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# Adaptability analysis of linear continuous control systems with reference model

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**Abstract.** The problem of analyzing the tuning parameters of linear continuous controllers related to the class of state models is studied. To do this, algorithms for adjusting the controller are used, which are based on estimates of the plant parameters. Within the framework of this study, the adaptability property of the system is considered, and the primary goal is to determine the possibility of finding an exact solution to the problem of tuning the control loop for a given reference transient. Moreover, the study extends to a whole class of input actions, such as piecewise continuous functions of time. Based on the analysis of the convergence properties of the identification residual in a closed control system, additional simplified conditions for the adaptability of control systems using an identifier and a reference model are put forward, which supplement the already existing knowledge on this topic.

## 1 Introduction

Adaptability is a unique property of the control loop structure, which reflects the potential ability to compensate for the influence of the variable parameters of the controlled object on the dynamic characteristics of the control system by adjusting the controller. This property is used to analyze the achievable tuning accuracy of controllers with a certain structure, to synthesize control systems and adaptive algorithms using an explicit or implicit reference model. Initially, the problem of analyzing the ability of the control loop to tune in to the reference response arose and was widely used for adaptive systems with a reference model [1-2]. Later, results were obtained that extended the concept of adaptability to all feedback control loops [3-4].

It was proved that the problem of adjusting the controller for a feedback control system in the case of using the procedure for identifying object parameters is equivalent to the problem of indirect adaptive control, which made it possible to apply the methods of adaptive control theory for automatic or automated tuning of the controller. In recent years, there has been an increase in research devoted to the development of energy-saving control systems. Since the methods for automating the adjustment of regulators are an effective tool for reducing energy consumption, the relevance of research in this area does not need to be confirmed [5].

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Adaptability property involves, first of all, determining the existence of an exact solution to the problem of tuning the control loop for a given reference transient process, and not only for one, but also for a whole class of input actions, for example, for piecewise continuous functions of time. Secondly, it is required to solve the problem of finding a solution to the tuning problem and determine the accuracy of the solution obtained. With regard to linear continuous control systems, these problems were solved in [2-5]. In an attempt was made to extend the obtained results to the class of linear control systems described by the Hammerstein model. In this paper, we consider the problem of analyzing the adaptability of linear continuous control systems in which the control object with many inputs and outputs is nonlinear, and the controller has a linear structure of a given type, although of a general nature.

## 2 Problem statement

In this problem, we consider the equation of motion of a linear continuous control system, presented in general form as follows [5-9]:

$$\begin{cases} \dot{x} = Ax + Bu, x_0(t) = x_0 \\ y = Cx, y_0(t) = y_0 \end{cases} \tag{1}$$

where $x \in R^n are$ the state vectors (outputs) of the object; $u \in R^n$- control vector (controllaw); $y$ - output, $A, B, C$ - matrices of plant parameters with unknown coefficients,

Let the desired motion of the object (1) be described by a reference model of the same order

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u_m, x_0(t) = x_0 \\ y_m = C_m x_m, y_0(t) = y_0 \end{cases} \tag{2}$$

where $x_m \in R^n$- the standard, respectively;$u_m$ - setting action at the input of the control system; $y_m$ - the output of the reference model, $A_m, B_m$- the given matrix of parameters of the reference models with constant coefficients (all relevant dimensions, $A_m$- Hurwitz matrix); $t$ - current time; $\dot{x}, x, u$are directly measured.

## 3 Results and discussion

The law of control, aimed at achieving the desired movement $x \equiv x_m$, is based on dependence [2-10].

$$u = B^+[(A_m - A)x + B_m u_m] \tag{3}$$

with pseudoinverse matrix $B^+$ under the necessary fulfillment of the Erzberger conditions

$$BB^+(A_m - A) = A_m - A, BB^+B_m = B_m, \tag{4}$$

which can be written in the form

$$rankB = rank(B, A_m - A) = rank(B, B_m). \tag{5}$$

The adaptation error is defined as a vector:

$$e = x - x_m \tag{6}$$

(1) and (2) using the difference between the derivatives of the first equations and (3).

$$\dot{e} = \dot{x} - \dot{x}_m = Ax + Bu - A_m x_m - B_m u_m$$

or

$$\begin{aligned} \dot{e} = Ax + BB^+[(A_m - A)x + B_m u_m] - A_m x_m - B_m u_m = \\ = Ax + A_m x - Ax + B_m u_m - A_m x_m - B_m u_m \end{aligned} \tag{7}$$

takes shape.

