New precoding blind channel estimation and channel order estimation algorithm in OFDM systems with cyclic prefix

Osamah Al-Qadhi[1, 2, *], *E.* Semenov[1], *Jaber* Talib[2], *Ali* Almufti[3], and *Maha* Al-Ghazali[4]

[1]Volgograd State University, 400062 Volgograd, Russia
[2]Al-Mustaqbal University, Babylon, Al Hillah, Iraq
[3]Kazan National Research Technical University n.a. A. N. Tupolev – KAI, Kazan, Russia
[4]Ministry of Oil, Baghdad, Iraq

Abstract. Using different precoding matrices, a new blind channel estimation in OFDM systems with cyclic prefix is presented in this paper. The proposed method employs one column of the correlation matrix directly, unlike the traditional precoding techniques where the elements of precoding matrix is been removed. Results show the impact of this method specially when using the type of precoding matrices which include the circulant property in its design. Since channel order estimation is an important task in blind methods, a new and simple algorithm is also investigated with no additional complexity been added to the system. A proof of diagonalizability property is mentioned which can be implemented for other investigations concerning this type of matrices.

1 Introduction

Due to its high data rate transmission capabilities with high bandwidth efficiency and its tolerance to multipath delay, orthogonal frequency division multiplexing (OFDM) has recently been widely used in wireless communication systems. Non blind, semi-blind, and blind approaches are the three categories of channel estimation techniques that can be distinguished [1]. In non-blind or pilot based channel estimation, the received value in the receiver is compared to the value of previously known pilot bits that are placed into the transmitted signal. One-dimensional (block and comb) and two-dimensional (comb) pilot insertion techniques are utilized [2]. Because fewer pilots are used, semi-blind channel estimate results in higher bandwidth efficiency than pilot-based channel estimation. Blind channel estimation techniques estimate the channel utilizing internal information in the received signals as well as the structural characteristics of the transmitted signals rather than pilot symbols, which use up valuable channel bandwidth. As a result, blind channel estimation has drawn a lot of interest and has grown to be an important field of study.

The subspace approach, which primarily depends on the design of redefined received symbol vector structure that results in reformation of the systems' channel matrix, is one of the extensively used techniques for blind channel estimation for OFDM systems. In order to

---

* Corresponding author: eng.usama2010@gmail.com

create an override system employing the redundancy introduced by CP for channel identification [3–6], as well as for systems without CP [7,8], the subspace-based technique is employed.

Precoding techniques also involve adding a precoding unit to the system at the transmitter, which allows for blind estimate of a channel. In [9,10] , a straightforward linear precoder that doesn't add redundancy to the system and doesn't alter block length was proposed. The algorithm's accuracy is greatly reduced because it only extracts a channel from one column of the covariance matrix, as opposed to a precoder with added redundancy [11] ,which employs autocorrelation and singular value decomposition (SVD) operations to estimate the frequency-selective fading channel.

Knowing the channel order at the receiver helps improve the system's performance in both data detection and channel estimation. In the literature, there are a few approaches for channel order estimate. Three common approaches are studied for the channel order estimation, which is sensitive to SNR fluctuations, based on information theoretic criteria: the minimal description length (MDL) [12], the Akaike information criteria (AIC) [13], and the Liavas algorithm[14], A deterministic approach for order detection and channel estimation is presented in [15] and provides flawless channel order estimation in the noise-free scenario.

In this paper we propose a new blind channel estimation for CP-OFDM system using a precoding matrix. Channel coefficients can be calculated using one column of the correlation matrix on the received symbol vectors before to FFT because of the unique design of precoding matrices. This method does not call for discarding the components of the precoding matrix, in contrast to the conventional precoding estimate methods in OFDM. For each of the precoding matrices utilized in the simulation, a demonstration of the existence of a dominant vector is stated. Since channel order estimation does not necessitate additional procedures or significantly increase system complexity, it may be done in a few easy stages and during the estimation process.

2 New channel estimation algorithims

In this section we solve the problem of blind channel estimation method in CP-OFDM system presenting two system models. Unlike the traditional estimation methods that uses precoding matrices and employs the correlation matrix of the received symbols after FFT.

