

# Principles of complex multi-stage processes decomposition into component subsystems

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**Abstract.** The article deals with the decomposition of complex multi-stage processes into components and interconnected by material and information flows. Decomposition is carried out to create micro-models for each subsystem. Simplifying the system management of the whole system. The main attention is paid to observance of principles, each subsystem has one input and two outputs according to the structural scheme. In addition, a material balance of input and output parameters is maintained.

## 1 Introduction

The high dimensionality and complexity of the structure of the general management task of multistage processes (MSP) make it practically impossible to develop a single algorithm of optimal management. Therefore, there is a problem of decomposing the general management task into a set of control tasks of accounting, forecasting, decision-making both in the main production activity of SME, and in the auxiliary sphere of preparation, service and production support.

At the same time, decomposition, making it possible to present a complex system in the form of a set of simpler constituent subsystems, makes it necessary to integrate the selected subsystems into a single complex. Therefore, when building MSP, methodological principles and methods of decomposition and their joint use in practice should determine the constituent subsystems of the system, their functions, ways of connection and coordination of subsystems, i.e. what determines the functional structure of the management system.

The principle of decomposition is the methodological basis for the system approach to the creation and design of MSP management system (MS). At the same time, it is possible to allocate such a part of the system, within which it is possible to build a single mathematical, in particular, optimisation model of functioning in the interests of the local objective and criterion. However, this principle gives a positive result only if all the previously mentioned principles of system construction are observed: system integrity, purposefulness of construction, hierarchy, i.e. the need to take into account external and internal connections, as well as the presence of a certain orderliness.

Decomposition raises the following main problems:

- formation of local subtasks as parts of a global task being solved as a whole;

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- -formulation of these problems in the form most convenient for coordinating the obtained solutions and efficient from a computational point of view;
- building a coordination procedure.

The formation of subtasks, first of all, entails the formation of subgoals. At the same time, the scope of information necessary to solve a specific subtask is determined, which makes it possible to exclude information that is not related to this subgoal, thereby reducing the uncertainty of the situation even before the start of the solution. The problem of forming local tasks is relatively isolated. As a rule, when solving it, proceeding from the actual structure of the control object. The following two problems are interrelated and have much in common. So, when forming local goals, formalizing subtasks, assigning criteria and comparing them, it is necessary to take into account not only the criteria for rational problem solving, but also the criteria for the optimality of the system under study as a whole, as well as the corresponding restrictions.

Thus, as a result of decomposition, a multi-level hierarchical functional structure is obtained, in which each task of the upper level has priority of action in relation to the tasks of the lower level associated with it, and the decision-making period for the upper level is longer than that of the lower one, and the tasks for the lower level are solving problems of the upper level.

The goal hierarchy is based on the coordinated assignment of optimality criteria for the entire technological complex and its individual control subsystems. Control actions of the upper level are implemented in the form of setting restrictions when forming the goals of managing subsystems.

When decomposing technological schemes into control subsystems, the problem of finding such a partition is solved, which would minimize the organization of interaction between subsystems with the condition that the specified restrictions are met [1, 2, 3, 15].

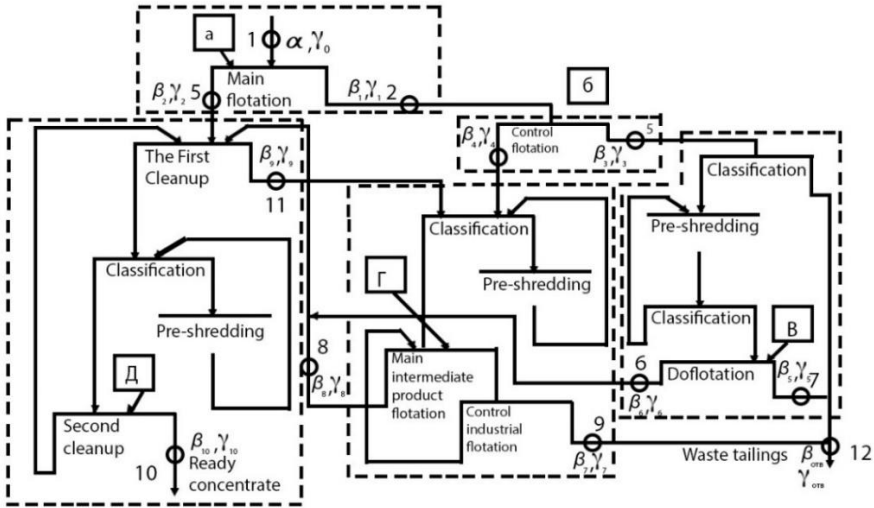
There are at least three principles that govern the development and application of decomposition methods.

1. The need to build a visual model of the system, the large dimension and the multitude of factors in the model are inconvenient for decision making, and this also includes the lack of information.
2. The need to reduce the amount of calculations when obtaining a solution on a model, most of the problems of technical cybernetics, when trying to solve with the help of similar, non-decomposed models (schemes), would be unsolvable at the current level of CT development.
3. The need to establish the relationship between macro and micro models, recently attempts have been made to observe the interaction between several control subsystems.