Equation (7) is a homogeneous differential equation based on (6).

$$\dot{e} = A_m(x - x_m) \text{ or } \dot{e} - A_m e = 0 \tag{8}$$

Vector representation of a homogeneous differential equation

$$\begin{pmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \\ \vdots \\ \dot{e}_m(t) \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix} \cdot \begin{pmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_m(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (9)$$

In that

$$e(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_m(t) \end{pmatrix}, A_m = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix}$$

The system of homogeneous linear equations is formed by applying the Laplace transform to (9).

$$\begin{pmatrix} pe_1(p) \\ pe_2(p) \\ \vdots \\ pe_m(p) \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix} \cdot \begin{pmatrix} e_1(p) \\ e_2(p) \\ \vdots \\ e_m(p) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\begin{cases} e_1(p)(p - a_{11}) + e_2(p)a_{12} + \dots + e_m(p)a_{1m} = 0 \\ a_{21}e_1(p) + e_2(p)(p - a_{22}) + \dots + e_m(p)a_{2m} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}e_1(p) + e_2(p)a_{m2} + \dots + e_m(p)(p - a_{mm}) = 0 \end{cases} \quad (10)$$

A system of homogeneous equations is always consistent (because $r(A_m) = r(F)$) and has a zero (trivial) solution

$$e_1 = e_2 = \dots = e_m = 0. \quad (11)$$

Thus, the use of relations (11) in (8) gives a new solution to the equation for the adaptation error $e(t)$.

The purpose of control is to achieve the required value of control error. Let's define a variable

$$\Delta(t) = y - y_m \quad (12)$$

as a setting error and rewrite the purpose of the setting in the form

$$\Delta(t) \equiv 0 \quad (13)$$

Of interest are control systems with unity feedback, consisting of a continuous linear controller of a general form and a continuous linear dynamic plant.

Subtracting equation (1) from equation (2) and taking into account (3), we obtain the equation

(1) and (2) using the difference between the derivatives of the second equations and (3).

$$\begin{aligned} \dot{\Delta} &= \dot{y}_m - \dot{y} = C_m(A_m x_m + B_m u_m) - C(Ax + Bu) = \\ &= C_m A_m x_m + C_m B_m u_m - CAx - CBu = \\ &= C_m A_m x_m + C_m B_m u_m - CAx - CBB^+[(A_m - A)x + B_m u_m] = \\ &= -CAx - CA_m x + CAx - CB_m u_m + C_m A_m x_m + C_m B_m u_m \end{aligned} \quad (14)$$

or

$$\dot{\Delta} = -A_m(Cx - C_m x_m) - B_m u_m(C - C_m)$$

From (12) one can

$$\dot{\Delta} + A_m \Delta = B_u u_m(C_m - C) \quad (15)$$

Taking into account the notation (15), the last equation can be represented as

$$\dot{\Delta} = A_m \Delta + u_m \Delta k_{u_m} \quad (16)$$

So, the adaptive control system in new variables is described by equations (16).

As a candidate for the Lyapunov function, we consider the quadratic form

$$V(\Delta, \Delta k_{u_m}) = \frac{1}{2}\left[\Delta^2 + \lambda \Delta k_{u_m}^2\right] \quad (17)$$

The derivative of this function with respect to time has the form.

$$\dot{V}\left(\Delta, \Delta k_{u_m}\right) = \Delta\dot{\Delta} + \lambda\Delta k_{u_m}\Delta\dot{k}_{u_m}$$

where

$$\Delta\dot{k}_{u_m} = -sign\left(\frac{1}{\lambda}\right)u_m\Delta \tag{18}$$

Substituting into the right side of the expression for the derivatives from the equations of the adaptive control system (18) and (16), we obtain

$$\dot{V}\left(\Delta, \Delta k_{u_m}\right) = \Delta\left(A_m\Delta + u_m\Delta k_{u_m}\right) + \lambda\Delta k_{u_m}\Delta\dot{k}_{u_m} =$$
$$= A_m\Delta^2 + \Delta u_m\Delta k_{u_m} + \lambda\Delta k_{u_m}\Delta\dot{k}_{u_m} =$$
$$= A_m\Delta^2 + \Delta u_m\Delta k_{u_m} + \lambda\Delta k_{u_m}\left(sign(\tfrac{1}{\lambda}u_m\Delta)\right) = -A_m\Delta^2.$$

