Non-axisymmetric problems of unsteady deformation of cylindrical shells with filler

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Abstract. Circular cylindrical shells, as structural elements, have found wide application in various fields of mechanical engineering. Thin-walled layered shells have also found wide application as components in underwater objects, air and space vehicles and in many other engineering structures. The aim of this work is to study the action of a non-axisymmetric moving wave of normal pressure on a cylindrical shell interacting with an ideal compressible fluid. The problem statement is given, solution methods are developed, and numerical results are obtained for new problems of stationary deformation of infinitely long viscoelastic cylindrical shells on a viscoelastic foundation when a non-axisymmetric normal pressure wave moves along the shell axis with up to resonant velocity. As an example, considered the action of a non-axisymmetric moving wave of normal pressure on a cylindrical shell interacting with an ideal compressible fluid. The solution methods are based on the joint application of the integral Fourier transform along the axial coordinate and the expansion of all given and desired values into Fourier series along the angular coordinate. An efficient algorithm for the joint calculation of integrals and Fourier series has been developed and implemented on a computer. It has been established that paying to attention the viscoelastic properties of the shell material reduces the deflections by 10–15%, and also makes it possible to evaluate the damping capabilities of the system.

1 Introduction

Circular cylindrical shells, as structural elements, have found wide application in various fields of mechanical engineering, and thin-walled layered shells have found wide application as components in underwater objects, air and space vehicles and in many other engineering structures. Of great practical interest is the study and elimination of resonant phenomena in shells with filler. A significant number of theoretical and experimental works have been devoted to the study of stationary deformation of cylindrical shells with a viscoelastic filler [1, 2]. However, there are still no reliable solution methods that allow determining the parameters of resonances in a wide range of changes in physico-mechanical and geometric parameters, paying to attention the rheological parameters of the system. There are also works in which dependences for determining the resonant frequencies [3] and the forms of vibrations of the shell of cylindrical panels with filler [4, 5] are obtained by theoretical and experimental method. Another method based mainly on hypotheses is used to study the dynamics that allow us to move from the stability equations of shells to the corresponding equations for cylindrical shells with a circular cross section.

In many works, the momentary and semi-momentary theory of shells is used [6, 7]. Approximate methods are also used to solve the problems of stationary deformation of cylindrical shells with a viscoelastic filler [8, 9, 10]. Of particular difficulty are the problems of vibrations of layered cylindrical shells in a geometrically nonlinear formulation, paying to attention the rheological properties of the material, solutions for which are practically absent. Thin-walled layered shells are also widely used as components in underwater objects, air and space vehicles and in many other engineering structures [11, 12]. One of the most important tasks in the design of such structures designed to operate in the stationary deformation mode is the problem of effective damping of excited vibrations, non-axisymmetric problems of stationary deformation of cylindrical shells with viscoelastic filler interacting with a liquid, which have been studied by many authors, in particular in [13, 14, 15]. At the same time, axisymmetric and non-axisymmetric problems were considered, various models for the liquid and shell were used. The question of the effect of a moving pressure wave on a cylindrical shell filled or surrounded by a liquid has been less studied, and only axisymmetric loading has been considered [16,17].

In this paper, the problems of the action of a non-axisymmetric moving wave of normal pressure on a cylindrical shell interacting with an ideal compressible fluid are considered. The problem is solved using an integral transformation along the axial coordinate and Fourier series along the angle. The solution of the problem of motion along an infinitely long viscoelastic cylindrical shell interacting with an ideal compressible fluid of normal pressure, arbitrary in length and circumference, but unchanged in time profile is obtained. The speed of movement of the load is constant. It is considered in the case when it moves at a speed lower than the speed of sound in a liquid. The liquid fills the cavity between the viscoelastic radius shell and the rigid cylindrical radius wall coaxial with it $b(b >< a)$.