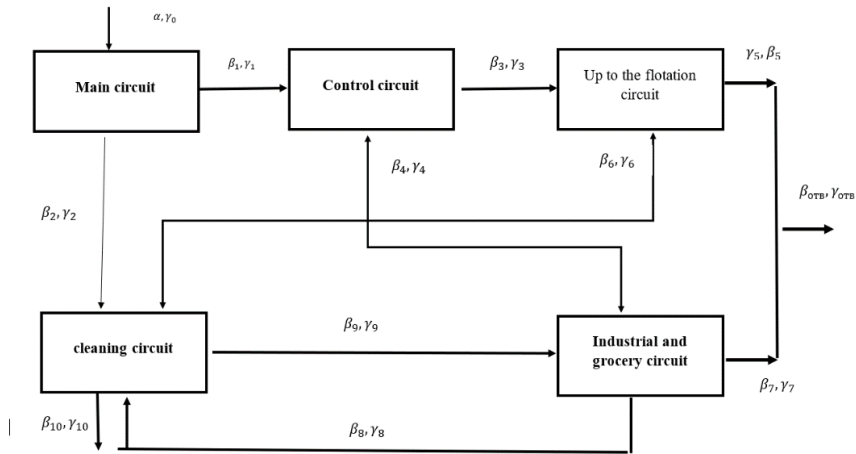
It should be noted that universal decomposition methods have not yet been developed, and the known methods are focused on objects of a certain class and do not cover the entire variety encountered in practice. For this reason, when carrying out decomposition, one often has to be guided by heuristic considerations, knowledge of the problem, intuition, and experience.

To ensure the effective functioning of the MSP flotation, a two-level optimization algorithm is proposed, which consists in solving the problem of interloop and loop optimization. At the same time, from the point of view of control, the technological scheme of the flotation process (Picture 1) is represented by five interconnected control loops: main flotation, control, pre-flotation, main and control middling flotation, first and second cleaning flotations (Picture 2). In this case, the technological scheme of the flotation process is divided into contours so that, when solving the system of balance equations [4, 11], the output product is relatively  $\gamma_i$  ( $i = \overline{1, k}$ ) finally  $\gamma_i$  expressed in terms of all intermediate  $\beta_{2n-1}$  and  $\beta_{2n}$  (where  $= \overline{1, k}$ ).

Based on the analysis of various block diagrams of the multi-stage flotation process, it can be noted that in order for the block diagram to be optimal, one of the necessary conditions is no more than two output products on each control loop. The ongoing decomposition of enrichment flow charts requires technologists to install sensors or measuring instruments at the output of each control loop to determine the content of a valuable component at the output (in tailings and concentrates).



**Fig. 1.** Technological scheme of the flotation process.



**Fig.2.** Interconnected control loops for flotation processes.

Based on the results of the study and analysis of enrichment schemes, we conducted a decomposition of the technological scheme of the process of flotation of copper-molybdenum ores of the eighth section of the copper processing plant of the Almalyk Mining and Metallurgical Combine (MOF AGMK). From the point of view of modeling and control, the technological scheme of flotation enrichment is represented as five control loops interconnected by material flows (Picture 2). Consider the procedure for constructing a macromodel of the flotation process.

According to the block diagram (Picture 2) of the multi-stage flotation process, a material balance model is built.

The correctness of the choice of the structural decomposition scheme is substantiated below:

$$\begin{aligned}
 \gamma_1 + \gamma_2 &= \gamma_0, \\
 \beta_1\gamma_1 + \beta_2\gamma_2 &= \alpha\gamma_0, \\
 \gamma_3 + \gamma_4 &= \gamma_1, \\
 \beta_3\gamma_3 + \beta_4\gamma_4 &= \beta_1\gamma_1, \\
 \gamma_5 + \gamma_6 &= \gamma_3, \\
 \beta_5\gamma_5 + \beta_6\gamma_6 &= \beta_3\gamma_3, \\
 \gamma_7 + \gamma_8 &= \gamma_4 + \gamma_9, \\
 \beta_7\gamma_7 + \beta_8\gamma_8 &= \beta_4\gamma_4 + \beta_9\gamma_9, \\
 \gamma_2 + \gamma_6 + \gamma_8 &= \gamma_9 + \gamma_{10}, \\
 \beta_2\gamma_2 + \beta_6\gamma_6 + \beta_8\gamma_8 &= \beta_9\gamma_9 + \beta_{10}\gamma_{10},
 \end{aligned} \tag{1}$$

where is  $\gamma$  the content of a valuable component (copper) in the original ore, %;  $\beta_{2i}$  and  $\beta_{2i-1}$  - the content of a valuable component (copper) in the concentrate and waste product of the  $i$  - th control loop, %;  $\gamma_0$  - quantitative factor characterizing the flow of incoming ore;  $\gamma_0 = 1$  or  $\gamma_0 = 100$ ;  $\gamma_{2i}, \gamma_{2i-1}$  - the output of the middling product into the concentrate  $\gamma_{\beta_i}$  and into the waste  $\gamma_{0_i}$  products of the  $i$  - th control loop.