The law of control, aimed at achieving the desired movement $x \equiv x_m$, is based on dependence [4-9].

$$u = B^{+}[(A_m - A)x + B_m u_m] \tag{19}$$

with pseudoinverse matrix $B^{+}$ under the necessary fulfillment of the Erzberger conditions

$$BB^{+}(A_m - A) = A_m - A, BB^{+}B_m = B_m, \tag{20}$$

which can be written in the form

$$rankB = rank(B, A_m - A) = rank(B, B_m). \tag{21}$$

The adaptation error is defined as a vector:

$$e = x - x_m \tag{22}$$

(21) and (22) using the difference between the derivatives of the first equations and (3).

$$\dot{e} = \dot{x} - \dot{x}_m = Ax + Bu - A_m x_m - B_m u_m$$

or

$$\begin{aligned}\dot{e} &= Ax + BB^{+}[(A_m - A)x + B_m u_m] - A_m x_m - B_m u_m = \\ &= Ax + A_m x - Ax + B_m u_m - A_m x_m - B_m u_m\end{aligned} \tag{23}$$

takes shape.

Equation (7) is a homogeneous differential equation based on (6).

$$\dot{e} = A_m(x - x_m) \text{ or } \dot{e} - A_m e = 0 \tag{24}$$

Vector representation of a homogeneous differential equation

$$\begin{pmatrix}\dot{e}_1(t)\\ \dot{e}_2(t)\\ \vdots\\ \dot{e}_m(t)\end{pmatrix} - \begin{pmatrix}a_{11} & a_{12} & \dots & a_{1m}\\ a_{21} & a_{22} & \cdots & a_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mm}\end{pmatrix}\cdot\begin{pmatrix}e_1(t)\\ e_2(t)\\ \vdots\\ e_m(t)\end{pmatrix} = \begin{pmatrix}0\\0\\ \vdots\\0\end{pmatrix} \tag{25}$$

In that

$$e(t) = \begin{pmatrix}e_1(t)\\ e_2(t)\\ \vdots\\ e_m(t)\end{pmatrix}, A_m = \begin{pmatrix}a_{11} & a_{12} & \dots & a_{1m}\\ a_{21} & a_{22} & \cdots & a_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mm}\end{pmatrix}$$

The system of homogeneous linear equations is formed by applying the Laplace transform to (9).

$$\begin{pmatrix}pe_1(p)\\ pe_2(p)\\ \vdots\\ pe_m(p)\end{pmatrix} - \begin{pmatrix}a_{11} & a_{12} & \dots & a_{1m}\\ a_{21} & a_{22} & \cdots & a_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mm}\end{pmatrix}\cdot\begin{pmatrix}e_1(p)\\ e_2(p)\\ \vdots\\ e_m(p)\end{pmatrix} = \begin{pmatrix}0\\0\\ \vdots\\0\end{pmatrix}$$

$$\begin{cases} e_1(p)(p - a_{11}) + e_2(p)a_{12} + \ldots + e_m(p)a_{1m} = 0\\ a_{21}e_1(p) + e_2(p)(p - a_{22}) + \ldots + e_m(p)a_{2m} = 0\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ a_{m1}e_1(p) + e_2(p)a_{m2} + \ldots + e_m(p)(p - a_{mm}) = 0\end{cases} \tag{26}$$

A system of homogeneous equations is always consistent (because $r(A_m) = r(F)$) and has a zero (trivial) solution

$$e_1 = e_2 = \ldots = e_m = 0. \tag{27}$$

Thus, the use of relations (11) in (8) gives a new solution to the equation for the adaptation error $e(t)$.

The purpose of control is to achieve the required value of control error. Let's define a variable

$$\varDelta(t) = y - y_m \tag{28}$$

as a setting error and rewrite the purpose of the setting in the form

$$\varDelta(t) \equiv 0 \tag{29}$$

Of interest are control systems with unity feedback, consisting of a continuous linear controller of a general form and a continuous linear dynamic plant.

