# Mathematical modeling formation of wole drainage under soil deformations 

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#### Abstract

In agriculture, the processing of land plots from underground is one of the modern problems. To solve this problem, new scientific and technical approaches to agricultural technology are required. This article presents data on the formation of mole drainage during washing of saline soils. The newly created technical device has a thick-walled conicalcylindrical shape. Here it is experimentally analyzed that the soil is deformed and does not lose its stability. Experimental data were obtained by numerical element and coordinate methods. It is compared with the results of a numerical solution obtained on the basis of an exact circuit of the device. The study of nonlinear processes of compression and supercritical deformation of the soil forming the mole drainage due to soil pressure is a complex and important scientific and technical problem. The discrepancy between the results of field and computational experiments, as well as the characteristics of the accepted mathematical model and the method of their solution, associated with the rough discretization of the original problem, are characterized by external pressure forces. Therefore, experimental and theoretical studies evaluating the accuracy of methods for numerical analysis of nonlinear problems of soil deformation during the formation of mole drainage under various types of pressure and loads, as well as the study of the influence of initial deficiencies on the results of solutions, are considered relevant issues. In this article, mathematical models were created for the relationships between deformations, stresses and coordinates of the solution of the above problems, as well as their numerical solutions were considered.


## 1 Introduction

The external influences that will be exerted on the conical part of the device for the formation of drainage holes, which is associated with a loss of stability, will differ significantly from the external influences that will be exerted on the cylindrical part. Compression of shells, as a rule, is accompanied not only by bending stresses, but also by the appearance of additional stresses (chain stresses) on the middle surface, while for cone and cylinder parts we could take into account only compression and bending stresses. Part of the load potential of external pressure and impact forces is spent in the shell state to increase the

[^0]compression energy, the other part - to change the energy of the middle surface. The ratio between these values depends on what configuration the shell will receive during compression.

## 2 Materials and methods

An experimental-theoretical method was used. In theoretical and numerical studies carried out in the work, general methods of soil mechanics and deformable solids are used. Methods of mathematical modeling were used in the analysis and generalization of the results $[1,4,6]$.

## 3 Discussion

When studying the stability of the forming drainage hole, attention is paid to round conecylindrical devices. Devices of this form, theoretically, meet the requirements of the complexity of their design and ease of manufacture, so they are widely used in various fields of technology. This article offers two approaches to the problem under consideration. The first one is based on the well-known solution of nonlinear equations by the Ritz method. This makes it possible to change the calculation methods, take into account the influence of geometric factors. The second approach is based on considering the problem of device stability in drainage systems, taking into account the dynamic processes occurring during the washing of land plots from salinization, which brings the theoretical formation of the problem closer to experimental observations (Fig.1).


Fig. 1. a) Devices of the mole drainage tool; 1-implement attachment; 2-P-gun shaped frame; 3-trace educator (markor); 4-work racks; 5-supporting spike; 6-chisel of working bodies; 7-cone-cylinder (drainer); 8-support wheel stands; 9-clamp; 10-cane; 11-compactor-screeder; 12-sprayers. Где Overall rack height, m; tillage depth, m; b) schematic view of the formation of a mole drainage; c) formation of mole drainage in the field.

The influence of maintaining the stability of the device on its shape and magnitude of critical impact forces is analyzed. The results of these studies are presented below. Coneshaped cylindrical devices are widely used in devices that form drainage holes when washing saline areas in agriculture.

Hooke's ratio can be represented as follows[11,12]:

$$
\sigma_{i}=\frac{E}{1-\mu^{2}}\left(\varepsilon_{1}+\mu \varepsilon_{2}\right), \sigma_{i 1}=\frac{E}{1-\mu^{2}}\left(\varepsilon_{2}+\mu \varepsilon_{1}\right), \tau=\frac{E}{2\left(1+\mu^{2}\right)}-\gamma,(1)
$$

Where $\boldsymbol{E}_{1}, \boldsymbol{\mathcal { E }}_{2}$ - tensile (compressive) strains in axial and circumferential directions; $\gamma$ - shear strain; $u, \mathcal{Q}, w$ - shell deformation components along the axes $x, y, z$; $\boldsymbol{k}_{1}, \boldsymbol{k}_{2}$ - principal surface curvatures; $\chi_{1}, \chi_{2}$ - bending deformation in axial and circumferential directions; normal and shear stresses in the middle surface of the shell; $E, \boldsymbol{\mu}$ - modulus of elasticity and coefficient[8,9,10].

