Propagation of purely shear viscose-elastic waves with allowance for the of particle rotation in a flat layer

Mukhsin Teshaev[1,2*], *Maxsud* Axmedov[3], *Mavluda* Ruziyeva[3], *Ra'no* Sobirova[3], and *Sharif* Axmedov[4]

[1]Asian International University, 74, Gijduvan str., Bukhara city, Uzbekistan
[2] Bukhara branch of Institute of Mathematics named after V.I. Romanovski, 11, M.Iqbol str., Bukhara city, Uzbekistan
[3] Bukhara Engineering-Technological Institute, 15, K. Murtazayev str., Bukhara city, Uzbekistan
[4] Bukhara Institute of Natural Resources Management, 32, highway Gazli, Bukhara city, Uzbekistan

Abstract. The paper studies the propagation of shear elastic waves in a layer on the basis of a simplified Cosserat model. One of the planes bounding the layer is loaded with inertial mass, the second boundary is fixed. The possibility of the appearance of a localized (surface) wave along the loaded surface of the layer is established. The velocity of wave propagation is determined in the short-wave and long-wave approximations.

1 Introduction

An analysis of the works of dedications on the moment theory of elasticity shows that taking into account the moment in the reduction equations in the work of Cosserat [1] is of great interest. Currently founding in the fields of seismology, there is a peak of interest in this topic. On the one hand, this interest is due to the rapid development of nanotechnologies and the need for models that describe the behavior of media with a microstructure. On the other hand, in connection with the further development of seismic research methods, there is a need for physical models that more accurately describe real geological environments. For example, with the development of the concepts of the micropolar continuum, which simultaneously take into account both translational displacements and kinematic independent microrotations of individual blocks, it turned out to be possible within the framework of one model to describe the propagation of tectonic solitary waves emitting seismic elastic precursors [2]. The result obtained is of fundamental importance, since it can be used to construct a model of the source of tectonic earthquakes within the framework of the micropolar continuum. In seismic and geophysical research at present.

Mechanical [3] or laser [4] sensors are used (although not very widely) to directly measure the rotational velocities of an elastic or seismic wave in three perpendicular directions. This indicates the fundamental possibility of measurements showing the nature of the relationship between the displacement and rotation vectors. In such experiments, it is

* Corresponding author: nasriddinaxmedov@gmail.com

often assumed that the rotation components are related to displacements by a relation that corresponds either to the classical theory of elasticity or to an asymmetric theory with constrained rotation. The localization of the energy of a propagating elastic wave in the vicinity of the boundary that bounds the medium has both negative (destruction) and positive (lossless energy transfer) significance in practice. The results of the study of localized waves (oscillations) lead to the creation of various kinds of devices and instruments, both for their damping and amplification [4,5,6].

In this paper, we consider the problem of the propagation of viscoelastic waves in a layer subordinating the Cosserat medium, i.e., taking into account the rotation of the stress and couple stress tensors, which are asymmetric [7,8]. In contrast to known works, the viscoelastic properties of the layer material are taken into account. Numerical results are obtained.

2 Methods

2.1 Problem statement

The propagation of purely shear viscoelastic waves is considered taking into account the inertia of rotation of particles in a flat layer. In a rectangular Cartesian coordinate system x, y, z layer occupies an area $-\infty < x < \infty, 0 \leq y \leq h, -\infty < z < \infty$. B In this paper, we consider an anti-planar problem $u = v = 0, w = w(x, y, t)$

The equation of motion in stresses has the form [9,10]

$$\frac{\partial \sigma_{13}}{\partial x} + \frac{\partial \sigma_{23}}{\partial y} = \rho \frac{\partial^2 w}{\partial t^2} \ , \tag{1}$$

where indexes 1,2,3 mean directions by coordinates x, y, z respectively, $\rho -$ density. The relationship between stresses and mixed for a viscoelastic Cosserat medium based on a simplified model is taken as [11,12]

$$\sigma_{13} = \tilde{c}_{44} \frac{\partial w}{\partial x} + I \frac{\partial^3 w}{\partial x \, \partial t^2}, \sigma_{31} = \tilde{c}_{44} \frac{\partial w}{\partial x} - I \frac{\partial^3 w}{\partial x \, \partial t^2},$$
$$\sigma_{23} = \tilde{c}_{44} \frac{\partial w}{\partial y} + I \frac{\partial^3 w}{\partial y \, \partial t^2}, \sigma_{32} = \tilde{c}_{44} \frac{\partial w}{\partial y} - I \frac{\partial^3 w}{\partial y \, \partial t^2}, \tag{2}$$

where

$$\tilde{c}_{44} f(t) = c_{044} \left[f(t) - \int_{-\infty}^{t} R_{c44}(t-\tau) f(\tau) d\tau \right]$$

