# Energy dissipation under natural vibrations of viscoelastic composite cylindrical shells

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**Abstract.** Structures consisting of three-layer shells are widely used in various fields of engineering and technology, the construction of nuclear power plants, power and chemical engineering and other areas of the national economy. The aim of the work is to study the issue of energy dissipation during natural vibrations of viscoelastic composite cylindrical shells. The paper considers a general technique for wave propagation in multilayer viscoelastic cylinders. Based on the energy approach, formulas are obtained for the energy dissipation coefficients corresponding to each vibration mode of multilayer cylindrical shells. As an example, a three-layer viscoelastic shell is considered. The relationship between stresses and strains satisfies the hereditary Boltzmann-Volterra integral. When solving the problem, the Green-Lemb expansion, the method of special functions of mathematical physics, and the Muller method are used. The non-monotonic nature of the dependence of the damping coefficients on the geometric and physical-mechanical parameters of three-layer structures is shown. A technique and algorithm have been developed for studying energy dissipation during natural vibrations of viscoelastic composite cylindrical shells. It is found that the energy dissipation depends on the number of layers, and the energy dissipation intensity can take minimum values depending on the angular coordinate.

## 1 Introduction

The dynamics of inhomogeneous shells under nonstationary loads was studied in [1]. Also, questions about the dynamic interaction of cylindrical shells with continuous media (gas, water, filler) are discussed in [2,3]. It follows from the above works that despite numerous studies on these problems, a number of issues related to the structural heterogeneity of a mechanical system have not been studied enough. In particular, it is very important for practice to study the dynamic behavior of structures, taking into account structural inhomogeneity under vibration effects. A significant number of works [4, 5] are devoted to this issue. However, until now, general methods for calculating structurally inhomogeneous layered cylindrical shells surrounded or filled with a linear continuous medium have not been

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developed. A sufficiently detailed analysis of the state of the issue of oscillations of shells interacting with a liquid medium is contained in [5, 6], and for shells with an elastic filler, in [7].

The problems of vibrations in a system of shells with filler are considered in [8,9]. In [10], the problem of free wave propagation in three-layer plates was considered in a refined formulation, when the filler motion is described by the Lame equations with inertial terms, and the Kirchhoff–Love hypothesis is used for skins. Dispersion equations are obtained and phase velocities are determined for symmetric and antisymmetric waves. Axisymmetric free waves in an infinitely long cylindrical shell with an elastic filler were studied in the article [11]. Studies of the dynamic state of three-layer shells are of theoretical as well as applied significance. Structures consisting of three-layer shells are widely used in various fields of engineering and technology, construction of nuclear power plants, energy and chemical engineering and other areas of the national economy [12,13,14]. For example, three-layer shells, formed by thin load-bearing outer layers (shells) and a filler (polymeric material) with a greater thickness, have less weight with equal rigidity in comparison with homogeneous structures. In addition, the middle layer can provide thermal insulation or structural strength [15, 16].

The paper [17] considers the problem of free wave propagation in three-layer plates in a refined formulation, when the filler motion is described by the Lame equations with inertial terms, and the Kirchhoff–Love hypothesis is used for the skins (shell). Dispersion equations are obtained and phase velocities are determined for symmetric and antisymmetric waves. Among the advantages of composite shells, in addition to low weight and high bearing capacity, are also high dissipation coefficients [18, 19]. A significant advantage in this case is the possibility of controlling the dissipation coefficients by purposefully changing the parameters of the structure of the composite material [20, 21]. The quality of three-layer fairings of modern launch vehicles is determined not only by their carrying capacity and mass characteristics, but also by the ability to provide acoustic protection for the payload launched into orbit under external acoustic impact with an acoustic pressure level of up to 160 dB [22].