2 Methods

2.1 Problem statement and solution methods

An infinite length deformable (viscoelastic) cylindrical shell with a constant thickness h_{0j}, density ρ_{0j}, Poisson's ratio $\nu_{0j}(j=1,2)$, filled into a liquid with a density in an equilibrium state is considered. The vibrations of such a shell under load, the density of which we denote $\vec{p}_j(p_{1j}, p_{2j}, p_{nj})$ accordingly, can be described, following [18], by the equations:

$$L\vec{u}_j - \int_{-\infty}^{t} LR_{Ej}(t-\tau)\vec{u}_j(\vec{r},\tau)d\tau = \frac{\left(1-\nu_{0j}^2\right)}{E_{0j}h_{0j}}\left(\vec{p}_j\right) + \rho_{0j}\frac{\left(1-\nu_{0j}{}^2\right)}{E_{0j}}\left(\frac{\partial^2\vec{u}_j}{\partial t^2}\right). \quad (1)$$

Here $\vec{u}_j = \vec{u}_j\left(u_{rj}, u_{\theta j}, u_{zj}\right)$ is the vector of displacements of the points of the median surface of the shell, and for Kirchhoff-Love shells it has a dimension equal to three $\left(u_{rj} = u_j; \quad u_{\theta j} = v_j; \quad u_{zj} = w_j\right)$, and for Timoshenko-type shells the dimension of the vector is five. Here, in addition to axial, circumferential and normal displacements, the angles of rotation of the normal to the median surface in the axial and circumferential directions are added [18]; $\{u_j, v_j, w_j\}^T$ - the displacement vector with axial, circumferential and radial components, respectively (the sign "+" before pn and the sign "–" before the last component of the inertial term indicates that the movement towards the center of curvature is considered positive); $R_{Ej}(t-\tau)$ –relaxation core; E_{0j} –instantaneous modulus of elasticity. The non-axisymmetric motion of the Timoshenko-type shell is described by equations (1), and in the components of the load vector, only the term [18] is different from zero: where $p_3 = -\dfrac{1-\nu}{2Gh}(q_r \mp p_r),$ the minus sign corresponds to the case $b > a$, and plus $-b < a$.

The motion of an ideal incompressible fluid is described by the wave equation

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{\partial^2\varphi}{\partial x^2} = \frac{1}{c_1^2}\frac{\partial^2\varphi}{\partial t^2}. \tag{2}$$

where φ is the velocity potential; c_1 - acoustic velocity of sound in a liquid; p_0 - density of the liquid.

The problem is reduced to the joint integration of equations (1) and (2) when the boundary conditions of the impermeability of the shell and the rigid wall are met.

$$\frac{d\varphi}{dr}\Big|_{r=a} = \frac{\partial w}{\partial t};\frac{d\varphi}{dr}\Big|_{r=b} = 0 \tag{3}$$

In this case, the pressure entering (3) from the liquid side is expressed in terms of the velocity potential according to the formula

$$q_r = -p_0\frac{\partial\varphi}{\partial t}\Big|_{r=a} \tag{4}$$

Considering the steady-state process, we proceed in the equations of motion of the shell and the fluid to the coordinate system moving with the load and apply the Fourier transform according to η [18]. In the image space θ, the solution of the transformed equations is sought in the form of Fourier series along the angular coordinate θ.

$$\begin{gathered}\left\{u^0, w^0, \psi_x^0, p_r^0, q_r^0\right\} = \sum_{n=0}^{\infty}\left\{u_n^0, w_n^0, \psi_{xn}^0, p_{rn}^0, q_{rn}^0\right\}e^{-i\omega t};\\ \left\{v^0, \psi_y^0\right\} = \sum_{n=1}^{\infty}\left\{v_n^0, \psi_{yn}^0\right\}e^{-i\omega t}.\end{gathered} \tag{5}$$