$\tilde{c}_{44}$ is the operator moduli of shear elasticity, $R_{c44}(t-\tau)$ is the relaxation nuclei and c_{044} is the instantaneous moduli of elasticity. $f(t)$-is the arbitrary function of time, I -is the coefficient characterizing the inertia of rotation of the particles of the medium.

Paying to attention (2), the equation for the propagation of shear waves in the medium will be [13,14]

$$c_{044} \Delta w - c_{044} \int_{-\infty}^{t} R_{c44} (t-\tau) \Delta w(x, y, \tau) d\tau + I \frac{\partial^2}{\partial t^2} \Delta w = \rho \frac{\partial^2 w}{\partial t^2}, \tag{3}$$

where Δ is the two-dimensional Laplace operator.

It is assumed that on the plane $y = 0$, bounding the layer, there is a boundary condition [15,16]

$$\sigma_{23} = m \frac{\partial^2 w}{\partial t^2} \tag{4}$$

Using the boundary condition (4), the influence of the presence of an inertial mass on the plane is modeled $y = 0$. The second plane bounding the layer is assumed to be fixed

$$w = 0 for y = h \tag{5}$$

It is required to find a solution to the integro-differential equation (3) that satisfies the boundary conditions (4) and (5).

2.2 Solution method

Solution equation (3) is presented as follows:

$$w = f(y)e^{i(\omega t - kx)} \quad , \tag{6}$$

where $\omega = \omega_R + i\omega_I$- complex circular frequency, k- wave number.

Substitution (6) into (3) leads to the solution of an ordinary differential equation with respect to the desired function $f(y)$

$$f''(y) - k^2\left(1 - \frac{\eta}{1-\chi\eta}\right)f(y) = 0, \tag{7}$$

where the following designations are accepted:

$$\eta = \frac{\omega^2}{k^2 c_t^2}, \bar{c}_t^2 = \frac{c_{440}}{\rho}\Gamma_{44}, \chi = \frac{k^2 I}{\rho}, \Gamma_{44} = 1 - \Gamma_{44}^c - i\Gamma_{44}^s \tag{8}$$

$$\Gamma_{44}^c(\omega_R) = \int_0^\infty R_{44}(\tau)\cos\omega_R\tau d\tau; \Gamma_{44}^s(\omega_R) = \int_0^\infty R_{44}(\tau)\sin\omega_R\tau d\tau -$$

Cosine and sine Fourier transforms of the material relaxation kernel respectively;ω_R- real value.

3 Result and discussion

The three-parameter relaxation kernel of Koltunov-Rzhanitsyn was used in the calculations:

$$R_{44}(t) = A_{44}e^{-\beta_{44}t}/t^{1-\alpha_{44}}.$$

According to (6), the boundary condition of the loaded edge (4) is reduced to a differential equation with complex coefficients in the form

$$(1-\chi\eta)\, f' + \frac{mk^2\eta}{\rho}f = 0 \text{ for } y = 0. \tag{9}$$

Here

$$\eta = \frac{\omega^2}{k^2 c_t^2}\frac{1-\Gamma_{44}^c}{[(1-\Gamma_{44}^c)^2 + (\Gamma_{44}^s)^2]} + i\frac{\omega^2(\Gamma_{44}^s)}{k^2 c_t^2[(1-\Gamma_{44}^c)^2 + (\Gamma_{44}^s)^2]}.$$

The solution to equation (7) satisfying the boundary condition (9) will be

$$f(y) = A\big((i-\rho_\eta)e^{-iky} - (i+\rho_\eta)e^{iky}\big), \rho_\eta = \frac{\rho(1-\chi\eta)p}{km\eta} \quad , \tag{10}$$

where A arbitrary constant,

$$p = \sqrt{1 - \frac{(\eta_R + i\eta_I)[(1-\chi\eta_R) - i\chi\eta_I]}{[(1-\chi\eta_R)^2 - (i\chi\eta_I)^2]}} \quad ,$$

$$\eta_R = \frac{\omega^2}{k^2 c_t^2}\frac{1-\Gamma_{44}^c}{[(1-\Gamma_{44}^c)^2 + (\Gamma_{44}^s)^2]}, \eta_I = \frac{\omega^2(\Gamma_{44}^s)}{k^2 c_t^2[(1-\Gamma_{44}^c)^2 + (\Gamma_{44}^s)^2]}. \tag{11}$$

