

# Force parameters of roller machines

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**Abstract.** To facilitate the force analysis of the roller machine under consideration, the previously obtained model of the pattern of distribution of normal contact forces is approximated by an empirical dependence. The moduli of the main vector of elementary normal and tangential forces of each roll, the main moments of elementary forces relative to the axis of each roll are determined. It is revealed that the qualitative distribution pattern of normal contact forces gives the magnitude of forces and moments, expressed through the maximum of the normal force.

## 1 Introduction

One of the universal machines performing various technological processes, used in many industries, including the mining and metallurgical industry is a roller machine.

Determining the forces acting on the links of any machine is of practical importance for calculating parts for strength and rigidity and calculating wear, etc. In view of the stationarity of the operating conditions of roller machines and the balance of their links, it is appropriate to use mainly static methods for force calculation [1].

To determine the power parameters, mathematical models of the shape of the roll contact curves and the patterns of distribution of contact forces are necessary [2].

The variety of purposes of machines, the difference in their parameters and the difference in the material being processed led to the publication of a large number of articles devoted to mathematical modeling of the shape of roll contact curves [2–6] and the patterns of distribution of contact forces [7–17]. The calculation of the power parameters of roller machines in these publications was performed using the mathematical models obtained.

The calculation formulas of the force parameters obtained so far are complex and approximate, which does not allow us to establish their values during the design and operation of roller machines with the accuracy required in the technology.

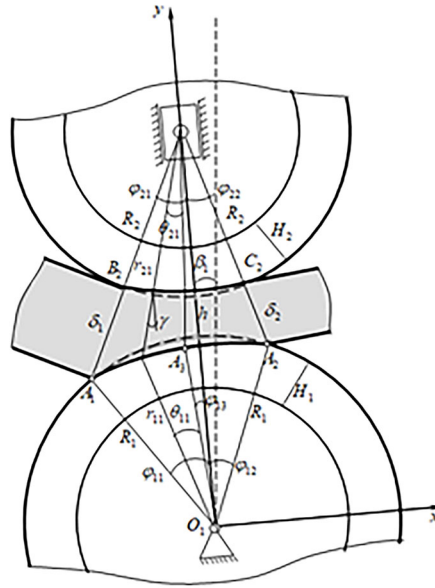
## 2 Materials and methods

Consider the generalized scheme of the roller machine [3], shown in Figure 1.

In the scheme under consideration, we consider the lower roll to be the first roll, and the upper roll to be the second roll. We also consider that in each roll the contact curve consists of compression and recovery zones.

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**Fig. 1.** Scheme of the roller machine in a steady state.

In polar coordinates with a pole in the center of the lower (or upper) roll, the shape of the roll contact curve and the patterns of distribution of contact forces can be expressed by the following equations:

$$r_{ij} = r_{ij}(\theta_{ij}), \quad n_{ij} = n_{ij}(\theta_{ij}), \quad t_{ij} = t_{ij}(\theta_{ij}),$$

where  $i$  – is the index indicating the roll number;  $j$  – is the index indicating the zone number;  $r_{ij}, \theta_{ij}$  – are the radius vector and polar angle;  $n_{ij}, t_{ij}$  – are normal and tangential contact forces.

In a steady process, each roll is subjected to the pressure force of clamping devices of roll  $\bar{Q}$ , the reaction of the roll supports  $\bar{F}$ , the moment in the roll supports  $M$ , elementary normal contact forces  $n$  and elementary tangential contact forces  $t$ .

Previously, in [3, 10, 11], mathematical models of the shape of roll contact curves and patterns of distribution of contact forces for the considered roller machine were obtained.