The project of an automated control system for the technological process of flotation provides for taking samples for analysis not only from the initial ore and the final product, but also at the internal points of the flotation scheme, which makes it possible to have information on the quantitative value of the parameters  $\beta_i$  ( $i = 1, 2, \dots, 10$ ). Therefore, assuming known, we solve system (1) with respect to  $\gamma_1$  (where  $i = 1, 2, \dots, 20$ ):

$$\begin{aligned}
 \gamma_1 &= \frac{\alpha - \beta_2}{\beta_1 - \beta_2}, \\
 \gamma_2 &= 1 - \gamma_1, \\
 \gamma_3 &= \frac{\beta_1 - \beta_4}{\beta_3 - \beta_4} \gamma_1, \\
 \gamma_4 &= \gamma_1 - \gamma_3, \\
 \gamma_5 &= \frac{\beta_3 - \beta_6}{\beta_5 - \beta_6} \gamma_3, \\
 \gamma_6 &= \gamma_3 - \gamma_5,
 \end{aligned}$$

$$\begin{aligned} \gamma_7 &= \frac{1}{\beta_7 - \beta_{10}} [(\beta_2 - \beta_{10})\gamma_2 + (\beta_6 - \beta_{10})\gamma_6 + (\beta_4 - \beta_{10})\gamma_4], \\ \gamma_8 &= \frac{1}{\beta_8 - \beta_9} [(\beta_4 - \beta_9)\gamma_4 + (\beta_7 - \beta_9)\gamma_7], \\ \gamma_9 &= \gamma_7 + \gamma_8 - \gamma_4, \\ \gamma_{10} &= \gamma_2 + \gamma_4 + \gamma_6 - \gamma_7, \end{aligned} \tag{2}$$

presenting the values  $\gamma_i$  sequentially, we represent (2) in the following form:

$$\begin{aligned} \gamma_1 &= \frac{\alpha - \beta_2}{\beta_1 - \beta_2}, \\ \gamma_2 &= \frac{\alpha - \beta_1}{\beta_2 - \beta_1}, \\ \gamma_3 &= \frac{(\beta_1 - \beta_4)(\alpha - \beta_2)}{(\beta_3 - \beta_4)(\beta_1 - \beta_2)}, \\ \gamma_4 &= \frac{(\beta_1 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_1 - \beta_2)}, \\ \gamma_5 &= \frac{(\beta_3 - \beta_6)(\beta_1 - \beta_4)(\alpha - \beta_2)}{(\beta_5 - \beta_6)(\beta_3 - \beta_4)(\beta_1 - \beta_2)}, \\ \gamma_6 &= \frac{(\beta_3 - \beta_5)(\beta_1 - \beta_4)(\alpha - \beta_2)}{(\beta_6 - \beta_5)(\beta_3 - \beta_4)(\beta_1 - \beta_2)}, \\ \gamma_7 &= \frac{1}{\beta_7 - \beta_9} \left\{ \frac{(\beta_4 - \beta_9)(\beta_2 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_2 - \beta_1)} - \frac{\beta_8 - \beta_9}{\beta_8 - \beta_{10}} \left[ \frac{(\beta_1 - \beta_{10})(\alpha - \beta_2)}{\beta_1 - \beta_2} + \frac{(\beta_5 - \beta_{10})(\beta_3 - \beta_6)(\alpha - \beta_2)}{(\beta_5 - \beta_6)(\beta_3 - \beta_4)(\beta_2 - \beta_1)} + \right. \right. \\ &\quad \left. \left. \frac{(\beta_4 - \beta_{10})(\beta_2 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_2 - \beta_1)} \right] \right\}, \\ \gamma_8 &= \frac{1}{\beta_8 - \beta_9} \left\{ \frac{(\beta_4 - \beta_9)(\beta_1 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_2 - \beta_1)} - \frac{\beta_7 - \beta_9}{\beta_7 - \beta_{10}} \left[ \frac{(\beta_2 - \beta_{10})(\alpha - \beta_1)}{\beta_2 - \beta_1} + \right. \right. \\ &\quad \left. \left. \frac{(\beta_6 - \beta_{10})(\beta_3 - \beta_5)(\beta_1 - \beta_4)(\alpha - \beta_2)}{(\beta_6 - \beta_5)(\beta_3 - \beta_4)(\beta_1 - \beta_2)} + \frac{(\beta_4 - \beta_{10})(\beta_1 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_1 - \beta_2)} \right] \right\}, \\ \gamma_9 &= \frac{1}{\beta_9 - \beta_7} \left\{ \frac{\beta_8 - \beta_7}{\beta_8 - \beta_{10}} \left[ \frac{(\beta_1 - \beta_{10})(\alpha - \beta_2)}{\beta_1 - \beta_2} + \frac{(\beta_5 - \beta_{10})(\beta_3 - \beta_6)(\beta_2 - \beta_4)(\alpha - \beta_1)}{(\beta_5 - \beta_6)(\beta_3 - \beta_4)(\beta_2 - \beta_1)} + \right. \right. \\ &\quad \left. \left. \frac{(\beta_4 - \beta_{10})(\beta_2 - \beta_3)(\alpha - \beta_1)}{(\beta_4 - \beta_3)(\beta_2 - \beta_1)} \right] \right\}, \\ \gamma_{10} &= \frac{1}{\beta_{10} - \beta_7} \left[ \frac{(\beta_2 - \beta_7)(\alpha - \beta_1)}{\beta_2 - \beta_1} + \frac{(\beta_6 - \beta_7)(\beta_3 - \beta_5)(\beta_1 - \beta_4)(\alpha - \beta_2)}{(\beta_6 - \beta_5)(\beta_3 - \beta_4)(\beta_1 - \beta_2)} + \frac{(\beta_4 - \beta_{10})(\beta_1 - \beta_3)(\alpha - \beta_2)}{(\beta_4 - \beta_3)(\beta_1 - \beta_2)} \right], \end{aligned} \tag{3}$$

Technological parameters  $\gamma_{\beta_i}$  and  $\gamma_{0_i}$  are used in the formation of the objective function and the constraints of the optimization problem, therefore, when automating the calculation of complex schemes of a multi-stage flotation process, it is necessary to set the matrix of connections //  $\mathcal{W}_{ij}$  // and //  $\mathcal{X}_{ij}$  //.