Substituting (5) into the transformed equations of motion of the shell, we obtain a system of algebraic equations for the Fourier coefficients of transformant displacements of the median surface. In this system, the coefficients of decomposition of the fluid pressure are unknown, which must be expressed in terms of the coefficients of normal displacement of the shell. Representing the transformant of the velocity potential in the form (5) and substituting it into the transformed equation (2), we come to the equation

$$\frac{\partial^2\varphi_b^0}{\partial r_*^2} + \frac{1}{r_*}\frac{\partial\varphi_n^0}{\partial r_*} - \left[\frac{n^2}{r_*^2} + \left[1-M^2\right]\xi^2\right]\varphi_n^0 = 0 \tag{6}$$

where $M = c / c_1$ is the Mach number. The solution of equation (6), in subsonic mode of motion $c < c_1$, has the form

$$\varphi_n^0 = A_n(\xi) K_n\left(\beta \xi r_*\right) + B_n(\xi) I_n\left(\beta \xi r_*\right); \beta = \sqrt{1 - M^2} \quad (7)$$

Substituting (7) into (3), (4), we find the connection between the reaction of the liquid and the normal displacement of the shell:

$$q_{r.n}^0 = p_0 c^2 k \xi^2 f(\xi, n, c) \frac{w_n^0}{h},$$

where for $c < c_1$

$$f(\xi, n, c) = \frac{ns_4 - \beta\xi\varepsilon - \left(ns_2 + \beta\xi\varepsilon s_3\right) s_5}{(n + \beta\xi s_1)(ns_4 - \beta\xi\varepsilon) - (ns_2 + \beta\xi\varepsilon s_3)(ns_5 - \beta\xi s_6)}; \quad (8)$$

$$s_1 = \frac{I_{n+1}(\beta\xi)}{I_n(\beta\xi)};\ s_2 = \frac{I_n(\beta\xi\varepsilon)}{I_n(\beta\xi)};$$
$$s_3 = \frac{I_{n+1}(\beta\xi\varepsilon)}{I_n(\beta\xi)};\ s_4 = \frac{I_n(\beta\xi\varepsilon)}{I_{n+1}(\beta\xi\varepsilon)};$$
$$s_5 = \frac{K_n(\beta\xi)}{K_{n+1}(\beta\xi\varepsilon)};\ s_6 = \frac{K_{n+1}(\beta\xi\varepsilon)}{K_{n+1}(\beta\xi\varepsilon)}; \varepsilon = \frac{b}{a}.$$

If the shell is completely filled with liquid, then formula (8) takes the form

$$f(\xi, n, c) = \left(n + \beta\xi s_1\right)^{-1}. \quad (9)$$

Substituting the found connection (8) into the system of algebraic equations for determining the coefficients of expansion of the transformants of the shell displacements, we find

$$\left\{u_n^0, \omega_n^0, \omega_{xn}^0, \psi_{xn}^0, \psi_{yn}^0\right\} = -\frac{1 - \nu}{2 G_0 \Gamma_{gn} k^2} p_{z,n} \frac{\{\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5\}}{\det_n \|a_{kl}\|}$$
$$(k, l = 1, \ldots, 5). \quad (10)$$

The elements of the determinants $\det_n \|a_{kl}\|$ are calculated by the formulas

$$a_{11} = -\left(1 - \frac{1 - \nu_0}{3} c_0^2\right) \xi^2 - \frac{1 - \nu_0}{3} n^2;$$

$$a_{12} = -a_{21} = a_{45} = -a_{54} = i\xi \frac{1 + \nu_0}{2} n;$$

$$a_{13} = a_{31} = i\xi \nu_0;$$

$$a_{22} = -\frac{1 - \nu_0}{2} (1 - \frac{2}{3} c_0^2) \xi^2 - n^2;$$

$$a_{23} = -\frac{2 + (1 + \nu_0) k_0^2}{2} n;\ a_{25} = k^{-1};\ a_{32} = n;$$

$$a_{33} = 1 + k_0^2 \frac{1 - \nu_0}{2} (n^2 + \xi^2) - \frac{1 - \nu_0}{3} c_0^2 \xi^2 \left[1 + \frac{p_0^*}{k} f(\xi, n, c)\right];$$

$$a_{34} = -i\xi k_0^2 \frac{1-\nu_0}{2k}; \;\; a_{35} = -k_0^2 \frac{1-\nu_0}{2k}\frac{n}{k};$$