For the lower roll, they have the following form:

1) model of the shape of roll contact curves

$$r_{11} = \frac{R_1}{1 + k_{11}\lambda_{11}} \left( 1 + k_{11}\lambda_{11} \frac{\cos \zeta_{11}}{\cos \vartheta_{11}} \right), \quad -\zeta_{11} \leq -\vartheta_{11} \leq 0, \quad (1)$$

$$r_{12} = \frac{R_1}{1 + k_{12}\lambda_{12}} \left( 1 + k_{12}\lambda_{12} \frac{\cos \zeta_{12}}{\cos \vartheta_{21}} \right), \quad -\zeta_{12} \leq -\vartheta_{12} \leq 0, \quad (2)$$

where  $k_{11} = \frac{m_{11}H_1}{m_1^*\delta_1}$ ,  $\varphi_{13} = \frac{\varphi_{11} - \varphi_{12}}{2}$ ,  $\vartheta_{11} = \theta_{11} + \varphi_{13} - \gamma$ ,  $\zeta_{11} = \varphi_{11} - \gamma_1$ ,  $\gamma_1 = c_1\varphi_{12}$ ,

$$c_1 = \sqrt{\frac{R_1(2R_1 - \delta_1 + h)}{(R_2 + \delta_1)(2R_2 + \delta_1 + h)}}, \quad k_{12} = \frac{m_{12}H_1}{m_2^* \delta_2}, \quad \gamma_2 = c_2 \varphi_{21}, \quad c_2 = \sqrt{\frac{R_1(2R_1 - \delta_2 + h)}{(R_2 + \delta_2)(2R_2 + \delta_2 + h)}}$$

$\vartheta_{12} = \theta_{12} + \varphi_{13} + \gamma$ ,  $\zeta_{12} = \varphi_{12} + \gamma_2$ ,  $m_{11}, m_{12}$  – are the hardening coefficients of the points of elastic coating of the lower roll in the compression and recovery zones,  $m_1^*, m_2^*$  – are the coefficients of strengthening of the points of the processed material in the compression and recovery zones;  $\lambda_{11}, \lambda_{12}$  are the indicators that determine the ratio of the rate of deformation of the lower roll coating to the rate of deformation of the processed material in the compression and recovery zones;

2) models of patterns of distribution of contact forces

$$n_{11} = B_{11} \left( 1 - \frac{\cos \zeta_{11}}{\cos \vartheta_{11}} \right)^{s_1} \cos \psi_{11}, \quad -\zeta_{11} \leq -\vartheta_{11} \leq 0, \quad (3)$$

$$n_{12} = B_{12} \left( 1 - \frac{\cos \zeta_{12}}{\cos \vartheta_{12}} \right)^{s_2} \cos \psi_{12}, \quad 0 \leq \vartheta_{12} \leq \zeta_{12}, \quad (4)$$

$$t_{11} = B_{11} \left( 1 - \frac{\cos \zeta_{11}}{\cos \vartheta_{11}} \right)^{s_1} \cos \psi_{11} \operatorname{tg}(\vartheta_{11} \psi_{11} + \xi_1), \quad -\zeta_{11} \leq -\vartheta_{11} \leq 0, \quad (5)$$

$$t_{12} = B_{12} \left( 1 - \frac{\cos \zeta_{12}}{\cos \vartheta_{12}} \right)^{s_1} \cos \psi_{12} \operatorname{tg}(\vartheta_{12} - \psi_{12} + \xi_1), \quad 0 \leq \vartheta_{12} \leq \zeta_{12}, \quad (6)$$

where  $B_{11} = C_1 \left( \frac{R_1}{\delta_1(1 + k_{11}\lambda_{11})} \right)^{s_1}$ ,  $\psi_{11} = \frac{k_{11}\lambda_{11} \cos \zeta_{11}}{\cos \vartheta_{11} + k_{11}\lambda_{11} \cos \zeta_{11}} \vartheta_{11}$ ,  $\xi_1 = \operatorname{arctg} \frac{F_1}{Q_1}$ ,

$$\xi_2 = \operatorname{arctg} \frac{F_2}{Q_2}, \quad B_{12} = C_2 \left( \frac{R_1}{\delta_2(1 + k_{12}\lambda_{12})} \right)^{s_2}, \quad \psi_{12} = \frac{k_{12}\lambda_{12} \cos \zeta_{12}}{\cos \vartheta_{12} + k_{12}\lambda_{12} \cos \zeta_{12}} \vartheta_{12}.$$

To analyze the distribution pattern of contact forces, the maximum value and the maximum point of the graph of the distribution of normal forces have special meanings.