For the considered flotation scheme (picture 1), Table 1 shows the matrix of relations //  $W_{ij}$  // and //  $X_{ij}$  //.

**Table 1.** The matrix of relations //  $W_{ij}$ // and //  $X_{ij}$ //

output parameter	$W_{ij}$					$X_{ij}$				
	output parameter									
	one	2	3	4	5	1	2	3	4	5
one	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	1	0	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	1	0	0	0	0	0	0	0	1
5	1	0	1	1	0	0	0	0	0	0

According to the formulas

$$\alpha_i = \frac{\sum_{j=1}^n (W_{ij} \beta_j \gamma_{\beta_i} + \varpi_{ij} \theta_j \gamma_{\theta_j}) + \alpha_i^0 \gamma_{\alpha_i}^0}{\sum_{j=1}^n (W_{ij} \gamma_{\beta_j} + \varpi_{ij} \gamma_{\theta_j}) + \gamma_{\alpha_i}^0} \quad (4)$$

$$\gamma_{\alpha_i} = \sum_{j=1}^n (W_{ij} \gamma_{\beta_j} + \varpi_{ij} \gamma_{\theta_j}) + \gamma_{\alpha_i}^0 \quad (5)$$

(4), (5) it is possible to build macromodels of the flotation process with respect to the selected output indicator. For example, if the content of a useful component (copper) in waste products is considered as an output parameter of the flotation process, then the model with respect to this parameter has the form:

$$\theta_{\text{отв}} = \beta_{10} + (\gamma_{\beta_{10}}) / \{ 1 - [ \beta_{10} - \beta_{7} ] / [ \beta_{10} - \beta_{7} ] \} \cdot [ ( \beta_{2} - \beta_{7} ) (\alpha_{\beta_1} - \beta_1) / ( \beta_2 - \beta_1 ) + ( \beta_6 - \beta_7 ) ( \beta_3 - \beta_5 ) / ( \beta_4 - \beta_2 ) ] (\alpha_{\beta_2} - \beta_2) / ( \beta_6 - \beta_5 ) \cdot [ ( \beta_3 - \beta_4 ) (\beta_1 - \beta_2) + ( \beta_4 - \beta_{10} ) (\beta_1 - \beta_3) ] (\alpha_{\beta_2} - \beta_2) / ( \beta_4 - \beta_3 ) (\beta_1 - \beta_2) \} \quad (6)$$

Similarly, with respect to any selected output indicator of the flotation process as a whole, it is possible to build its macromodels that characterize the connections between local contours.

As a result, it should also be noted that the number of selected circuits and the efficiency of control depend on the structure of the technological scheme and its hardware, on the point of supply of reagents and on the complexity of determining (calculating) inter-circuit connections. A large number of circuits complicates the connections between them, making it difficult to implement the first stage of solving the problem, makes it difficult to manage the circuits themselves, and in some cases, the assessment of their performance.

The division of flotation RSP into separate interconnected controlled circuits implies the formation of a certain hierarchy of optimization criteria. A hierarchical approach to managing the flotation process, in which the optimal values are first determined for individual flotation cycles, and then I solve the control problems for each circuit, is considered in the work. The development of such an approach will lead to optimal control of the entire process. The most interesting results of the use of computers in control systems according to a hierarchical principle were obtained at a processing plant processing copper-molybdenum ores [4, 7, 11].