$$a_{43} = 12a_{34}; \; a_{44} = a_{11} - 6(1-\nu)\frac{k_0^2}{k^2};$$

$$a_{53} = -12a_{35}; \; a_{55} = a_{22} - 6(1-\nu)\frac{k_0^2}{k^2};$$

$$a_{14} = a_{15} = a_{24} = a_{41} = a_{12} = a_{51} = a_{52} = 0;$$

$$p_0^* = \frac{p_0}{p}; \; c_0 = c(\frac{3p}{2G})^{1/2}.$$

The determinants $\Delta_j \, (j = 1,...,5)$ are obtained from replacing the $\det_n \|a_{kl}\| \; j-$ column with elements $\{0,0,1,0,0\}$. Substituting (10) into formula (7), we find the Fourier coefficients of the fluid pressure transformants

$$q_{r,n}^0 = -\frac{1-\nu}{3}\frac{p_0^* c_0^2}{k}\xi^2 f(\xi,n,c)\frac{\Delta_3}{\det_n \|a_{kl}\|} p_{r.n}^0 .$$

For the bending moment and the transverse force in the shell , we obtain

$$M_{x,n}^0 = -\frac{ha}{12} p_{r,n}^0 \frac{i\xi\Delta_4 n\nu\Delta_5}{\det_n \|a_{kl}\|}; \; Q_{x,n}^0 = -\frac{(1-\nu)k_0^5}{2k} a p_{r,n}^0 \frac{i\xi\Delta_4 n\nu\Delta_5}{\det_n \|a_{kl}\|}. \quad (11)$$

As an example, the motion in the case a >b of a system exponentially decreasing in length and concentrated along the circumference of self-balanced external loads of the same intensity is considered:

$$p_r(\eta,\theta) = p_2 \exp(a\eta) H(-\eta) \sum_{k=1}^{l} (\theta - \theta_k).$$

Here $H(x), \delta(x)$ are the Heaviside and Dirac functions. In this case $p_{r,n}^0 = \frac{p_2 a_n}{a - i\xi}$, where a_n are the Fourier coefficients of the function $\sum_{k=1}^{l}(\theta - \theta_k)$. If we take where $p_2 = 2\pi p_1 / l, \;\; p_1$ is the intensity of the corresponding loads, then the mixing and the corresponding load take the following form

$$w_1^* = \frac{\omega \Gamma_{g1} G}{p_1 a} - \frac{1-\nu}{kl} \sum_{n=0}^{\infty} \left\{ \int_0^{\infty} \frac{\Delta_3 \left[a\cos(\xi\eta) - \xi \sin(\xi\eta) \right]}{\left(a^2 + \xi^2\right)} \right\} \times a_n \cos(n\theta); \quad (12)$$

$$q^* = \frac{q_r}{p_1} = w_1^* = -\frac{2(1-\nu)p_0^* c_0^2}{3kl} \sum_{n=0}^{\infty} \left\{ \int_0^{\infty} \frac{f(\xi,n,c)\xi^2 \Delta_3 \left[a\cos(\xi\eta) - \xi\sin(\xi\eta)\right]}{\left(a^2+\xi^2\right)\det_n \|a_{kl}\|} \right\} \times a_n \cos(n\theta).$$

where $\Gamma_{g1}^{\bullet} = 1 - \Gamma_{g1}^{C}(\omega_R) - i\Gamma_{g1}^{S}(\omega_R)$, $\Gamma_{\lambda\mu\kappa}^{C}(\omega_R)$, $\Gamma_{\lambda\mu\kappa}^{S}(\omega_R)$, $\Gamma_{\mu\kappa}^{C}(\omega_R)$, $\Gamma_{\mu\kappa}^{S}(\omega_R)$ - accordingly, the cosine and sine of the Fourier image of relaxation kernels, which are defined as follows

$$\Gamma_{g1}^{c}(\omega_R) = \int_0^{\infty} R_E(\tau)\cos\omega_R \tau \, d\tau, \quad \Gamma_{g1}^{s}(\omega_R) = \int_0^{\infty} R_G(\tau)\sin\omega_R\tau \, d\tau.$$

Similarly, using (12), it is possible to write formulas for $M_x, Q_x.$ for the system paying to attention the damping of vibrations of the mechanical system at different numbers of waves in the circumferential direction.

