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Mathematical modeling and numerical calculation of an epidemic with medical vaccination in account

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Abstract. This article describes and analyzes mathematical models of the spread of infectious diseases, on the basis of the considered model, a mathematical model of the spread of epidemics is built, it is studied to what extent receiving medical vaccination affects the spread of infectious diseases in this process, and the mathematical models considered in this paper are compared with others. epidemic models.

1 Introduction

Mathematical disease modeling is a powerful tool for studying disease transmission mechanisms. Epidemiological models serve as a basis for predicting and evaluating the dynamics of disease spread [1,2].

There are various mathematical models of epidemics. This article mainly discusses the SIR model, its modifications and additions.

In the book of mathematical models in biology [3, p. 242-254] The SIR-model, as well as its modifications, are considered in sufficient detail, for example: SI-model, SIS-model, SIRS-model. A peculiarity of the models is that after recovery, a person can again enter the class of susceptible individuals and be prone to re-infection.

Herbert W. Hethcote's article [4] analyzed many mathematical models of the spread of infectious diseases in populations and applied them to specific diseases. The models considered in this article are based on the classic SIR model. The MSEIR model takes into account birth and death. In the MSEIR model, the population is divided into 5 classes: passively immune, susceptible, latent, infected and non-susceptible. If the mother is infected, some antibodies pass through the placenta, so the newborn has a temporary passive immunity to the infection. There are M children with passive immunity in the class. After the mother's antibodies disappear from the newborn's body, the child passes into the class of susceptible individuals [4].

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As mentioned above, various mathematical models of epidemic development have been developed and used in the analysis of the spread of various diseases (typhoid, cholera, ebola, etc.). Many models are based on the SIR model (S - Susceptible, I - infected, R - recovered) and its modifications [5].

The model considered in the article is also considered a modification of the SIR model, in which the state of medical vaccination is additionally considered.

2 Mathematical model of an epidemic

In the immune epidemic model based on the SIR model, the population is divided into three classes. $S(t)+I(t)+R(t)=N=const$, $S(t)$ – healthy persons prone to disease, $I(t)$ – infected, disease-spreading persons, $R(t)$ – persons who have recovered from infection, have immunity or died, who do not spread the disease, N – total population, unchanged.

$$u_1(t) = \frac{S(t)}{N}, \quad u_2(t) = \frac{I(t)}{N}, \quad u_3(t) = \frac{R(t)}{N} \tag{1}$$

we introduce a notation like [6].

A mathematical model of an epidemic taking into account medical vaccination:

$$\begin{cases} \frac{du_1}{dt} = -au_1u_2 \\ \frac{du_2}{dt} = au_1u_2 - bu_2 \\ \frac{du_3}{dt} = bu_2 + \boldsymbol{c}u_2 \end{cases} \tag{2}$$

at t = 0 with initial conditions:

$$u_1(0) = u_{10} \geq 0, \quad u_2(0) = u_{20} \geq 0, \quad u_3(0) = u_{30} \geq 0. \tag{3}$$

Here, a, b – positive coefficients usually have a size of 1/day;

$a = \frac{1}{T_k}$ – probability of contracting the disease of a susceptible person in contact with an infected person, the coefficient of the rate of infection, T_k -period of illness;

$b = \frac{1}{T_{td}}$ – recovery rate without taking into account medical vaccination, T_{td} – period of recovery from illness;

$c = \frac{1}{T_v}$ – recovery rate after medical vaccination, T_v – vaccination period.

3 Calculation methods

Numerical solution is found using the following formula:

$$y_i = y_{i-1} + hf(x_{i-1}, y_{i-1}). \tag{4}$$

The exact solution can be expanded into a Taylor series:

$$y(x_{i-1} + h) = y(x_{i-1}) + hy'(x_{i-1}) + O(h^2). \tag{5}$$

Local error L is obtained by subtracting the third from the fourth equation:

$$L = y(x_{i-1} + h) - y_i = O(h^2). \tag{6}$$

This is appropriate only if the function y has a derivative of the second order.
We use Euler's method to numerically solve model (1):

$$\begin{cases} (u_1)_i = (u_1)_{i-1} - ha(u_1)_{i-1}(u_2)_{i-1} \\ (u_2)_i = (u_2)_{i-1} + ha(u_1)_{i-1}(u_2)_{i-1} - hb(u_2)_{i-1} \\ (u_3)_i = (u_3)_{i-1} + hb(u_2)_{i-1} + h\boldsymbol{c}(u_2)_{i-1} \end{cases} \quad (7)$$

Global error, the global error, is the error at the end point of an arbitrary final step of integrating the equation. Calculate the solution at this point S/h requires steps, where S is the step length. Hence the global error of the method $G = O(h^2 S/h) = O(h)$.

4 Computing experiment

Incoming information:
- Time T=365 days.
- Time steps K=700.
- Disease rate coefficient a=0.128.
- Recovery rate without medical vaccination b=0.0963.
- The population is 6,380,000 people.