Experience number	Contours and Factors										Optimization parameters, %				
	I			II		III		IV		V	$\beta_2$	$\beta_4$	$\beta_6$	$\beta_8$	$\beta_{10}$
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>1</sub>					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	-	-	-	+	-	-	-	+	+	+	5,97	5,16	5,76	4,00	20,15
2	-	-	-	+	-	-	-	+	+	+	11,83	5,12	5,76	3,36	21,83
3	-	-	-	+	-	-	-	+	+	+	10,94	5,44	4,96	3,20	20,94
4	-	-	-	+	-	-	-	+	+	+	6,57	5,50	2,68	3,20	19,80
5	-	-	-	+	-	-	-	+	+	+	4,07	4,80	5,60	4,16	20,17
6	+	-	-	-	+	+	+	+	+	-	8,59	2,80	2,80	7,86	18,59
7	+	-	-	-	+	+	+	+	+	-	12,57	3,84	6,80	7,12	18,11
8	+	-	-	-	+	+	+	+	+	-	8,67	3,44	3,12	4,96	19,73
9	+	-	-	-	+	+	+	+	+	-	9,06	3,84	7,14	4,16	20,15
10	+	-	-	-	+	+	+	+	+	-	8,99	5,28	3,76	4,08	19,15
11	+	+	-	+	+	-	+	-	-	-	14,62	3,80	6,76	3,04	22,46
12	+	+	-	+	+	-	+	-	-	-	8,71	1,44	7,96	4,56	20,18
13	+	+	-	+	+	-	+	-	-	-	7,40	3,96	6,48	3,36	21,70
14	+	+	-	+	+	-	+	-	-	-	15,76	5,20	4,52	2,24	19,47
15	+	+	-	+	+	-	+	-	-	-	11,86	3,04	6,48	4,16	18,52
16	-	+	-	-	-	+	-	-	+	-	10,98	2,01	3,32	1,28	19,09
17	-	+	-	-	-	+	-	-	+	-	9,38	1,68	3,46	1,28	17,14
18	-	+	-	-	-	+	-	-	+	-	9,95	2,24	2,96	1,12	19,03
19	-	+	-	-	-	+	-	-	+	-	10,40	2,83	6,62	1,28	19,15
20	-	+	-	-	-	+	-	-	+	-	12,11	2,72	5,32	1,12	18,90
21	-	+	+	-	-	-	-	+	-	+	9,12	2,96	4,32	6,88	22,53
22	-	+	+	-	-	-	-	+	-	+	11,48	1,44	6,32	5,12	18,82
23	-	+	+	-	-	-	-	+	-	+	9,12	2,24	5,96	8,64	20,87
24	-	+	+	-	-	-	-	+	-	+	13,08	1,12	5,96	7,62	17,23
25	-	+	+	-	-	-	-	+	-	+	10,21	2,96	4,04	8,16	17,27
26	-	-	+	+	+	+	+	-	+	-	12,76	1,92	6,08	5,32	18,90
27	-	-	+	+	+	+	+	-	+	-	8,29	2,96	5,76	5,28	20,40
28	-	-	+	+	+	+	+	-	+	-	5,20	5,68	5,60	4,68	19,57
29	-	-	+	+	+	+	+	-	+	-	13,08	5,36	4,80	6,64	19,97
30	-	-	+	+	+	+	+	-	+	-	11,23	3,88	4,96	6,72	17,86
31	+	-	+	-	+	-	+	-	-	+	7,98	2,32	5,30	6,24	18,88
32	+	-	+	-	+	-	+	-	-	+	7,98	2,32	5,30	6,24	18,88
33	+	-	+	-	+	-	+	-	-	+	5,42	2,32	5,44	4,52	20,83
34	+	-	+	-	+	-	+	-	-	+	7,40	2,12	3,04	4,72	18,16
35	+	-	+	-	+	-	+	-	-	+	6,76	2,64	7,04	5,36	17,28
36	+	+	+	+	-	+	-	+	+	+	8,17	4,08	3,36	5,76	21,86
37	+	+	+	+	-	+	-	+	+	+	5,11	2,12	6,24	6,08	19,25
38	+	+	+	+	-	+	-	+	+	+	12,63	3,16	3,84	2,72	21,48
39	+	+	+	+	-	+	-	+	+	-	8,29	2,08	3,88	3,44	21,74
40	+	+	+	+	-	+	-	+	+	-	10,40	3,68	5,92	4,48	20,88

**Fig. 3.** Planning matrix and experimental results.

To solve interloop optimization problems according to this scheme, it is necessary: to carry out the decomposition of technological schemes of the flotation process into interconnected control loops; determine the functional relationship between the control loops; find the values of the input and output variables of the control loops; compose systems of material balance equations of control loops; solve systems of balance equations with respect to  $\gamma_i$ ; draw up systems of restrictions on the dump products of control loops; define two-sided constraints on variables  $\gamma_i$ , etc.  $\beta_{2n}$  for outlets and their copper content; choose an efficiency criterion for solving the problem of interloop optimization; solve the problem of interloop optimization using linear programming methods; to determine the optimal values  $\beta_{2i}$  of the control loops, at which the goal function of the entire flotation circuit will reach an extremum; give the operator-technologist recommendations on the limits of maintaining the values  $\beta_{2i}$ ;

Stabilize and regulate process parameters (water consumption, pulp, its level and alkalinity, etc.).

For the practical implementation of the above procedure, the data obtained during the active experiment according to the developed plan (see Table 2) at the MFO section of the AGMK were used.

Based on the processing of experimental data, the system of balance equations was solved and the values  $\gamma_i$  were determined, the numerical values of the constraints (7) and (8) were determined, which are summarized in Table. 3

The problem of inter-loop optimization for the studied RSP flotation is formulated as follows. It is necessary to determine such values  $\beta_i$  ( $i=2, 4, \dots, 8$ ) that would provide the  $\alpha = 0.4 \div 0.8$  minimum value for the variable

$$\theta_{OTB} = \beta_{10} + \frac{\alpha - \beta_{10}}{1 - \gamma_{10}}, \quad (7)$$

The content of a useful component in a separate product at a given value  $\beta_{10}$  and the fulfillment of certain restrictions [5, 6, 12, 13]. Under optimal control, for each  $i$ th circuit, there is an area of acceptable values for the content of a useful component in tails or middlings, which are represented as restrictions "from below" by inequalities

$$\begin{aligned} \beta_1 &\geq a_{10} + a_{11}\beta_2 + a_{12}\alpha, \\ \beta_3 &\geq a_{20} + a_{21}\beta_1\gamma_1 + a_{22}\beta_4, \\ \beta_5 &\geq a_{30} + a_{31}\beta_3\gamma_3 + a_{32}\beta_6, \\ \beta_7 &\geq a_{40} + a_{41}(\beta_4\gamma_4 + \beta_9\gamma_9) + a_{42}\beta_8, \\ \beta_9 &\geq a_{50} + a_{51}(\beta_2\gamma_2 + \beta_6\gamma_6 + \beta_8\gamma_8) + a_{52}\beta_{10}, \end{aligned} \quad (8)$$

In addition, two-way restrictions can be imposed on the parameters

$$\beta_i^- \leq \beta_i \leq \beta_i^+, \quad \gamma_i^- \leq \gamma_i \leq \gamma_i^+, \quad (9)$$

g de  $i = 1, 2, \dots, 10$ .