## 2 Materials and methods

The calculation scheme of the problem is shown in (fig. 1). The z axis is directed along the axis of the cylinder, and the r axis is in the radial direction. The general method of wave propagation in multilayer viscoelastic cylinders is considered. We will look for layer displacements for an $\vec{u}_k(r,z,t)(k=1,.....,N)$ axisymmetric cylinder ($|z|<\infty$, $0 \le r \le R_N$). Consider the equations of viscoelasticity in displacements [23]

$$(\lambda_{0\kappa}+\mu_{0\kappa})\Gamma^{(k)}_{\lambda\mu} graddiv\,\vec{u}_k+\mu_{0\kappa}\Gamma^{(k)}_{\mu}(\nabla^2\vec{u}_k)=\rho_\kappa\frac{\partial^2\vec{u}_k}{\partial t^2}, \tag{1}$$

where $\Gamma^{(k)}_{\lambda\mu}=1-\Gamma^{c}_{\lambda\mu k}\left(\omega_R\right)-i\Gamma^{s}_{\lambda\mu k}\left(\omega_R\right), \Gamma^{(k)}_{\mu}=1-\Gamma^{c}_{\mu k}\left(\omega_R\right)-i\Gamma^{s}_{\lambda\mu k}\left(\omega_R\right).$

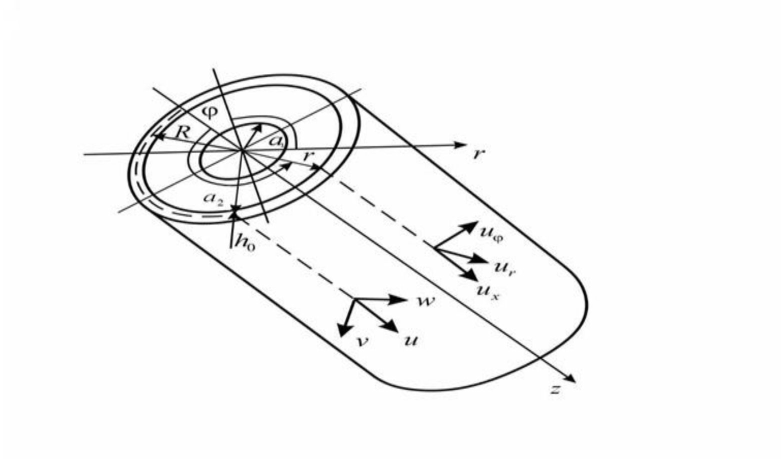


**Fig. 1.** Calculation scheme.

In the case under consideration, the displacements (1) obey the Green expansion [24]

$$\vec{u}_\kappa = grad\,\phi_\kappa + rot\vec{\psi}_\kappa, div\vec{\psi}_\kappa = 0 \tag{2}$$

Then expression (2) in cylindrical coordinates takes the following form:

$$\begin{aligned} u_{rk} &= \frac{\partial \phi_k}{\partial r} + \frac{1}{r}\frac{\partial \psi_{zk}}{\partial \theta} - \frac{\partial \psi_{\theta k}}{\partial z}; \\ u_{\theta k} &= \frac{1}{r}\frac{\partial \phi_k}{\partial \theta} + \frac{\partial \psi_{rk}}{\partial z} - \frac{\partial \psi_{zk}}{\partial r}; \\ u_{zk} &= \frac{\partial \phi_k}{\partial z} + \frac{1}{r}\frac{\partial (r\psi_{\theta k})}{\partial r} - \frac{1}{r}\frac{\partial \psi_{rk}}{\partial \theta}, \end{aligned} \tag{3}$$

In the case of an axisymmetric viscoelastic cylindrical body of the displacement component $u_{\theta k}(r,z,t)=0$. Then the displacements do not change when the angle $\theta$ changes and the derivatives are $\theta$ also zero. In this regard, in expression (3), the vector $\vec{\psi}_k$ has one component $\psi_{\theta k}$ .

$$\begin{aligned} u_{rk} &= \frac{\partial \phi_k}{\partial r} - \frac{\partial \psi_k}{\partial z}; \\ u_{zk} &= \frac{\partial \phi_k}{\partial z} + \frac{1}{r}\frac{\partial \psi_k}{\partial r} + \frac{\psi_k}{r}, \end{aligned} \tag{4}$$

In order for the Green decomposition (2) to be carried out, it is necessary to satisfy the equations

$$\begin{aligned} &(\lambda_{0\kappa} + \mu_{0\kappa})\Gamma^{(k)}_{\lambda\mu}\Delta\phi_\kappa - \rho_\kappa \frac{\partial^2 \phi_\kappa}{\partial t^2} = 0, \\ &\mu_{0\kappa}\Gamma^{(k)}_{\mu}\Delta\vec{\psi}_k - \rho_\kappa \frac{\partial^2 \psi_\kappa}{\partial t^2} = 0, \end{aligned} \tag{5}$$