Poisson shell material. The equilibrium equations for an arbitrary shell element can be writtenas:

$$
\begin{align*}
& \frac{\partial \tau}{\partial x}+\frac{\partial \sigma}{\partial y}=0  \tag{2}\\
& D \nabla^{2} \nabla^{2} w=\sigma_{i} \delta\left(k_{i}+\frac{\partial^{2} w}{\partial x^{2}}\right)+\sigma_{i} \delta\left(k_{i}+\frac{\partial^{2} w}{\partial y^{2}}\right)+2 \tau \delta \frac{\partial^{2} w}{\partial x \partial y}+q
\end{align*}
$$

where $\boldsymbol{\delta}$ - shell thickness; $q$ - intensity of the acting transverse load.
Here $D=\frac{E h^{3}}{12\left(1-\mu^{2}\right)}$ - cylindrical rigidity of the shell during its bending. When solving the stability problem as $q$ consider the total projection of the main forces $p_{x}, p_{y}, S$ to the direction of the normal to the surface of the shell. Then

$$
\begin{equation*}
q=-\delta\left(p_{x} \frac{\partial^{2} w}{\partial x^{2}}+p_{y} \frac{\partial^{2} w}{\partial y^{2}}+2 s \frac{\partial^{2} w}{\partial x \partial y}\right) \tag{3}
\end{equation*}
$$

Forces that increase the parameters of curvature are considered positive, in particular, forces $p_{x}$ and $p_{y}$ positive if they are contractive.

Here $D=\frac{E h^{3}}{12\left(1-\mu^{2}\right)}-$ cylindrical rigidity of the shell during its bending. The first quadratic form of the midsurface will be:

$$
\left\{\begin{array}{l}
l=d x^{2}+d y^{2}+d z^{2},  \tag{4}\\
Q=q_{x}+q_{y}+q_{z} \\
T_{T} ;=\tau_{x}+\tau_{y}+\tau_{z} \\
F_{T x} \gg\left(Q+T_{T x}\right)
\end{array}\right.
$$

Let's move on to the case when the shell is subjected to the action of an external pressure uniformly distributed over the lateral surface $q$ (Fig. 2).


Fig. 2. The study of the formation of mole drainage during deformation and for washing saline soils.
Here $\sigma_{x}, \sigma_{y}, \sigma_{z}$ - normal stresses in coordinate areas passing through a given point, $\tau_{x y}, \tau_{y z}, \tau_{z x}-$ shear stresses, $\varepsilon_{x}, \varepsilon_{y}, \varepsilon_{z}-$ elongation strain, $\gamma_{x y}, \gamma_{y z}, \gamma_{z x}-$ shear deformations.

This type of loaded is typical for the body of underground washing of saline soils and also underground tillage.

Let us consider the problem of the stability of such a shell in a linear formulation. If the circular shell is subjected to external pressure $q$ and there is no shell bending, then from the equation (Fig. 1):

$$
\begin{equation*}
\varepsilon_{x}=\frac{\partial u}{\partial x}, \varepsilon_{y}=\frac{\partial u}{\partial y}-\frac{w}{R}, \gamma=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x} \tag{5}
\end{equation*}
$$