2.1 Channel estimation of OFDM using a single precoding matrix

2.1.1 System model

The proposed CP-OFDM system is shown in Fig. 1. The modulated signal is turned into a series of blocks that take the form $\boldsymbol{d}_i = [d_{i,0},...,d_{i,N-1}]^T$, where $d_{i,m}$, $m = 0,1,...,N-1$ is a zero mean (Independent identically distributed random variables), temporarily white, unit variance, spatially uncorrelated, and N is the number of subcarriers in the OFDM symbol, $i = 1,2,....,$ OFDM symbols , then the precoded signal is obtained as follows:

$$\boldsymbol{s}_i^{(k)} = \boldsymbol{A}^{(k)}\boldsymbol{d}_i \tag{1}$$

where $\boldsymbol{s}_i^{(k)} = [s_{i,0}^{(k)},...,s_{i,N-1}^{(k)}]^T$, $k = 1,2,3$ is the precoded data symbols using the precoders $\boldsymbol{A}^{(1)}, \boldsymbol{A}^{(2)}$ and $\boldsymbol{A}^{(3)}$. In this system the matrices does not add any redundancy to the system since the all of size $N \times N$.

Each block $s_i^{(k)}$ is modulated using the Inverse Fourier Transform, CP portion of length L is inserted at the biggening of each block to form final vector of length $N + L$.

$$\boldsymbol{x}_i^{(k)} = \boldsymbol{F}_{CP} \boldsymbol{F}^H \boldsymbol{s}_i^{(k)} \tag{2}$$

where $\boldsymbol{F}_{CP} = \begin{bmatrix} \boldsymbol{0}_{L \times (N-L)} & \boldsymbol{I}_L \\ \boldsymbol{I}_N \end{bmatrix}$ of size $(N+L) \times N$ and $\boldsymbol{F}$ is a normalized Discrete Fourier Transform matrix of the from in Eq. **Ошибка! Источник ссылки не найден.**:
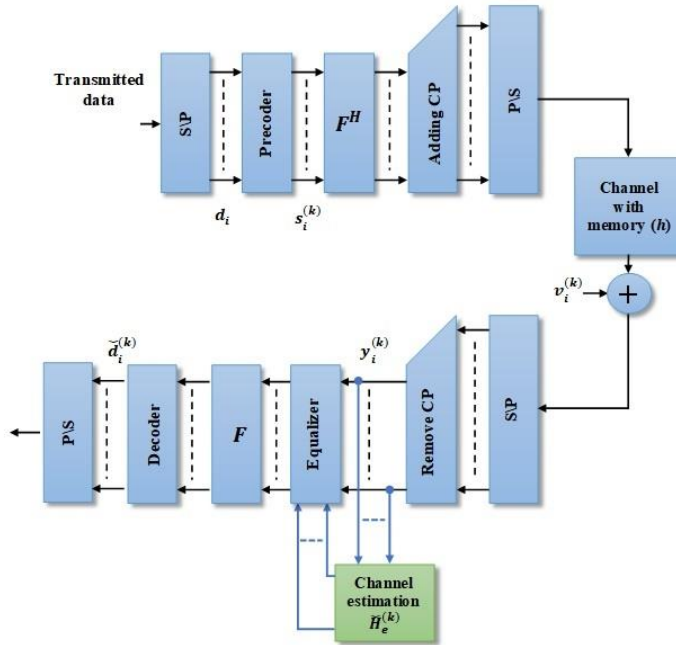
Fig. 1. Block diagram of blind channel estimation.

$$\boldsymbol{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \tag{3}$$

where $W_N = e^{-j\frac{2p}{N}}$. The block $\boldsymbol{x}_i^{(k)}$ is transformed into a serial data sequence for transmission. The data is transmitted through a multipath channel $\boldsymbol{h} = [h_0, ..., h_L]^T$ of order $L+1$ and $h_i, i = 0, ..., L$ is the samples of channel impulse response.

The matrix representation of the received signal $\boldsymbol{y}_i^{(k)}$ after convolution with the channel coefficients and removing the CP portion is as follows:

$$\boldsymbol{y}_i^{(k)} = \boldsymbol{H}\boldsymbol{F}^H \boldsymbol{A}^{(k)} \boldsymbol{d}_i + \boldsymbol{v}_i^{(k)} \tag{4}$$

where $\boldsymbol{y}_i^{(k)}$ is of length N, $\boldsymbol{v}_i^{(k)} = [v_{i,0}^{(k)}, ..., v_{i,N-1}^{(k)}]^T$ is additive white gaussian noise, $\boldsymbol{H}$ is the circulant channel matrix of size $N \times N$ as in Eq. **Ошибка! Источник ссылки не найден.**:

$$
\boldsymbol{H} = \begin{bmatrix}
h_0 & 0 & \cdots & 0 & h_L & \cdots & h_1 \\
h_1 & h_0 & \ddots & \vdots & \ddots & \ddots & \vdots \\
\vdots & h_1 & \ddots & \ddots & \vdots & \ddots & h_L \\
h_L & \vdots & \ddots & \ddots & \ddots & \vdots & 0 \\
0 & h_L & \vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & h_L & \cdots & h_1 & h_0
\end{bmatrix} \tag{5}
$$

We assume that the noise is a complex Gaussian, zero mean with variance of $s_{v^{(k)}}^2$.