It should be noted that the value  $\gamma_{10}$  in formula (6) is determined from formula (2). When solving the problem of inter-loop optimization, along with (6), the efficiency criteria are:

maximum recovery of useful metal

$$\varepsilon_{10} = \gamma_{10} \frac{\beta_{10}}{\alpha}, \quad (10)$$

n for a given  $\beta_{10}$ ;

the maximum content of useful metal in the concentrate

$$\beta_{10} = \theta_{OTB} + \frac{\alpha - \theta_{OTB}}{\gamma_{10}}, \quad (11)$$

n for a given value  $\theta_{OTB}$  and the fulfillment of conditions (7), (8).

As a result of solving the problem of interloop optimization, the optimal values are determined  $\beta_{ionm}$  ( $i=2, 4, \dots, 8$ ), which give an extremum for the goal function and are further used as given when solving the problem of contour optimization.

The task of contour optimization is to maintain the content of a valuable component in concentrates, respectively, for each circuit, within the specified value  $\beta_{ionm}$  determined when solving the problem of inter-circuit optimization.



**Table 2.**

№	$a_{i0}$	$a_{i0}$	$a_{i0}$	$\beta_i^-$	$\beta_i^+$	$\gamma_i^-$	$\gamma_i^+$
1	-0.0181	0.0032	0.2005	0.12	0.18	0.88	0.97
2	0.0804	0.1500	0.0028	4.50	10.50	0.03	0.12
3	0.0301	0.3214	0.0085	0.08	0.13	0.86	0.96
4	0.0920	0.0636	-0.0004	2.10	3.60	0.02	0.06
5	0.4932	-0.0203	-0.0108	0.055	0.075	0.85	0.65
6				1.60	5.00	0.01	0.02
7				0.055	0.085	0.02	0.05
8				1.50	5.50	0.01	0.04
9				0.15	0.40	0.02	0.05
10				15.00	21.00	0.01	0.04

The problem of interloop optimization of a multi-stage physicochemical flotation process is solved using a random search algorithm for the global minimum of the objective function [7, 8], which can be taken as the minimum of the square deviation from the given value

$$|\theta_{OTB} - \theta_{3ад}| \rightarrow \min, \tag{12}$$

$$|\varepsilon_{10} - \varepsilon_{3ад}| \rightarrow \min, \tag{13}$$

$$|\beta_{10} - \beta_{3ад}| \rightarrow \min, \tag{14}$$

Respectively for (6), (9), (10).

Results of software implementation of the random search algorithm for finding optimal values  $\beta_i$  ( $i=2, 4, \dots, 10$ ) are given in table. 4. As the objective function is chosen

$$|\theta_{OTB} - \theta_{3ад}| = \left| \beta_{10} + \frac{\alpha - \beta_{10}}{1 - \gamma_{10}} - \theta_{3ад} \right| \rightarrow \min, \tag{15}$$

At given values  $\beta_{10} = 20$  and  $\theta_{3ад} = 0,065$ . When the content of the useful component (copper)  $\alpha$  in the original ore changes, its optimal value at the output of the circuits  $\beta_i$  also changes. The obtained values for various  $\alpha$  ( $\alpha = 0.4 \div 0.8$ ) play a role  $\beta_{i3ад}$  in solving the problem of contour optimization.

n	$\alpha$	$\varepsilon_k$	$\gamma_1$	$\beta_1$	$\gamma_2$	$\beta_2$	$\gamma_3$	$\beta_3$	$\gamma_4$	$\beta_4$	$\gamma_5$
1	0,40	83,52	97,050	0,1213	2,95	9,570	96,59	0,107	0,464	3,19	95,47
2	0,45	85,40	97,010	0,1232	2,99	9,709	96,12	0,109	0,469	3,26	94,31
3	0,50	86,89	96,963	0,1251	3,04	9,863	96,11	0,112	0,479	3,33	94,27
4	0,55	88,11	96,010	0,1255	3,08	10,032	96,09	0,114	0,491	3,41	94,22
5	0,60	89,13	96,854	0,1278	3,14	10,217	96,08	0,117	0,504	3,49	94,16
6	0,65	89,99	96,792	0,1303	3,21	10,417	96,07	0,121	0,518	3,59	94,11
7	0,70	90,73	96,730	0,1330	3,27	10,633	96,05	0,124	0,533	3,69	94,05
8	0,75	91,37	96,655	0,1359	3,34	10,864	96,04	0,128	0,549	3,80	94,00
9	0,80	91,93	96,579	0,1390	3,42	11,111	96,03	0,133	0,567	3,92	93,93
Continued Table 4											
n	$\beta_5$	$\gamma_6$	$\beta_6$	$\gamma_7$	$\beta_7$	$\gamma_8$	$\beta_8$	$\gamma_9$	$\beta_9$	$\gamma_{10}$	$\varepsilon_{sb}$
1	0,0631	1,12	3,85	2,860	0,0671	0,170	5,080	2,57	0,390	1,670	16,48
2	0,0653	1,16	3,98	2,689	0,0673	0,170	5,065	2,40	0,391	1,922	14,60
3	0,0678	1,20	4,13	2,689	0,0675	0,171	5,112	2,38	0,392	2,172	13,11
4	0,0704	1,25	4,29	2,688	0,0677	0,171	5,130	2,37	0,394	2,423	11,89
5	0,0732	1,30	4,46	2,687	0,0680	0,172	5,150	2,36	0,395	2,674	10,87
6	0,0751	1,36	4,65	2,686	0,0683	0,173	5,170	2,34	0,397	2,925	10,01
7	0,0780	1,41	4,86	2,685	0,0686	0,173	5,205	2,33	0,398	3,176	09,27
8	0,0815	1,48	5,08	2,685	0,0689	0,174	5,220	2,31	0,401	3,426	08,63
9	0,0852	1,54	5,31	2,684	0,0694	0,175	5,250	2,30	0,403	3,677	08,07

**Fig. 4.**

When solving this problem, a random search algorithm and programs [9, 10] were used.