When considering isotropic shells, we will introduce into the main relations not the forces $N_{x}, N_{y}, T$, and directly stresses in the middle surface $\delta_{x}=\frac{N_{x}}{h}, \delta_{y}=\frac{N_{y}}{h}, \tau=\frac{T}{h}$. equation of equilibrium in projections onto the axis $x$, tangent on the line $y$, and axis $Z$ rewrite in mind:

$$
\left\{\begin{array}{l}
\frac{\partial \sigma_{x}}{\partial x}+\frac{\partial \tau}{\partial y}+\frac{1}{h} q_{x}=0  \tag{6}\\
\frac{\partial \tau}{\partial x}+\frac{\partial \sigma_{y}}{\partial y}-\frac{Q_{y}}{R h}+\frac{1}{h} q_{y}=0 \\
\frac{\partial Q_{x}}{\partial x}+\frac{\partial Q_{y}}{\partial y}+\frac{h \sigma_{y}}{R}+q_{z}=0
\end{array}\right.
$$

The moment equations take the form:

$$
\left\{\begin{array}{l}
\frac{\partial M_{x}}{\partial x}+\frac{\partial H}{\partial y}-Q_{x}=0  \tag{7}\\
\frac{\partial H}{\partial x}+\frac{\partial M_{y}}{\partial y}-Q_{y}=0
\end{array}\right.
$$

Find $Q_{x}$ and $Q_{y}$ from (7) and substitute into (6)

$$
\left\{\begin{array}{l}
\frac{\partial \sigma_{x}}{\partial x}+\frac{\partial \tau}{\partial y}+\frac{1}{h} q_{x}=0  \tag{8}\\
\frac{\partial \tau}{\partial x}+\frac{\partial \sigma_{y}}{\partial y}-\frac{1}{R h}\left(\frac{\partial H}{\partial x}+\frac{\partial M_{y}}{\partial y}\right)+\frac{1}{h} q_{y}=0 \\
\frac{\partial^{2} M_{x}}{\partial x^{2}}+2 \frac{\partial^{2} H}{\partial x \partial y}+\frac{\partial^{2} M_{y}}{\partial y^{2}}+\frac{h \sigma_{y}}{R}+q_{z}=0
\end{array}\right.
$$

Let us determine, further, the stresses in the middle surface:

$$
\left\{\begin{array}{l}
\sigma_{x}=\frac{E}{1-\mu^{2}}\left[\frac{\partial u}{\partial x}+\mu\left(\frac{\partial \vartheta}{\partial y}-\frac{w}{R}\right)\right]  \tag{9}\\
\sigma_{y}=\frac{E}{1-\mu^{2}}\left[\frac{\partial u}{\partial y}-\frac{w}{R}+\mu \frac{\partial u}{\partial x}\right] \\
\tau=\frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x}\right)
\end{array}\right.
$$

and the moment will be in the following form:

$$
\left\{\begin{array}{l}
M_{x}=-D\left[\frac{\partial^{2} w}{\partial x^{2}}+\mu\left(\frac{1}{R} \frac{\partial \vartheta}{\partial y}+\frac{\partial^{2} w}{\partial y^{2}}\right)\right]  \tag{10}\\
M_{y}=-D\left[\frac{\partial^{2} w}{\partial y^{2}}+\frac{1}{R} \frac{\partial \vartheta}{\partial y}+\mu \frac{\partial^{2} w}{\partial x^{2}}\right] \\
H=-D(1-\mu)\left(\frac{1}{2 R} \frac{\partial \vartheta}{\partial x}+\frac{\partial^{2} w}{\partial x \partial y}\right)
\end{array}\right.
$$

We introduce expressions (9) and (10) into the equilibrium conditions (8), then we have the following system:

$$
\left\{\begin{array}{l}
\frac{\partial^{2} u}{\partial x^{2}}+\frac{1-\mu}{2} \frac{\partial^{2} u}{\partial y^{2}}+\frac{1+\mu}{2} \frac{\partial^{2} \vartheta}{\partial x \partial y}-\frac{\mu}{R} \frac{\partial w}{\partial x}+\frac{1-\mu^{2}}{E h} q_{x}=0  \tag{11}\\
\frac{\partial^{2} \vartheta}{\partial y^{2}}+\frac{1-\mu}{2} \frac{\partial^{2} \vartheta}{\partial x^{2}}+\frac{h^{2}}{12 R^{2}}\left(\frac{\partial^{2} \vartheta}{\partial y^{2}}+\frac{1-\mu}{2} \frac{\partial^{2} \vartheta}{\partial x^{2}}\right)+ \\
+\frac{1-\mu}{2} \frac{\partial^{2} u}{\partial x \partial y}-\frac{1}{R} \frac{\partial w}{\partial y}+\frac{h^{2}}{12 R}\left(\frac{\partial^{3} w}{\partial x^{2} \partial y}+\frac{\partial^{3} w}{\partial y^{3}}\right)-\frac{1-\mu^{2}}{E h} q_{y}=0 \\
\frac{h^{2}}{12} \nabla^{4} w+\frac{w}{R^{2}}-\frac{\mu}{R} \frac{\partial u}{\partial x}-\frac{1}{R} \frac{\partial \vartheta}{\partial y}+\frac{h^{2}}{12 R}\left(\frac{\partial^{3} \vartheta}{\partial x^{2} \partial y}+\frac{\partial^{3} \vartheta}{\partial y^{3}}\right)-\frac{1-\mu^{2}}{E h} q_{z}=0
\end{array}\right.
$$

Where $\nabla^{4}=\nabla^{2} \nabla^{2}-$ double Laplace operator. We have obtained one of the variants of the equations of the theory of cylindrical shells in displacements. Let us now turn to a simplified version of the linear theory of shells, a new expression for changes in curvature has the following form:

$$
\begin{equation*}
\chi_{x}=-\frac{\partial^{2} w}{\partial x^{2}}, \chi_{y}=\frac{\partial^{2} w}{\partial y^{2}}, \chi=-\frac{\partial^{2} w}{\partial x \partial y} \tag{12}
\end{equation*}
$$

The equilibrium equations in the projections of the axis tangent to the line and the axis will be rewritten in the form:

$$
\left\{\begin{array}{l}
\frac{\partial \vartheta_{x}}{\partial x}+\frac{\partial \tau}{\partial y}+\frac{1}{h} q_{x}=0  \tag{13}\\
\frac{\partial \tau}{\partial x}+\frac{\partial \vartheta_{y}}{\partial y}-\frac{Q_{y}}{R h}+\frac{1}{h} q_{y}=0 \\
\frac{\partial Q_{x}}{\partial x}+\frac{\partial Q_{y}}{\partial y}+\frac{h \sigma_{y}}{R}+q_{z}
\end{array}\right.
$$

we get at $Q_{[ }=Q_{y}=0$ stresses along the arc equal to $\sigma_{y}=-q \frac{R}{h}$. Thus, the action of the transverse load $q$ equivalent to the action of compressive stresses $P_{y}=q \frac{R}{h}$. Therefore, we can use the homogeneous equation:

$$
\begin{equation*}
\frac{D}{h} \nabla^{8} w+\frac{E}{R^{2}} \frac{\partial^{4} w}{\partial x^{4}}+p_{x} \nabla^{4}\left(\frac{\partial^{2} w}{\partial x^{2}}\right)+p_{y} \nabla^{4}\left(\frac{\partial^{2} w}{\partial y^{2}}\right)+2 s \nabla^{4}\left(\frac{\partial^{2} w}{\partial x \partial y}\right)=0 \tag{14}
\end{equation*}
$$

considering only the efforts $P_{y}$ : then we get

$$
\begin{equation*}
\frac{D}{h} \nabla^{8} \omega+\frac{E}{R^{2}} \frac{\partial^{4} w}{\partial x^{4}}+\frac{q R}{h} \nabla^{4}\left(\frac{\partial^{2} w}{\partial y^{2}}\right)=0 \tag{15}
\end{equation*}
$$

Accept for $\omega$ an expression that also satisfies the boundary conditions:
$\omega=f \sin \frac{m \pi x}{L} \sin \frac{\pi y}{R}$; that in the case of compression we get the following equation:

$$
\begin{equation*}
\frac{D}{h}\left(\frac{m^{2} \pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{4}+\frac{E}{R^{2}} \frac{m^{4} \pi^{4}}{L^{4}}-\frac{q R}{h} \nabla^{4}\left(\frac{\partial^{2} w}{\partial y^{2}}\right)=0 \tag{16}
\end{equation*}
$$

from here:

$$
\begin{equation*}
q=D R\left(\frac{m^{2} \pi^{2}}{L^{2} n}+\frac{n}{R^{2}}\right)^{2}+\frac{E h}{R n^{2}} \frac{1}{\left(1+\frac{n^{2} L^{2}}{R^{2} m^{2} \pi^{2}}\right)^{2}} \tag{17}
\end{equation*}
$$