The obtained partial differential equations or wave equations (in cylindrical coordinates) are used in solving problem (4). As an example, consider the propagation of natural waves in a viscoelastic solid cylinder. In this case k=1. The solution of equation (4) is sought in the complex form [25]

$$\phi_k = \Phi_k(r)e^{i(\alpha z-\omega t)}, \psi_k = \Psi_k(r)e^{i(\alpha z-\omega t)}, \tag{6}$$

Substituting (5) into (4) we obtain the Bessel or Helmholtz equation

$$\begin{aligned} &\Delta\phi_\kappa + \delta_p^2\phi_\kappa = 0, \\ &\Delta\psi_k + \delta_s^2\psi_\kappa = 0, \end{aligned} \tag{7}$$

where $\delta_{pk}^2 = \dfrac{\omega^2}{c_{0pk}^2\Gamma_{\lambda\mu}^{(k)}}, \delta_{sk}^2 = \dfrac{\omega^2}{c_{0sk}^2\Gamma_{\mu}^{(k)}}$

The solution of equations (6) is expressed in terms of the Bessel or Hankel functions [26]:

$$\begin{aligned} \Phi_k &= A_k J_0\left(\delta_{pk} r\right) + B_k N_0\left(\delta_{pk} r\right), \\ \Psi_k &= C_k J_1\left(\delta_{sk} r\right) + D_k N_1\left(\delta_{sk} r\right), \end{aligned} \tag{8}$$

where are $J_0, N_0$ respectively, the Bessel and Neumann functions of zero order; $A_k, B_k, C_k, D_k$ - arbitrary constants.

For a solid cylinder, the arbitrary constants $B_k, D_k$ are zero. The solution consists of arbitrary constants $A_k, B_k$, which are determined from the boundary conditions. The mixtures for the case under consideration have the following form:

$$\begin{aligned} u_r(r,z,t) &= (A_k \frac{dJ_0\left(\delta_{pk} r\right)}{dr} + C_k ikJ_1\left(\delta_{sk} r\right))e^{i(kz-\omega t)}, \\ u_z(r,z,t) &= (C_k(\frac{dJ_1\left(\delta_{sk} r\right)}{dr} + \frac{J_1\left(\delta_{sk} r\right)}{r}) + A_k ikJ_0\left(\delta_{pk} r\right))e^{i(kz-\omega t)}. \end{aligned} \tag{9}$$

Arbitrary $A_k, B_k$ constants are determined from the boundary conditions, i.e. radial and shear stresses of a cylindrical rod at r=R1

$$\begin{aligned} \sigma_{rr} &= \lambda_0\Gamma_\lambda^{(k)}(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial r} + \frac{u_r}{r}) + 2\mu_0\Gamma_\mu^{(k)}\frac{\partial u_r}{\partial r}\Bigg|_{r=R_1} = 0, \\ \sigma_{r\theta} &= \mu_0\Gamma_\mu^{(k)}(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r})\Bigg|_{r=R_1} = 0. \end{aligned} \tag{10}$$

Substituting (8) into conditions (9), we obtain homogeneous algebraic equations with two unknowns ( $A_k, B_k$ ) and two equations

$$\left|\begin{matrix} (\lambda_0+2\mu_0)\Gamma_{\lambda\mu}\delta_{pk}^2 J_0''\left(\delta_{pk}R_0\right)+\frac{\lambda_0}{R}\delta_{pk}\Gamma_{\lambda}J_0'\left(\delta_{pk}R_0\right)-\lambda k^2\Gamma_{\lambda}J_0\left(\delta_{pk}R_0\right) & -2\mu_0 i\delta_{sk}\Gamma_{\mu}J_0'\left(\delta_{sk}R_0\right) \\ 2\mu_0 ik\delta_{pk}\Gamma_{\mu}J_0'\left(\delta_{pk}R_0\right) & \delta_{sk}^2 J_1''\left(\delta_{sk}R_0\right)+\frac{\delta_{sk}}{R_0}J_1'\left(\delta_{sk}R_0\right)+(k^2-\frac{1}{R_0^2})J_1 \end{matrix}\right|\left|\begin{matrix} A_k \\ C_k \end{matrix}\right|=0 \tag{11}$$