Let this point be determined by the angle  $\varphi_{14}$ .

According to the condition of the function maximum, differentiating expressions (3), we find  $n'_{11}$  and equate it to zero at point  $-\vartheta_{11} = -\zeta_{14} = -\varphi_{14} + \varphi_{13} + \gamma$ .

Since the angles  $\varphi_{14}$  and  $\varphi_{13}$  are close to zero [2, 10], we can assume that  $\sin \zeta_{14} \approx \zeta_{14}$ ,  $\cos \zeta_{14} \approx 1$ . Then, taking into account expression (3), we obtain

$$\varphi_{14} = -\varphi_{13}, \quad n_{1\max} = B_{11} \left( \frac{\varphi_{11} + \varphi_{21}}{2} \right)^{2s_1}. \quad (7)$$

To facilitate the force analysis, models (3) and (4) are approximated by simpler empirical dependences.

In [2, 4], the distribution patterns of normal contact forces obtained in experimental and theoretical studies are analyzed and described by empirical formulas (uniform, elliptical, parabolic, harmonic, and exponential ones). A comparative analysis of the graphs of these patterns with the graphs of dependences (3) and (4) showed that models (3) and (4) correspond to the hyperbolic law of distribution of normal contact forces, described by dependences of the following form [2, 4]:

$$n = \frac{n_{\max}}{2} \left( 1 + \cos \frac{\pi}{\varphi} \theta \right).$$

Proceeding from this, we approximate the patterns of distribution of normal contact forces (3) and (4) by the following formulas:

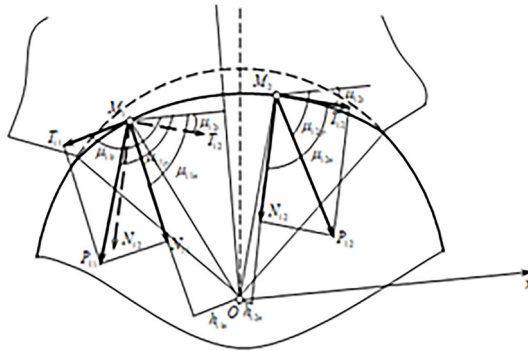
$$n_{11} = \frac{n_{1\max}}{2} \left( 1 + \cos \frac{\pi}{\zeta_{11}} \vartheta_{11} \right), \quad -\zeta_{11} \leq -\vartheta_{11} \leq 0, \quad (8)$$

$$n_{12} = \frac{n_{1\max}}{2} \left( 1 + \cos \frac{\pi}{\zeta_{12}} \vartheta_{12} \right), \quad 0 \leq \vartheta_{12} \leq \zeta_{12}, \quad (9)$$

where  $n_{1\max}$  is determined by formula (7).

Projecting elementary forces onto the  $Ox$  and  $Oy$  axes, we obtain (Figure 2):

$$\begin{aligned} dN_{11x} &= dN_{11} \sin(\vartheta_{11} - \psi_{11}), & dN_{11y} &= -dN_{11} \cos(\vartheta_{11} - \psi_{11}), \\ dT_{11x} &= -dT_{11} \cos(\vartheta_{11} - \psi_{11}), & dT_{11y} &= -dT_{11} \cos(\vartheta_{11} - \psi_{11}). \end{aligned}$$



**Fig. 2.** Scheme for determining the value of elemental forces of the lower roll.

According to [10], the moduli of elementary contact forces of the compression zone of the lower roll are expressed as:

$$dN_{11} = n_{11} \sqrt{r_{11}^2 + r_{11}'^2} d\vartheta_{11}, \quad dT_{11} = t_{11} \sqrt{r_{11}^2 + r_{11}'^2} d\vartheta_{11}.$$