Obviously, when determining the critical pressure, it is necessary to take $m=1$.
Therefore, in contrast to the case of axial compression, under external pressure, the shell must always buckle along the generatrix along one half-wave: this conclusion is confirmed by experiments. We introduce the notation

$$
\left\{\begin{array}{l}
Q=\frac{q_{x}}{E}\left(\frac{R}{h}\right)^{2}+\frac{q_{y}}{E}\left(\frac{R}{h}\right)^{2}+\frac{q_{z}}{E}\left(\frac{R}{h}\right)^{2}=\hat{q}  \tag{18,19}\\
\hat{q}=\frac{q}{E}\left(\frac{R}{h}\right)^{2}, \\
\text { Instead (17) we find } \\
\hat{q}=\frac{h}{R} \frac{n^{2}}{12\left(l-\mu^{2}\right)}\left(l+\frac{\pi^{2} R^{2}}{n^{2} L^{2}}\right)^{2}+\frac{\pi^{4} R^{5}}{L^{4} h n^{6}} \frac{1}{\left(1+\frac{\pi^{2} R^{2}}{n^{2} L^{2}}\right)^{2}}
\end{array}\right.
$$

This formula is simplified if we accept the conditions:

$$
\left\{\begin{array}{l}
\left(\frac{\pi R}{n L}\right)^{2} \ll 1  \tag{20}\\
\widehat{q}=\frac{1}{12\left(l-\mu^{2}\right)} \frac{n^{2} h}{R}+\frac{\pi^{4} R^{4}}{L^{4} h n^{4}}
\end{array}\right.
$$

minimize $\widehat{q}$ on $n$, we get

$$
\begin{align*}
& n=\sqrt[4]{6 \pi^{2} \sqrt{1-\mu^{2}}} \sqrt{\frac{R}{L}} \sqrt[4]{\frac{R}{h}}  \tag{21}\\
& \mu=0.3, \quad n \approx 2.7 \sqrt{\frac{R}{L}} \sqrt[4]{\frac{R}{h}} \tag{22}
\end{align*}
$$

Substituting (21) into expression (19), we determine the upper critical value: $\widehat{q}_{n}$ (Fig. 3).

$$
\begin{align*}
& \widehat{q}_{n}=\frac{\sqrt{6}}{9\left(1-\mu^{2}\right)^{0.75}} \frac{\pi R}{L}\left(\frac{h}{R}\right)^{0.5}  \tag{23}\\
& \mu=0.3, \quad \mathrm{q}_{\mathrm{n}}=0.92 \frac{L}{R} \sqrt[4]{\frac{h}{R}} \tag{24}
\end{align*}
$$

the corresponding circumferential stress is

$$
\begin{equation*}
p_{y}=0.92 E \frac{h}{L} \sqrt{\frac{h}{R}} \tag{25}
\end{equation*}
$$

Now let's write the moment of rotation:

$$
M_{y}=-D\left(\frac{\partial^{2} w}{\partial y^{2}}+\frac{w}{R^{2}}+\mu \frac{\partial^{2} w}{\partial x^{2}}\right)
$$

If we use new expressions for the moments, and the load $q_{z}$ for stability problems is determined by the following equations[13,14]:

$$
\begin{equation*}
\frac{D}{R}\left[\nabla^{8} w+\frac{1}{R^{2}} \nabla^{4}\left(2 \mu \frac{\partial^{2} w}{\partial x^{2}}+2 \frac{\partial^{2} w}{\partial y^{2}}+\frac{w}{R^{2}}\right)\right]+\frac{E}{R^{2}} \frac{\partial^{4} w}{\partial x^{4}}+\frac{q R}{h} \nabla^{4}\left(\frac{\partial^{2} w}{\partial y^{2}}+\frac{w}{R^{2}}\right)=0 \tag{26}
\end{equation*}
$$