3 Results and analysis

As the relaxation core of the viscoelastic shell material $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$, we take the three-parameter Rzhanitsyn-Koltunov core [19], which has a weak singularity, where are the parameters A, α, β of the materials [20]. Let's take the following parameters: $A = 0{,}048; \quad \beta = 0{,}05; \quad \alpha = 0{,}1$. The roots of the dispersion equation are determined by the Muller method ( ), at each iteration of the Muller method, the Gauss method is applied with the allocation of the main element.

The calculations used the representation of the delta function by a finite Fourier series with improved convergence

$$\delta(\theta) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_{n=1}^{N-1}\frac{N}{n\pi}\sin\left(\frac{n\pi}{N}\right)\cos(n\theta)\right].$$

Calculations were carried out for a steel shell interacting with a layer of water. At the same time , the following parameter values were taken:

$$k_0^2 = 0.65,\ k = 0.0035, \in = 0.45, \nu_0 = 0.25,$$

$$a = 1.0, \rho_0 = 0.13, c_0 = 0.1, M_0 = 1.7.$$

Figure 1 shows the distribution of dimensionless deflections of a shell with a compressible liquid along a circle $\eta = -0.3$ in cross section for two (line1), four (line 2), three (line 3) and twenty uniformly concentrated loads (in the second octant), which are described by the formula (12). Calculations have shown that the series in formulas of the form (12) for both displacements and forces in the shell converge quickly enough and to achieve the necessary accuracy, it is enough to take n=2.

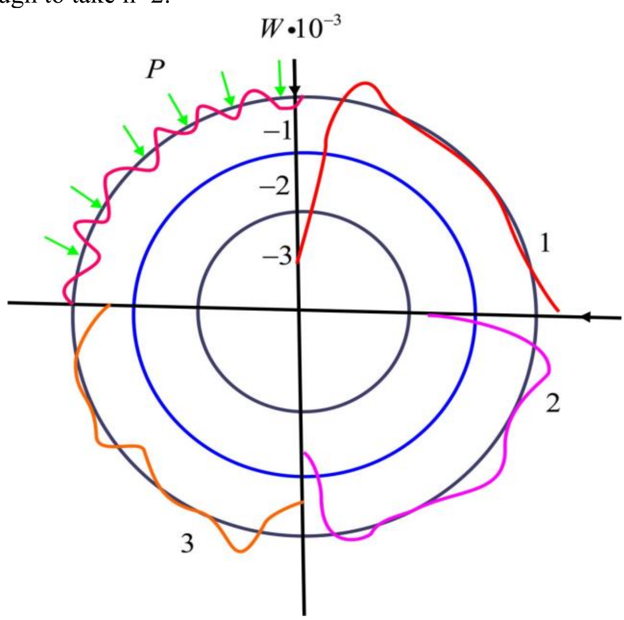

Fig. 1. Distribution of deflections of the shell with liquid at different numbers of concentrated forces.

Since the total pressure on the shell remains constant for any number of loads, with an increase in the number of forces, the maximum deflections decrease rapidly (the effect of discrete application of the load is mitigated) and the nature of the deflection distribution approaches the corresponding axisymmetric one. It is established that paying to attention the viscoelastic properties of the shell material deflections by 10-15%, and also allows us to evaluate the damping abilities of the system.

4 Conclusions

Thus, the paper presents a mathematical formulation and methods for solving the problem of motion along an infinitely long viscoelastic cylindrical shell interacting with an ideal compressible fluid of normal pressure, arbitrary in length and circumference, but unchanged in time, wave propagation profile in flat and extended mechanical systems. The equations of motion of piecewise homogeneous mechanical systems are described by linear equations of shell theory. It is established that paying to attention the viscoelastic properties of the shell material reduces deflections by 10-15%, and also allows us to evaluate the damping capabilities of the system.

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