Transforming the functions of the difference of angles taking into account  $\cos \psi_{11} = \frac{r_{11}}{\sqrt{r_{11}^2 + r_{11}'^2}}$  and  $\sin \psi_{11} = \frac{r_{11}'}{\sqrt{r_{11}^2 + r_{11}'^2}}$  we obtain expressions for the projections of elementary forces onto the coordinate axes:

$$dN_{11x} = n_{11} (r_{11} \sin \vartheta_{11} - r_{11}' \cos \vartheta_{11}) d\vartheta_{11}, \quad dN_{11y} = -n_{11} (r_{11} \cos \vartheta_{11} + r_{11}' \sin \vartheta_{11}) d\vartheta_{11},$$

$$dT_{11x} = -t_{11} (r_{11} \cos \vartheta_{11} + r_{11}' \sin \vartheta_{11}) d\vartheta_{11}, \quad dT_{11y} = -t_{11} (r_{11} \sin \vartheta_{11} - r_{11}' \cos \vartheta_{11}) d\vartheta_{11}.$$

Differentiating (1), we obtain

$$r_{11}' = \frac{dr_{11}}{d\vartheta_{11}} = \frac{k_{11} \lambda_{11} R_{11}}{1 + k_{11} \lambda_{11}} \cdot \frac{\cos \zeta_{11}}{\cos^2 \vartheta_{11}} \sin \vartheta_{11}. \quad (10)$$

After substitution of  $r_{11}$  and  $r_{11}'$  in expressions of elementary forces, we determine:

$$dN_{11x} = n_{11} \frac{R_{11}}{1 + k_{11}} \sin \vartheta_{11} d\vartheta_{11}, \quad dN_{11y} = -n_{11} \frac{R_{11}}{1 + k_{11} \lambda_{11}} \left( \cos \vartheta_{11} + k_{11} \lambda_{11} \frac{1}{\cos^2 \vartheta_{11}} \right) d\vartheta_{11},$$

$$dT_{11x} = -t_{11} \frac{R_{11}}{1+k_{11}\lambda_{11}} \left( \cos \vartheta_{11} + k_{11}\lambda_{11} \frac{1}{\cos^2 \vartheta_{11}} \right) d\vartheta_{11}, \quad dT_{11y} = -t_{11} \frac{R_{11}}{1+k_{11}} \sin \vartheta_{11} d\vartheta_{11}. \quad (11)$$

We substitute expressions  $n_{11}$  from equation (8) into the first equality of expression (11)

$$dN_{11x} = \frac{n_{1\max} R_1}{2(1+k_{11}\lambda_{11})} \left( 1 + \cos \frac{\pi}{\zeta_{11}} \vartheta_{11} \right) \sin \vartheta_{11} d\vartheta_{11}.$$

Hence, we find

$$N_{11x} = \frac{n_{1\max} R_1 (\pi^2 - 4)}{4(1+k_{11}\lambda_{11})}. \quad (12)$$

We substitute expressions  $n_{11}$  from equation (8) into the second equality of expression (11)

$$dN_{11y} = -\frac{n_{1\max} R_{11}}{1+k_{11}\lambda_{11}} \left( 1 + \cos \frac{\pi}{\zeta_{11}} \vartheta_{11} \right) \left( \cos \vartheta_{11} + k_{11}\lambda_{11} \frac{1}{\cos^2 \vartheta_{11}} \right) d\vartheta_{11}.$$

Let us perform an integration

$$N_{11y} = -\frac{n_{1\max} R_1 \pi^2 (\varphi_{11} - \gamma_1)}{6}. \quad (13)$$

From expressions (3) and (5), we have  $t_{11} = n_{11} \operatorname{tg}(\vartheta_{11} + \xi_1)$  or in the first approximation

$$t_{11} = n_{11} (\operatorname{tg} \vartheta_{11} + D_1), \quad (14)$$

where  $D_1 = \frac{F_1}{Q_1}$  – is the dynamic coefficient of the lower roll [11].