A necessary and sufficient condition for the existence of a nontrivial solution of this system is the equality of its main determinant to zero. The last condition gives the dispersion equation for the considered dissipative mechanical system

$$\begin{gathered}[(\lambda_0+2\mu_0)\Gamma_{\lambda\mu}\delta_{pk}^2 J_0''\left(\delta_{pk}R_0\right)+\frac{\lambda_0}{R}\delta_{pk}\Gamma_{\lambda}J_0'\left(\delta_{pk}R_0\right)-\lambda k^2\Gamma_{\mu}J_0\left(\delta_{pk}R_0\right)]\times \\ \times\left[\delta_{sk}^2 J_1''\left(\delta_{sk}R_0\right)+\frac{1}{R_0}\delta_{sk}J_1'\left(\delta_{sk}R_0\right)+(k^2-\frac{1}{R_0^2})J_1\right]-4\mu_0\delta_{pk}\delta_{sk}\Gamma_{\mu}J_0'J_1'=0\end{gathered} \tag{12}$$

To solve the transcendental equation (11), one of the effective methods is the Muller method. At the same time, at each iteration of the Muller method, the Gauss method is applied with the selection of the main element.

The dispersion characteristics are understood as the dependences of the phase ( $C=C_R+iC_I$ ) and group velocity on the wave number ( $V=V_R+iV_I$ ) for various parameters of the mechanical system. This technique is also suitable for multilayer cylindrical bodies (Fig. 1).

## 3 Results

For closed cylindrical shells, along with those described, axisymmetric vibration modes are also possible, in the analysis of which n=0 should be substituted into the above formulas. The dependences of the energy dissipation characteristics on the structural parameters of the composite structure are very complex. For an example in fig. 2 shows graphs of the coefficients and dissipation powers for a composite shell formed by cross-reinforced layers of carbon fiber. Shell radius 0.5 m, length 0.5 m, wall thickness 2 mm. Here, along the abscissa axis, the values of the angle of orientation of the layers $\pm\varphi$ counted from the shell axis are plotted. The graphs show the characteristics of natural forms with the number of half-waves $m=1$ along the axis and the number of waves $n=3...10$ in the circumferential direction. The characteristics of the unidirectional material are taken from [23].

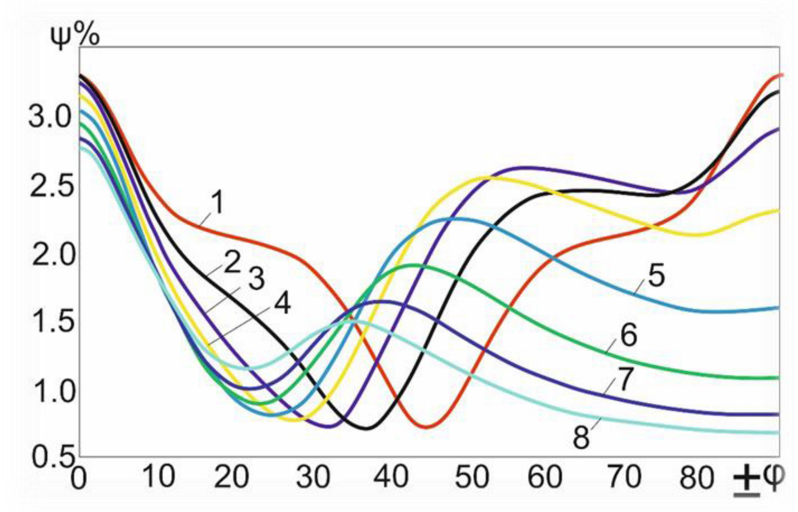


**Fig. 2.** Damping coefficients for predominantly flexural modes of vibrations made of carbon fiber depending on the angle of reinforcement at.

## 4 Conclusion

Thus, a technique and an algorithm for studying the energy dissipation during natural vibrations of viscoelastic composite cylindrical shells have been developed in this paper. It is found that the energy dissipation strongly depends on the number of layers. The intensity of energy dissipation can take on minimum values depending on the angular coordinate.

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