Subtracting equation (1) from equation (2) and taking into account (3), we obtain the equation (1) and (2) using the difference between the derivatives of the second equations and (3).

$$\begin{aligned}\dot{\varDelta} &= \dot{y}_m - \dot{y} = C_m(A_m x_m + B_m u_m) - C(Ax + Bu) = \\ &= C_m A_m x_m + C_m B_m u_m - CAx - CBu = \\ &= C_m A_m x_m + C_m B_m u_m - CAx - CBB^{+}[(A_m - A)x + B_m u_m] = \\ &= -CAx - CA_m x + CAx - CB_m u_m + C_m A_m x_m + C_m B_m u_m \end{aligned} \tag{30}$$

or

$$\dot{\varDelta} = -A_m(Cx - C_m x_m) - B_m u_m(C - C_m)$$

From (12) one can

$$\dot{\varDelta} + A_m \varDelta = B_u u_m (C_m - C) \tag{31}$$

Taking into account the notation (15), the last equation can be represented as

$$\dot{\varDelta} = -A_m \varDelta + u_m \varDelta k_{u_m} \tag{32}$$

So, the adaptive control system in new variables is described by equations (16).

As a candidate for the Lyapunov function, we consider the quadratic form

$$V\left(\varDelta, \varDelta k_{u_m}\right) = \frac{1}{2}\left[\varDelta^2 + \lambda \varDelta k_{u_m}^2\right] \tag{33}$$

The derivative of this function with respect to time has the form.

$$\dot{V}\left(\varDelta, \varDelta k_{u_m}\right) = \varDelta\dot{\varDelta} + \lambda \varDelta k_{u_m} \varDelta \dot{k}_{u_m}$$

where

$$\varDelta \dot{k}_{u_m} = -sign\left(\frac{1}{\lambda}\right) u_m \varDelta \tag{34}$$

Substituting into the right side of the expression for the derivatives from the equations of the adaptive control system (18) and (16), we obtain [10-18]

$$\begin{aligned}\dot{V}\left(\varDelta, \varDelta k_{u_m}\right) &= \varDelta\left(A_m \varDelta + u_m \varDelta k_{u_m}\right) + \lambda \varDelta k_{u_m} \varDelta \dot{k}_{u_m} = A_m \varDelta^2 + \varDelta u_m \varDelta k_{u_m} + \lambda \varDelta k_{u_m} \varDelta \dot{k}_{u_m} = \\ &= A_m \varDelta^2 + \varDelta u_m \varDelta k_{u_m} + \lambda \varDelta k_{u_m}\left(sign\left(\frac{1}{\lambda} u_m \varDelta\right)\right) = -A_m \varDelta^2.\end{aligned}$$

In the synthesized system, the Lyapunov function in the form of a quadratic form (17) ensures its Lyapunov stability. The positive definiteness of $V\left(\Delta, \Delta k_{u_m}\right) > 0$and $\dot{V}\left(\Delta, \Delta k_{u_m}\right) \leq 0$, as well as the boundedness of the function $V\left(\Delta, \Delta k_{u_m}\right)$and variables e, $\Delta, \Delta k_{u_m}$ (see (17)), ensure that they remain bounded. In addition, over time, the quadratic form $V\left(\Delta, \Delta k_{u_m}\right)$tends to a finite limit at $t \to \infty$ [18-20].

## 4 Conclusion

Taking into account the limitedness of the driving action $u_m$, it follows from $\dot{\Delta} = -A_m \Delta + u_m \Delta k_{u_m}$ that the derivative $\dot{V}$ and the second derivative $\ddot{V}\left(\Delta, \Delta k_{u_m}\right) = 2A_m \Delta\dot{\Delta}$ are also limited. Therefore, the first derivative $\dot{V}$is uniformly continuous, according to Barbalata's lemma $\dot{V} \to 0$, and as a result,$\Delta(t) \to \infty$ at $t \to \infty$

From the analysis performed, it follows that the boundedness of the variables and the convergence of the tracking error $\Delta(t)$ to zero are guaranteed for arbitrary positive values $A_m$of and $\lambda$.

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