Prerequisite: $u_{10} = 0.999N, \quad u_{20} = 0.001N, \quad u_{30} = 0.$

Case 1. When the rate of recovery after medical vaccination is equal to c=0, the calculation result is as follows:

Table 1. Calculated result at c=0.

№	$(\boldsymbol{u_1})$ steps	Numbers	$(\boldsymbol{u_2})$ steps	Numbers	$(\boldsymbol{u_3})$ steps	Numbers
1	$(u_1)_0$	6.37362e6	$(u_2)_0$	6.38e3	$(u_3)_0$	0
2	$(u_1)_{100}$	6.2697371659e6	$(u_2)_{100}$	3.1392213226e4	$(u_3)_{100}$	7.8870620831e4
3	$(u_1)_{200}$	5.836065666e6	$(u_2)_{200}$	1.2115584978e5	$(u_3)_{200}$	4.2277848423e5
4	$(u_1)_{300}$	4.8305309326e6	$(u_2)_{300}$	2.1989975693e5	$(u_3)_{300}$	1.3295693105e6
5	$(u_1)_{400}$	3.9589611572e6	$(u_2)_{400}$	1.3736713192e5	$(u_3)_{400}$	2.2836717109e6
6	$(u_1)_{500}$	3.6167166447e6	$(u_2)_{500}$	4.5838636768e4	$(u_3)_{500}$	2.7174447185e6
7	$(u_1)_{600}$	3.5198076347e6	$(u_2)_{600}$	1.2399755675e4	$(u_3)_{600}$	2.8477926097e6
8	$(u_1)_{700}$	3.4947586157e6	$(u_2)_{700}$	3.1687677924e3	$(u_3)_{700}$	2.8820726165e6

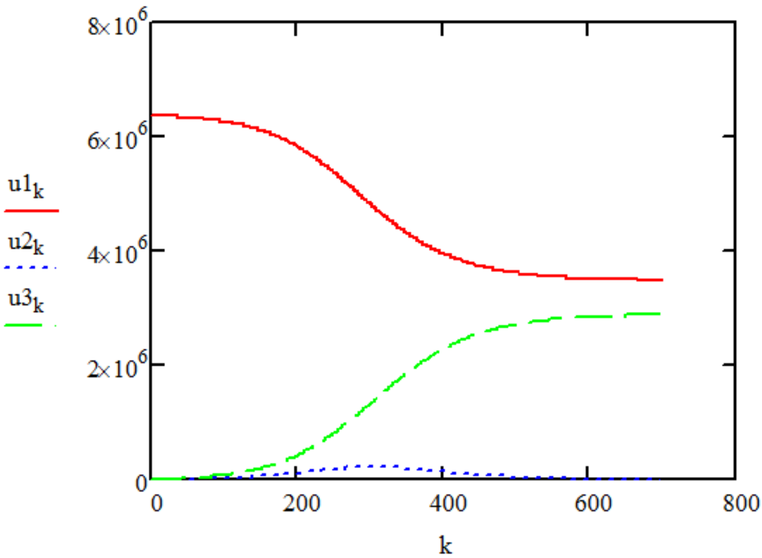

Fig. 1. Red line -$(u_1)_k$, blue line -$(u_2)_k$, green line - $(u_3)_k$. Here, k is the time step.

According to the graph in Fig. 1, when the recovery rate after vaccination is c = 0, the number of people who have recovered from infection, have immunity or died, and who do not spread the disease, does not increase rapidly.

Case 2. When the recovery rate after medical vaccination is equal to c=0.1, the calculation result is as follows:

Table 2. Calculated result at c=0,1.

№	$(\boldsymbol{u_1})$ **steps**	**Numbers**	$(\boldsymbol{u_2})$ **steps**	**Numbers**	$(\boldsymbol{u_3})$ **steps**	**Numbers**
1	$(u_1)_0$	6.37362e6	$(u_2)_0$	6.38e3	$(u_3)_0$	0
2	$(u_1)_{100}$	6.2697371659e6	$(u_2)_{100}$	3.1392213226e4	$(u_3)_{100}$	1.6077157704e5
3	$(u_1)_{200}$	5.836065666e6	$(u_2)_{200}$	1.2115584978e5	$(u_3)_{200}$	8.6180079391e5
4	$(u_1)_{300}$	4.8305309326e6	$(u_2)_{300}$	2.1989975693e5	$(u_3)_{300}$	2.7102228001e6
5	$(u_1)_{400}$	3.9589611572e6	$(u_2)_{400}$	1.3736713192e5	$(u_3)_{400}$	4.655085741e6
6	$(u_1)_{500}$	3.6167166447e6	$(u_2)_{500}$	4.5838636768e4	$(u_3)_{500}$	5.5392980088e6
7	$(u_1)_{600}$	3.5198076347e6	$(u_2)_{600}$	1.2399755675e4	$(u_3)_{600}$	5.8050019655e6
8	$(u_1)_{700}$	3.4947586157e6	$(u_2)_{700}$	3.1687677924e3	$(u_3)_{700}$	5.8748790719e6

