial for signal reconstruction, the sampling frequency is determined by the methodological error of the type of polynomial, its order, and the boundary frequency which it divides by the maximum derivative [9].

Comparisons of the two approaches are clearly in favor of the countdown theorem and an overwhelming number of publications recommend this theoretically achievable sampling rate, not for the justification of the theoretically achievable sampling rate, but for the selection of the operating frequency. Having a rigorous mathematical proof, the reference theorem in the above formulation often becomes a trap for users. If when choosing the sampling rate one does not operate with the notion of methodical error from series truncation, then there is no problem with the instrumental error of reconstruction which indirectly affects the sampling rate and depends on the reconstruction method. It is not difficult to show that in terms of spectrum overlap error, the counting theorem is formulated as: If a continuous time function does not contain spectral components with a frequency of  $\omega_{gr}$ , then when it is sampled with a frequency greater than or equal to  $2 \omega_{gr}$  the spectrum superposition error is zero. This is what the counting theorem says, and the sampling rate itself can be determined by specifying the methodological and instrumental error of the reconstruction. But the Kotelnikov series is not very convenient, in comparison with other series, in software and hardware reconstruction, which most often determines the choice in favour of power series, Lagrange polynomials, etc. [10-15].

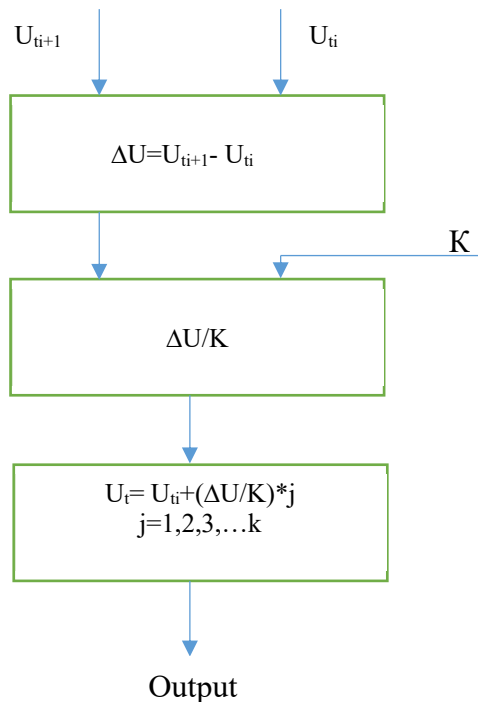
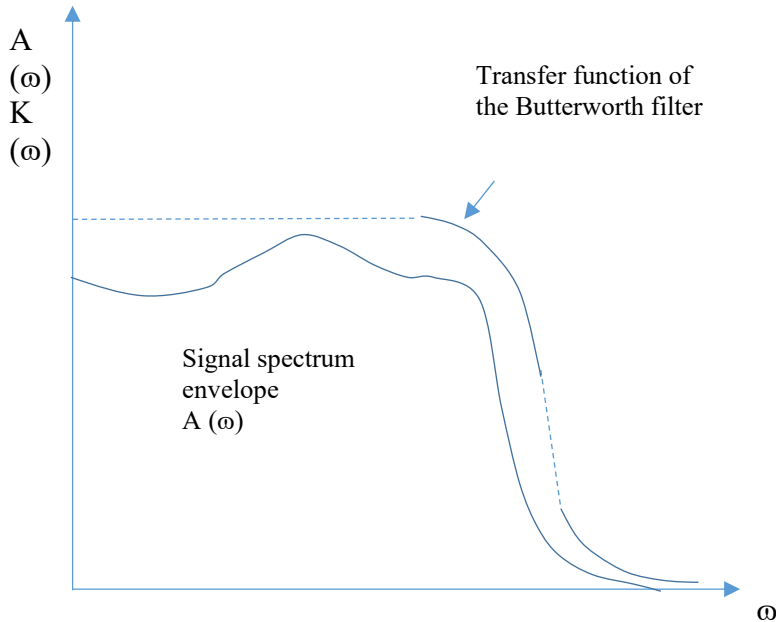


Fig. 4. Signal discretization algorithm.



**Fig. 5.** Spectral response of the signal.

In the proof of the sampling theorem it is assumed that an infinite Kotelnikov series is used to recover the information. Even in real-world software reconstruction, the series will be finite. And in hardware recovery, the number of row terms can be small and this significantly affects the recovery process and the choice of sampling rate. Thus, we can conclude that the count theorem algorithm does not account for the recovery process. When finding the boundary frequency for the real spectrum, an uncertainty arises where the boundary frequency is needed to find the sampling frequency, which requires the sampling frequency to be found through the spectrum overlap error.

## 4 Conclusion

In conclusion, there are two approaches to sampling rate selection, which in practice exist separately from each other. This creates uncertainty in the choice of sampling rate. Approximation can only be applied to recover signals whose spectra are finite and a boundary frequency can be specified. But real signals have infinite spectra and describing the sampling-recovery process by approximating polynomials alone does not allow to take into account all the error components.

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