## 4 Results

Substituting here the expression $\omega=f \sin \frac{m \pi x}{L} \sin \frac{\pi y}{R}$; we get:

$$
\begin{aligned}
& \frac{D}{R}\left(\frac{m^{2} \pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{2}\left\{\left(\frac{m^{2} \pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{2}+\frac{1}{R^{2}}\left[1-2\left(\mu \frac{m^{2} \pi^{2} R^{2}}{L^{2}}+n^{2}\right)\right]\right\}+ \\
& +\frac{E}{R^{2}} \frac{m^{4} \pi^{4}}{L^{4}}-\frac{q}{R h}\left(\frac{m^{2} \pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{2}\left(n^{2}-1\right)=0
\end{aligned}
$$

From here at $m=1$ :

$$
\begin{align*}
& q_{n}=\frac{D R^{2}}{n^{2}-1}\left\{\left(\frac{\pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{2}+\frac{1}{R^{4}}\left[1-2\left(\mu \frac{\pi^{2} R^{2}}{L^{2}}+n^{2}\right)\right]\right\}+  \tag{27}\\
& +\frac{E h}{R} \frac{\pi^{4}}{L^{4}} \frac{1}{\left(\frac{\pi^{2}}{L^{2}}+\frac{n^{2}}{R^{2}}\right)^{2}\left(n^{2}-1\right)}
\end{align*}
$$

Consider the case of a very long shell, when $L \gg R$; expression (27) becomes the following:

$$
\begin{equation*}
q_{n}=-\frac{\left(n^{2}-1\right) D}{R^{3}} \tag{28}
\end{equation*}
$$

Numerical solutions of cylindrical shells:
At normal pressure, due to the symmetry of the problem, the shape of the shell must remain a cylindrical shell. That is, the system equation (28) must satisfy the solution on which the equalities.

$$
\begin{equation*}
\lambda_{1}=\text { const }, T_{1}=\text { const }, \varphi=\frac{s}{R}(0 \leq \varphi \leq 2 \pi \tag{29}
\end{equation*}
$$

$\lambda_{1}-$ The relative change in the shell radius, taking this into account, from (2) follows the equation for the multiplicity of elongation $\lambda_{1}$

$$
\begin{equation*}
q \lambda_{1}-\frac{1}{R} T_{1}=R \rho h \frac{d^{2} \lambda_{1}}{d t^{2}} \tag{29.1.}
\end{equation*}
$$

In the static case, the relationship between pressure and the relative measurement of the shell radius is determined from the relationship $q R-\frac{1}{\lambda_{1}} T_{1}=0$.
This dependence may have maximum points. So, for example, for the elastic potential, Fig. 2 shows the dependence of the dimensionless pressure $Q=\frac{q R}{\mu h}=Q\left(\lambda_{1}\right)$ from the relative increase in the shell radius for values $n=1, n=2, n=3$, in the elastic potential (4). In $n=2$, this case, the dependence has an asymptote, and at $n \prec 2-$ the maximum points the maximum points are reached at the values $\lambda_{1}^{2 \pi}=\frac{(n+2)}{(2-n)}$ (Fig.3).


Fig.3. Dependences of the external pressure on the relative change in the shell radius.
The roots of the right side of this equation $\lambda_{1}=1$, и $\lambda_{1} \leq \frac{4-Q-\sqrt{Q^{2}-24 Q+16}}{2 Q}$, The second root will be positive provided that, how $Q \leq Q=12-8 \sqrt{2}$, so it $\lambda_{1}$ changes in the range $\left[1, \frac{4-Q}{2 Q}\right]$.

The dependence of the oscillation amplitude on the parameter $Q$ under the initial conditions (7) for the elastic potential (1) с $n=1, n=2, n=3$, is reflected (in Fig. 3). The dotted lines for mark $n=1, n=2$ the boundaries of the values $Q$ beyond which the fluctuations stop (Fig. 4).