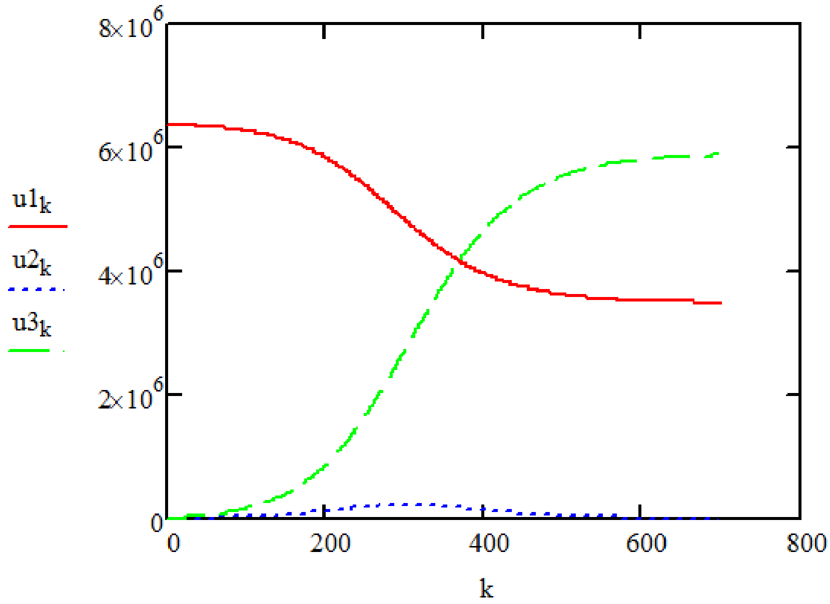

Fig. 2. Red line -$(u_1)_k$, blue line -$(u_2)_k$, green line - $(u_3)_k$. Here, k is the time step.

According to the graph in Figure 2, when the rate of recovery after vaccination is c=0.1, the number of people who have recovered, become immune, or die, and who do not spread the disease, after contracting the infection, appears quickly.

5 Conclusion

Medical vaccination is a simple, safe and effective way to protect a person from diseases before they come into contact with people who spread the disease. Vaccination activates the body's natural defense mechanisms and provides resistance to various infectious diseases and strengthens the immune system. Like diseases, medical vaccinations train the immune system to produce specific antibodies. However, medical vaccinations contain only killed or weakened forms of the causative agent of a specific disease - viruses or bacteria, which do not cause the disease and do not create the risk of complications related to it.

In the above results, the difference between the situation when the medical vaccination coefficient is c=0 and the situation when it is c=0.1, i.e., a sharp increase in the number of recovered and immune persons can be seen. This means that during an epidemic, the use of medical vaccination measures in addition to quarantine restrictions ensures that the epidemic does not last for a long time.

References

1. Y. Chen, J. Cheng, Y. Jiang, K. Liu, Journal of Inverse and Ill-posed Problems **28(2)**, 243 (2020).
2. W. O. Kermack, A. G. McKendrick, Proceedings of the Royal Society **115**, 700 (1927).
3. L. Edelstein-Keshet, *Mathematical Models in Biology. Society for Industrial and Applied Mathematics* (2005).
4. H. W. Hethcote, SIAM Review **42(4)**, 599 (2000).

5. V. L. Sokolovsky, G. B. Furman, D. A .Polyanskaya, E. G. Furman, Health Risk Analysis **1**, 23 (2021).
6. F. A. Kabiljanova, N. I. Boltaboyeva, *O'zbekistonda COVID-19 koronavirus epidemiyasining tarqalishini matematik modellashtirish. Innovative developments and research in education* (Canada, 2022), pp. 132-136.
7. L. Euler, *Integral calculus* (1956).
8. D. K. Muhamediyeva, O. B. Ruzibaev, Yu. Baxromova, IOP Conf. Series: Journal of Physics: Conference Series **2182**, 012026 (2022).
9. D. K. Muhamedieva, A.Yu. Nurumova, Ferpi scientific and technical journal **26(2)**, 9 (2022).
10. D. K. Muhamediev, Ferpi scientific and technical journal **26(3)**, 158 (2022).
11. D. Ziyadullaev, D. Muhamediyeva, Z. Abdullaev, S. Aynaqulov, K. Kayumov, E3S Web of Conferences **365**, 01019 (2023).
12. D. Sh. Ziyadullaev, D.T. Mukhamedieva, G.Ye. Ziyodullaeva, Journal of Advanced Research in Dynamical and Control Systems **10 (14)**, 1850 (2018).
13. Z. Abdullaev, D. Sh. Ziyadullaev, D. T. Muhamediyeva, Journal of Physics: Conference Series **2176(1)**, 012071 (2022).