It follows from (11) that Eq. (7) has a solution in the form of hyperbolic functions (in the form (10)) if the condition

$$0 < |\eta| < \frac{1}{1+\chi} \tag{12}$$

Otherwise, the solution of equation (7) will be expressed in terms of trigonometric functions. If equation (7) has a solution that satisfies condition (12), the shear wave is localized in the vicinity of the layer surface $y = 0$ [9,10]. In particular, for the half-space $0 \leq y < \infty$ condition (12) will be the conditions for the damping of the wave amplitude at $y \to \infty (w \to 0)$. The requirement that solution (10) satisfy the fixed edge boundary condition (5) leads to the solution of the dispersion equation with respect to the dimensionless parameter η wave propagation speed

$$tg(kph) = \frac{1-\chi\eta}{\theta\eta}p, \theta = \frac{mk}{\rho} \tag{13}$$

In the general case, it is difficult to find a solution to (13), because the argument of the trigonometric functions is complex. Therefore, we consider some special cases. Let $R_{44}(t) =$

0, then in the shortwave approximation $kh \approx 1$or $t\,g\,(kph) \approx 1$ equation (reduced) to the form

$$\sqrt{1-\chi\eta}\sqrt{1-\eta(1+\chi)} = \theta\eta \tag{14}$$

The solutions of equation (14) satisfying the damping condition (12) will be

$$\eta = \frac{1+2\chi-\sqrt{1+4\theta^2}}{2(\chi(1+\chi)-\theta^2)} \tag{15}$$

It is easy to check that the velocity of propagation of a localized wave, determined by formula (15), coincides with the velocity of a surface shear wave (of the Love type) for a half-space with a loaded surface and taking into account the internal rotation of particles. Equation (13) has a solution $\eta = (1+\chi)^{-1}$ which sets the layer thickness at which the transition from solutions in the form of trigonometric functions (to localized waves) occurs. The conditions for the appearance of localized waves are obtained from Eq. (13) when passing to the limit

$$\eta \to (1+\chi)^{-1}(p \to 0)$$

$$kh = \frac{1}{\theta} \tag{16}$$

From equation (13) in the long-wavelength approximation $(kh)^2 \approx 1$ at $kh < \frac{1}{\theta}$ it turns out

$$\eta = (\chi + kh\theta)^{-1}\,. \tag{17}$$

In the approximation adopted here ω does not depend on the wavenumber, so it is necessary to consider the following approximation.

When the layer boundary $y = h$ free

$$\sigma_{23} = 0 \text{ at } \; y = h\,, \tag{18}$$

the dispersion equation is obtained in the form

$$ctg(kph) = \frac{1-\chi\eta}{\theta\eta}p. \tag{19}$$

In the short-wavelength approximation, from (19) an equation is obtained that coincides with equations (14). In the long wave approximation

$$\eta = \frac{kh}{(1+\chi)kh+\theta} \tag{20}$$

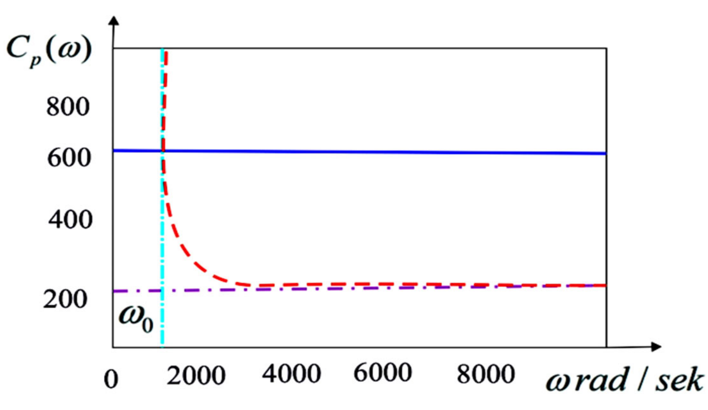

Fig. 1. Dependences of phase velocities on frequency in the Cosserat medium - longitudinal wave.