2.1.2 Blind channel estimation

We suggest the following methods for implementing a subspace approach in order to estimate the impulse response of the channel:

Considering the autocorrelation matrix of the received signal $\boldsymbol{y}_i^{(k)}$ as shown below.

$$
\boldsymbol{R}^{(k)} = \boldsymbol{E}\left[\boldsymbol{y}_i^{(k)}\boldsymbol{y}_i^{(k)H}\right] = \boldsymbol{E}\left[\left(\boldsymbol{HF}^H\boldsymbol{A}^{(k)}\boldsymbol{d}_i + \boldsymbol{v}_i^{(k)}\right)\left(\boldsymbol{d}_i^H\boldsymbol{A}^{(k)H}\boldsymbol{FH}^H + \boldsymbol{v}_i^{(k)H}\right)\right] \tag{6}
$$

$$
\boldsymbol{R}^{(k)} = \boldsymbol{HF}^H\boldsymbol{A}^{(k)}\boldsymbol{E}\left[\boldsymbol{d}_i\boldsymbol{d}_i^H\right]\boldsymbol{A}^{(k)H}\boldsymbol{FH}^H + \boldsymbol{E}\left[\boldsymbol{v}_i^{(k)}\boldsymbol{v}_i^{(k)H}\right]
$$

We propose that the data symbols has an average power equal to one, then the data and noise auto-correlation matrices has the following characteristics: $\boldsymbol{E}\left[\boldsymbol{d}_i\boldsymbol{d}_i^H\right] = s_s^2\boldsymbol{I} = \boldsymbol{I}$, $\boldsymbol{E}\left[\boldsymbol{v}_i^{(k)}\boldsymbol{v}_i^{(k)H}\right] = s_{v^{(k)}}^2\boldsymbol{I}$

Since for all k cases of precoding matrices, we implement a matrix that reduces Eq. **Ошибка! Источник ссылки не найден.** to the following:

$$
\boldsymbol{R}^{(k)} = \boldsymbol{HP}^{(k)}\boldsymbol{H}^H + s_{v^{(k)}}^2\boldsymbol{I} \tag{7}
$$

where $\boldsymbol{P}^{(k)}$ is represented by the following equation:

$$
\boldsymbol{P}^{(k)} = \boldsymbol{F}^H\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}\boldsymbol{F} \tag{8}
$$

Precoders are designed by assuming that $\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}$ is a circulant matrix. Due to the diagonalization performed by matrix $\boldsymbol{F}$ over $\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}$, the matrix $\boldsymbol{P}^{(k)}$ is a diagonal matrix with the main diagonal's elements .

It is difficult to analyze the first term in Eq. **Ошибка! Источник ссылки не найден.** since the product leads to so many distinct conclusions, but thanks to the features of $\boldsymbol{P}^{(k)}$ (Appendix A), we can view the first $L+1$ vectors of length $L+1$ as follows:

$$
\boldsymbol{HP}^{(k)}\boldsymbol{H}^H\left(1:L+1, 1:L+1\right) = \left(\overline{\boldsymbol{h}}^{(1,k)} \quad \overline{\boldsymbol{h}}^{(2,k)} \quad \cdots \quad \overline{\boldsymbol{h}}^{(L+1,k)}\right) \tag{9}
$$

where $\overline{\boldsymbol{h}}^{(i,k)}$, $i = 1, 2, ..., L+1$ consists of three vectors:

$$\bar{\boldsymbol{h}}^{(i,k)} = \boldsymbol{h}_d^{(i,k)} + \boldsymbol{h}_u^{(i,k)} + \boldsymbol{h}_L^{(i,k)} \tag{10}$$