With expressions (8) and (14) from the third equality of expression (11), we obtain

$$T_{11x} = -\frac{n_{1\max} R_{11}}{1+k_{11}\lambda_{11}} \int_{-\zeta_{11}}^0 \left( 1 + \cos \frac{\pi}{\zeta_{11}} \vartheta_{11} \right) \left( \cos \vartheta_{11} + k_{11}\lambda_{11} \frac{1}{\cos^2 \vartheta_{11}} \right) (\operatorname{tg} \vartheta_{11} + D_1) d\vartheta_{11}.$$

After integration, we find

$$T_{11x} = -\frac{n_{1\max} R_1}{12(1+k_{11}\lambda_{11})} (3(\pi^2 - 4)(1 - k_{11}\lambda_{11}) - 2\pi^2 D_1 (\varphi_{11} - \gamma_1)). \quad (15)$$

Substituting expressions  $t_{11}$  from equation (14) into the fourth equality of expression (11) and after integration, we have

$$T_{11y} = -\frac{n_{1\max} R_1 (\pi^2 - 4)}{4(1+k_{11}\lambda_{11})} D_1.$$

Using formulas (12) and (13) we find the modulus of the main vector of normal forces for the compression zone of the lower roll:

$$N_{11} = \frac{n_{1\max} R_1}{12(1+k_{11}\lambda_{11})} \sqrt{9(\pi^2 - 4)^2 + 4\pi^4 (1+k_{11}\lambda_{11})^2 (\varphi_{11} - \gamma_1)^2}. \quad (16)$$

By analogy, we find the modulus of the main vector of tangential forces for the compression zone of the lower roll:

$$T_{11} = \frac{n_{1\max} R_1}{12(1+k_{11}\lambda_{11})} \sqrt{(3(\pi^2 - 4)(1 - k_{11}\lambda_{11}) - 2\pi^2 D_1 (\varphi_{11} - \gamma_1))^2 + 9(\pi^2 - 4)^2 D_1^2}. \quad (17)$$

The moments of elementary normal and tangential forces relative to the roll axis are defined as [2]:

$$dM_n = nrr'd\theta, \quad dM_t = tr^2 d\theta.$$

Therefore, the main moment of elementary forces relative to the axis of the lower roll for the roller machine under consideration is determined by the following formula:

$$M_{1e} = M_{11e} + M_{12e} = \int_{-\zeta_{11}}^0 (n_{11}r'_{11}r'_{11} + t_{11}r_{11}^2) d\mathcal{G}_{11} + \int_0^{\zeta_{22}} (n_{12}r'_{12}r'_{12} + t_{12}r_{12}^2) d\mathcal{G}_{12}.$$

Solution for compression zone is:

$$M_{11e} = \frac{n_{1\max}}{2} \int_{-\zeta_{11}}^0 \left( 1 + \cos \frac{\pi}{\zeta_{11}} \mathcal{G}_{11} \right) (r_{11}r'_{11} + (D_1 + tg \mathcal{G}_{11})r_{11}^2) d\mathcal{G}_{11};$$

$$M_{11e} = \frac{n_{1\max}R_1^2}{8(1+k_{11}\lambda_{11})} ((1+k_{11}\lambda_{11})(\pi^2+4) + 2D_1(1+k_{11}\lambda_{11})(\varphi_{11}-\gamma_1) - k_{11}\lambda_{11}(\pi^2+4)(\varphi_{11}-\gamma_1)^2).$$

The moduli and the main moment of elementary forces for the recovery zone, as well as for the upper roll, are determined similarly.

### 3 Results

To facilitate the force analysis of the considered roller machine, the previously obtained model of the distribution pattern of normal contact forces is approximated by an empirical dependence.

As a result of the force analysis of the generalized scheme of the roller machine, the following values were determined:

- the modulus of the main vector of elementary normal forces of each roll;
- the modulus of the main vector of elementary tangential forces of each roll;
- the modulus of the resultant elementary forces (normal and tangential) of each roll;
- the main moments of elementary forces relative to the axis of each roll.

### 4 Conclusions

On the basis of comparative analysis, it was revealed that models (3) and (4) correspond to the hyperbolic law of distribution of normal contact forces.

The qualitative pattern of distribution of normal contact forces gives the magnitude of the forces and moments expressed by  $n_{\max}$ .

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