According to (20), the shear wave propagation velocity satisfies the damping condition (12), i.e. in this case, a localized wave always exists (regardless of the layer thickness or kh).

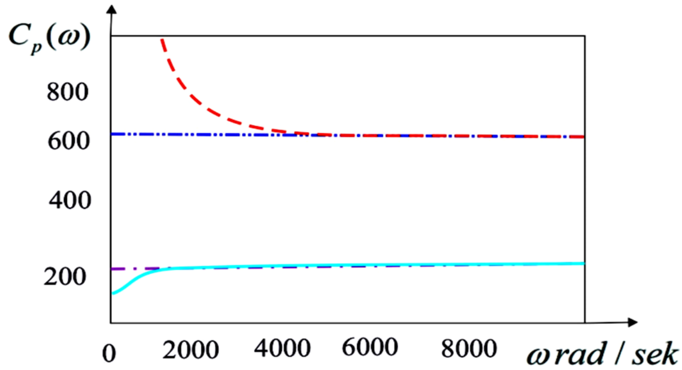

Fig. 2. Dependences of phase velocities on frequency in a Cosserat medium - a transverse wave

Figures 1 and 2 show the frequency dependences of the phase velocities for the longitudinal and transverse waves in the Cosserat medium, respectively. On Figure 1, the solid line denotes the phase velocity of the body wave of displacement, the dashed line is the phase velocity of the body wave of rotations, which has an asymptotic value, at $\omega\to\infty$, as well as the vertical asymptote at $\omega\to\omega_0$. In Figure 2, the solid and dashed lines denote two phase velocities for two bulk shear modes. They have, respectively, two asymptotic values at $\omega\to\infty$.

4 Conclusions

The study of the propagation of shear elastic waves in a layer based on the simplified Cosserat model, when one of the planes bounding the layer is loaded with an inertial mass, and the second boundary is fixed, revealed the possibility of the appearance of a localized (surface) wave along the loaded layer surface. The wave propagation velocity and damping coefficients are determined in the short-wave and long-wave approximations.

Reference

1. R.L. Nigbor, Bulletin of the Seismological Society of America **84(5)**, 1665-1669 (1994)
2. G.N. Savin, Applied. Mechanics **6(6)**, 37-41 (1970)
3. A.E. Lyalin, V.A. Pirozhkov, R.D. Stepanov, Acoustic journal. **28(6)**, 236-241 (1982)
4. V. Novatsky, *Theory of elasticity* (Mir, Moscow, 1975)
5. M.A. Kulesh, Applied Mechanics and Technical Physics **46(4)**, 116-124 (2005)
6. M.A. Kulesh, Izvestiya RAN. Mechanics solid body **4**, 100-113 (2007)
7. M.A. Kulesh, V.P. Matveenko, I.N. Shardakov, Acoustic journal **52(2)**, 227-235 (2006)
8. D.N. Mikhailov, Physics of the Earth **10**, 3-10 (2000)
9. L.M. Schwartz, D.L. Johnson, S. Feng, Physical eview Letters **52(10)**, 831-834 (1984)
10. M.A. Kulesh, Applied mechanics and technical physics **48(1)**, 143-150 (2007)
11. G. Korn, *Handbook of mathematics* (for scientists and engineers) (St. Petersburg, Lan, 2003)
12. I.I. Safarov, M.Kh. Teshaev, Z.I. Boltaev, Journal of Critical Reviews **7(12)**, 893-904 (2020). https://www.doi.org/10.31838/jcr.07.12.157

13. I.I. Safarov, M.Kh. Teshaev, J.A. Yarashev, IOP Conference Series: Materials Science and Engineering **1030(1)**, 012073 (2021). https://www.doi.org/10.1088/1757-899X/1030/1/012073
14. I.I. Safarov, Sh. N. Almuratov, M.Kh. Teshaev, F.F. Homidov, D.G. Rayimov, Indian Journal of Engineering **17(47)**, (2020)
15. I.I. Safarov, F.F. Homidov, B.S. Rakhmonov, Sh.N. Almuratov, Journal of Physics Conference Series **1706(1)**, 012125 (2020). https://www.doi.org/10.1088/1742-6596/1706/1/012125
16. I.I. Safarov, M.Kh. Teshaev, Z.I. Boltayev, T.R. Ruziev, International Journal of Engineering Trends and Technology **70(6)**, 252-256 (2022). https://doi.org/10.14445/22315381/IJETT-V70I6P226