The software implementation of interloop optimization procedures made it possible to determine such values of the parameter  $\beta_{2i}$  at which  $\theta_{OTB}$  decreased from 0.071 to 0.065, and the value  $\beta_{10}$  from 19.74 increased to 20.00.

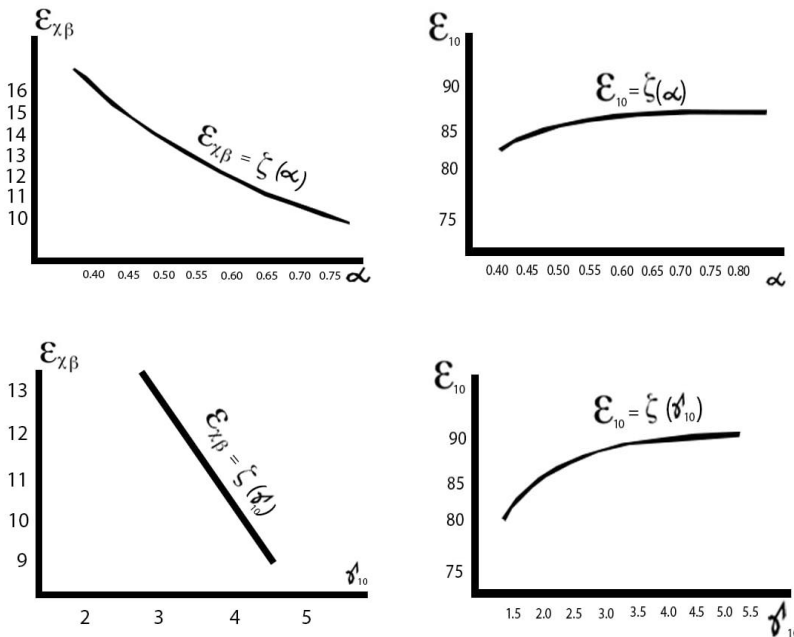
As a result of applying the algorithm of interloop optimization procedures, the recovery value  $\epsilon_{10}$  increased by 1.1% by reducing copper losses in waste products and maintaining the value within specific limits.

Comparison of the values  $\beta_{2i}$  u200bgiven in table. 5 shows that as a result of the software implementation of optimization procedures, the content of the valuable component in the concentrate increases. The number of valuable components in the tails for each control loop is as follows: before optimization  $\beta_1$  it is 0.118%, after optimization it is 0.116%,  $\beta_3$  respectively 0.106 and 0.102,  $\beta_5$ -0.102 and 0.101,  $\beta_7$ -0.097 and 0.090,  $\beta_9$ -0.262 and 0.225,  $\theta_{OTB}$ -0.0710 and 0, 0661%.

**Table 3.**

Process Flotations	Notation				
	$\beta_2$	$\beta_4$	$\beta_6$	$\beta_8$	$\beta_{10}$
Before optimization	9.683	3.253	4.032	4.611	19.741
After optimization	9.893	3.430	4.254	5.270	20,000

The results of the studies carried out made it possible to identify a number of significant relationships between the most important parameters of the flotation process. The dependence of the parameters of the multi-stage flotation process is shown in picture 3.



**Fig. 5.** Dependence of the parameters of the multi-stage flotation process.

The greatest interest in many cases is the determination of the optimal yield of the product in the concentrate and the content of useful metal in the feed to obtain the maximum recovery at the factory.

Thus, the solution of the optimization problem by the multi-stage flotation process is divided into two stages. At the first stage, the optimal connections between the contours are determined, i.e. optimal values of the output variables of each of them, restrictions of the type (7), (8). At the second stage, on the basis of mathematical models, the control problem is solved - obtaining at the output the values of the variables determined at the first stage.

At the heart of a flexible automated control system for MSP flotation are some general fundamental control principles that determine how the functioning and control algorithms are linked to the actual operation or the reasons that cause it to deviate from the specified one.

The state of the control object is described by a number of variables. For a multi-stage flotation process, the variables inherent in its state are the level, the flow rate of solids, water, the speed of the impeller and skimming, etc.

Of the variables, those that determine the goals of management are distinguished, they are called controlled variables. For example, the goal of managing a copper flotation process is characterized by the quality of the concentrate (copper content  $\beta_{cu}$ ), its output  $\gamma_{cu}$ , as well as the losses of copper in the tailings  $\beta_{xb}$  and their output  $\gamma_{xb}$ , which are controlled coordinates.

When managing the process, the process operator evaluates the parameters and indicators according to instrument data, express analysis and visual observation of the process, determines the control parameters in accordance with technical and economic conditions, and controls the process using control tools and mechanisms.

Due to the well-known multidimensionality, inertia, high noise level, a large number of restrictions, complexity and variability of the process, the process operator does not always lead the process in the optimal technological mode, but only approaches it more or less closely. Different levels of training and experience of process operators to significant differences in flotation results. Therefore arises to significant differences in the results of flotation. Therefore, there is a need for a control system that allows you to determine and maintain the optimal technological regime for the process under study.