Fig.4. Dependences of the external pressure on the amplitude of the cylindrical shell Internal force factors for axisymmetric shell deformation under longitudinal force:

$$
\begin{equation*}
N_{x}=\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{x} d z=\frac{E}{\left(1-\mu^{2}\right)} \int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\left(\varepsilon_{0}-z \frac{d^{2} w}{d x^{2}}\right)+\mu \frac{w}{R}\right] d z=\frac{E h}{\left(1-\mu^{2}\right.}\left[\varepsilon_{0}+\mu \frac{w}{R}\right], \tag{30}
\end{equation*}
$$

Internal force factors for axisymmetric deformation of the shell with a circumferential force:

$$
\begin{equation*}
N_{y}=\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{y} d z=\frac{E}{\left(1-\mu^{2}\right)} \int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\frac{w}{R}+\mu\left(\varepsilon_{0}-z \frac{d^{2} w}{d x^{2}}\right)\right] d z=\frac{E h}{\left(1-\mu^{2}\right.}\left[\frac{w}{R}+\mu \varepsilon_{0}\right] \tag{31}
\end{equation*}
$$

Equilibrium equation of a cylindrical shell in displacements, its integration.

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}}\left(D \frac{d^{2} w}{d x^{2}}\right)+\frac{1}{R}\left(\mu N_{x}+\frac{E h}{R} w\right)=p, \quad D \frac{d^{4} w}{d x^{4}}+\frac{E h}{R^{2}} w=\left(p-\mu \frac{N_{x}}{R}\right) \tag{32}
\end{equation*}
$$

Here $D=\frac{E h^{3}}{12\left(1-\mu^{2}\right)}$ - cylindrical rigidity of the shell during its bending.
Numerical solutions normal stresses in the cross section of the shell at a distance $Z$ from the middle surface of the shell are determined according to Hooke's law (Fig. 1 and Fig. 5)

$$
\begin{align*}
& \sigma_{x}=\frac{E}{\left(1-\mu^{2}\right)}\left[\varepsilon_{x}+\mu \varepsilon_{y}\right]=\frac{E}{\left(1-\mu^{2}\right)}\left[\left(\varepsilon_{0}-h \frac{d^{2} w}{d x^{2}}\right)+\mu \frac{w}{R}\right]  \tag{33}\\
& \sigma_{y}=\frac{E}{\left(1-\mu^{2}\right)}\left[\varepsilon_{y}+\mu \varepsilon_{x}\right]=\frac{E}{\left(1-\mu^{2}\right)}\left[\frac{w}{R}+\mu\left(\varepsilon_{0}-h \frac{d^{2} w}{d x^{2}}\right)\right]  \tag{34}\\
& \frac{E_{n p}}{2} w_{b}^{2}\left[\frac{R_{H}+R_{\theta}}{2 \gamma_{n p}}+\frac{1}{4} \ln \left(\frac{R_{b}+\gamma_{n p}}{R_{b}}\right)\right] d \varphi d z=\frac{1}{2} K w_{b}^{2} R_{b} d \varphi d z, \tag{35}
\end{align*}
$$

Hence it follows that

$$
\begin{equation*}
E_{n p}=\frac{K R_{b}}{\frac{2 R_{b}+\gamma_{n p}}{2 \gamma_{n p}}+\frac{1}{4} \ln \left(\frac{R_{b}+\gamma_{n p}}{R_{b}}\right)} \tag{36}
\end{equation*}
$$

For example $R_{6}=1 \mathcal{M}, \gamma_{n p}=0,08 \mathcal{M}, K=2,2 \cdot 10^{8} \frac{H}{\mathcal{M}^{3}}$, The found value of the reduced modulus of elasticity of the layer was $E_{n p}=1,691 \cdot 10^{7} \frac{H}{\mathcal{M}^{2}}$,

Note that a formula similar to (36) but not taking into account the deformation of the layer in the circumferential direction is given in [7]. It lacks a term containing the natural logarithm.

It should be noted that for elastic layers with a thickness of $0,08 \partial o 0,4 \mathcal{M}\left(0,08 R_{\sigma} \leq \lambda_{n p} \leq 0,4 R_{g}\right)$. The calculated values of the maximum displacements and stresses in the shell turned out to be quite close and agree well with similar results obtained using other soil models. This confirms the validity of formula (36) and the assumptions made in its derivation table 1 and [1] (Fig. 5).

Table 1. Maximum displacements and equivalent sheath stresses.

| Options <br> NDS | Optio n 1 | $\begin{gathered} \text { Optio } \\ \text { n } 2 \end{gathered}$ | $\begin{aligned} & \text { Option 3 } \\ & \gamma_{n p}=0,08 \mathrm{M} \end{aligned}$ | Option 3 $\gamma_{n p}=0,1 м$ | Option 3 $\gamma_{n p}=0,2 \mu$ | Option 3 $\gamma_{n p}=0,3 \mu$ | Option 3 $\gamma_{n p}=0,4 \cdot M$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $w_{\text {max }}=10^{-5} \mathrm{M}$ | $\begin{gathered} 7,816 \\ 4 \end{gathered}$ | $\begin{gathered} 7,817 \\ 4 \end{gathered}$ | 7,0483 | 7,0454 | 8,0316 | 8,0176 | 8,0628 |
| $\sigma_{\text {экє }}^{6}=10^{5} \frac{N}{M^{2}}$ | $\begin{gathered} 5,138 \\ 1 \end{gathered}$ | $\begin{gathered} 5,098 \\ 6 \end{gathered}$ | 5,1358 | 5,1376 | 5,1366 | 5,1282 | 5,1389 |
| $\sigma_{\text {эxg }}^{H}=10^{5} \frac{N}{M^{2}}$ | $\begin{gathered} 5,479 \\ 9 \end{gathered}$ | $\begin{gathered} 5,540 \\ 0 \end{gathered}$ | 5,6483 | 5,6484 | 5,6440 | 5,6338 | 5,6443 |



Fig. 5. Dependencies of circular transverse stress on circular residual and elastic deformations.

## 5 Conclusion

1. In general, the results of numerical and experimental studies allow us to conclude that the computational model $[1,2,3,4,6]$ qualitatively correctly and quantitatively satisfactorily describes the deformation and buckling of thin-walled cylindrical shells during bending.
2. Static problems for long-cylinder shells were solved by the German scientist A. L. Goldenweiser, who was seen in the theory. In the proposed article, analytical and numerical solutions were obtained for external pondomotor forces and stresses of soil deformation in agriculture.
3. A technique has been developed that takes into account the one-sided contact interaction of the shell and the soil base and allows comparing three types of soil models surrounding the shell: the Fuss-Winkler base, the model of the elastic layer and the volumetric array. As a result of the analysis of these models, it was found that the SSS characteristics of the "shell - surrounding soil" system turned out to be relatively close both qualitatively and quantitatively in all three cases.
4. The speed of a loaded shell significantly affects the critical dynamic pressure, increasing the value of the critical load during short-term loaded.
5. In problems of the stability of a round cylindrical shell, a linear statement is considered. Note that when $L \gg R ; q=-\frac{n^{2} D}{R^{3}}$, and at $n=2$, will $q_{n}=\frac{4 D}{R}$ which exceeds the external pressure by $33 \%$.
6. This illustrates that the scope of application of the approximate equations of the theory of shells of average length is limited and that the error of the calculation results in some cases can be significant..
7. However, from a practical point of view, these cases are rather exceptions; for the case of external pressure, surfaces isothermal to the cylinder can be constructed in a similar way. The results obtained can be used as test examples in the development of numerical methods for solving nonlinear equations of deformation of external pressure shells.
8. An analysis of the stress-strain state of the shell showed that in the subcritical stage, its deformation occurs in the elastic zone. After the loss of stability in the zone of corrugations, plastic deformations of the order of $4-7 \%$ are formed, which corresponds to the experimental data.
9. The proposed method consists in calculating the dynamic stability of cylindrical shells when loaded with external overpressure distributed over the shell surface. As an example, the case is considered when the pressure changes in accordance with the linear law.

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