This is a combined, multidimensional control system that operates with a large amount of information about input actions, output indicators of the process and external conditions for the functioning of the object, the content of the CM, which allows the operator to participate in the control process and has ample opportunities to present information to the operator about the progress of the flotation process.

The basis for the construction of a flexible automated control system for SME flotation should be based on the hierarchical principle of control within the main technological divisions of the factory, as the most fully consistent with the specifics of the flotation process.

The continuous variability of the properties of the enriched raw materials, a large number of external disturbing influences significantly reduce the efficiency of automatic stabilization systems for individual flotation parameters. In such cases, the process operator evaluates the process parameters and its indicators according to instrument data, express analysis and visual observation of the process, determines the control parameters in accordance with the technical and economic conditions, and changes the system task, determining the optimal flotation RMF mode.

Process control is mainly carried out at the second stage of the optimization problem, i.e. with contour optimization.

The control scheme for individual circuits of the flotation process is shown in picture 4. Each circuit is considered as a separate control object and has its own control, input and output variables, which, during the normal operation of the process, can be found using sensors or measuring instruments (analyzers). With the help of specially designed programs, the accumulated information about the process parameters is processed on a computer, then the results of processing are given in the form of a table of optimal values of control actions. Here, mathematical expectations, variances, standard deviations, auto- and cross-correlation

relationships between variables, moments and the transition from a natural (dimensional) scale to a standardized (dimensionless) scale are mainly calculated.

For the mathematical description of the characteristics of the process, goals and objectives are determined and, on their basis, a criterion for the effectiveness of the control loop is developed. With respect to this criterion, a mathematical model of the control loop is built, and at each step of the control, the correspondence of the model to the process is checked. The non-stationarity of the process and a large number of disturbing influences can lead to a discrepancy between the model and the real process. In such cases, a specially designed program of the adaptation algorithm is connected, which, by adjusting the coefficients, establishes the adequacy of the model to the process.

The choice of the criterion for contour optimization is carried out according to the results of the first stage of optimization (inter-contour). When solving the problem of contour optimization, the results of the inter-loop optimization are used as given ones, and we strive to maintain the output indicators of the control loops  $\beta_{2i}$  within the optimal values. This is achieved by varying the control parameters of the process in the allowable region with stable values of the input parameters of the circuit.

In the presence of an adequate control loop model for solving the loop optimization problem, the criterion (goal function) is the minimum control cost

$$\sum_{k=1}^n C_k U_{ik} \rightarrow \min, \quad (16)$$

on condition

$$\beta_{2i \text{зад}} - \beta_{2i}(X_{ij}, U_{ik}, \theta_{il}) = 0, \quad (17)$$

$$U_{ik}^- \leq U_{ik} \leq U_{ik}^+, \quad (18)$$

where  $\beta_{2i \text{зад}}$  is the specified value of the content of a valuable component (copper) in the concentrate at the output of the control loops, determined as a result of solving the problem of interloop optimization;  $x, u$  - vectors of input disturbing and control actions;  $\theta$  - the vector of outputs to the tails:  $-\beta(x, u, \theta)$  the value of the output parameter determined on the basis of the mathematical model.

In the flotation process, the main perturbing parameter is the content of the valuable component in the original ore. Therefore, a change in its share in the composition of the ore significantly affects the management system, i.e. the values of the output function change accordingly. Based on this, in order to obtain a mathematical model of the process of copper extraction, the range of copper content in the diet was chosen  $\alpha = 0.4 \div 0.8$ . In studies, the value of the parameter  $\alpha$  was changed in a discrete step  $h = 0.05$ , and for each  $\alpha$  value, the values of other process parameters necessary for optimization and control were determined.

The study of the flotation process showed that with a significant increase in oxides in the ore, the extraction of copper in the middling product increases. When lime is supplied to ore grinding to  $pH = 8.5 \div 10.5$ , the extraction of copper in the concentrate is maximum, in the middling product is minimal. When the  $pH$  value is outside the specified range, the extraction of copper in the middling increases and, accordingly, decreases in the concentrate.

In our experiment, the value changed in the optimal region, the density of the pulp was within  $1.12 \div 1.26$ , the temperature in the flotation machines was  $8 \div 10$  °C.

Studies of the relationship between flotation parameters, carried out as a result of industrial testing of the MSP, showed that the main disturbing influences are the  $\alpha$  amount of solids in the total feed  $\theta_{TB}$ , as well as fluctuations in the pulp level in the flotation machines. Note that copper flotation is already formed in the main flotation circuit, since here the copper floatability reaches 60% of the total recovery ( $Na_2S, Kst$ ) in the main flotation circuit has the greatest influence on the flotation performance as a whole. Based on this, when

developing the functional structure of the control system, the main attention was paid to managing the specific costs of flotation reagents, and the copper content in the concentrates of the control loops was taken as the resulting parameter. For the entire flotation process, the copper content in tailings was taken as the objective function. Similarly, the rest of the process circuits were investigated and their optimal technological ( reagent ) regimes were determined. The supply of flotation reagents to the circuits of the flotation process is regulated in accordance with the control model. Therefore, at each control step, the compliance of the model with the process is checked.

As a result, we note that the functional structure of the MSP flotation control system is determined by the location of the main technological circuits and the nature of their internal interconnection. Consequently, the efficiency of the functioning of the flotation control system is largely determined by the correct choice of control criteria and the completeness of the control algorithms for each process circuit.

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