Each vector in Eq. **Ошибка! Источник ссылки не найден.** is represented as follows:

$$\boldsymbol{h}_d^{(i,k)} = P_0^{(k)} h_{i-1}^* \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L+1} \end{bmatrix} = P_0^{(k)} h_{i-1}^* \boldsymbol{h} \tag{11}$$

$$\boldsymbol{h}_u^{(i,k)} = \begin{cases} \sum_{j=1}^{L+1-i} P_{N-j}^{(k)} h_{j+i-1}^* \begin{bmatrix} \boldsymbol{h}(j+1:L+1,1) \\ 0_{j\times 1} \end{bmatrix} &, 1 \le i \le L \\ 0 & i = L+1 \end{cases} \tag{14}$$

$$\boldsymbol{h}_L^{(i,k)} = \begin{cases} \sum_{j=1}^{i-1} P_j^{(k)} h_{i-1-j}^* \begin{bmatrix} 0_{j\times 1} \\ \boldsymbol{h}(1:L+1-j,1) \end{bmatrix} &, 2 \le i \le L+1 \\ 0 & i = 1 \end{cases} \tag{13}$$

Equations **Ошибка! Источник ссылки не найден.-Ошибка! Источник ссылки не найден.**) provide a number of observations as follows:

1. Since $P_0^{(k)}$ is the largest element in $\boldsymbol{P}^{(k)}$ (equations of elements of $\boldsymbol{P}^{(k)}$ in Appendix A), the vector $\boldsymbol{h}_d^{(i,k)}$ has dominance over the other two vectors.

2. Both $\boldsymbol{h}_u^{(i,k)}$ and $\boldsymbol{h}_L^{(i,k)}$ include an upper and lower shifted version of $\boldsymbol{h}$.

3. The existence of vector $\boldsymbol{h}_d^{(i,k)}$ depends on the characteristics of channel impulse response which can be zero for the indexes i where $h_i = 0$.

4. The system is assumed to be synchronized, means that $h_0 \ne 0$.

Based on the foregoing, we have suggested that the estimated channel vector $\breve{\boldsymbol{h}}^{(k)}$ in this study be as follows:

$$\breve{\boldsymbol{h}}^{(k)} = \bar{\boldsymbol{h}}^{(1,k)} + s_{v^{(k)}}^2 \boldsymbol{e}_1 = \boldsymbol{h}_d^{(1,k)} + \boldsymbol{h}_u^{(1,k)} + s_{v^{(k)}}^2 \boldsymbol{e}_1 \tag{15}$$

The effect of using precoding is obvious in the previous equation where the channel vector is scaled by $P_0^{(k)}$. The normalized estimated channel vector $\breve{\boldsymbol{h}}_n^{(k)}$ can be derived directly by normalizing $\breve{\boldsymbol{h}}^{(k)}$: $\breve{\boldsymbol{h}}_n^{(k)} = \frac{\breve{\boldsymbol{h}}^{(k)}}{\|\breve{\boldsymbol{h}}^{(k)}\|}$

A common problem with estimating a channel blindly by subspace approach is known as scalar ambiguity, in which a scalar complex value a_k is required for the final estimation of the channel coefficients. a_k can be estimated from Eq. **Ошибка! Источник ссылки не найден.**.

$$\alpha_k = \frac{hp_m^{(k)}}{\breve{h}_n^{(k)}(m)} \tag{16}$$

where *m* is the index that represents the position of pilot signal in $s_i^{(k)}$, $hp_m^{(k)}$ calculated by Eq. **Ошибка! Источник ссылки не найден.**.

$$hp_m^{(k)} = \frac{1}{M} \sum_{i=1}^{M} \frac{Y_{i,m}^{(k)}}{s_{i,m}^{(k)}} = \frac{1}{M} \sum_{i=1}^{M} \frac{F(m,:)y_i^{(k)}}{s_{i,m}^{(k)}} \tag{17}$$

where M is number of transmitted OFDM symbols. The final estimated channel vector $\breve{h}_e^{(k)}$ are derived as follows:

$$\breve{h}_e^{(k)} = a_k \breve{h}_n^{(k)} \tag{18}$$

2.1.3 Data detection

As long as the estimation process is accomplished at the pre-FFT stage, we propose that data detection is performed a simple zero forcing equalizer and continue with the rest procedure in Fig.1 as in Eq. **Ошибка! Источник ссылки не найден.**:

$$\breve{d}_i^{(k)} = \left(A^{(k)}\right)^{-1} F \left(\breve{H}_e^{(k)}\right)^{-1} y_i^{(k)} \tag{19}$$

where $\breve{d}_i^{(k)}$ is the estimated symbols vector, $\breve{H}_e^{(k)}$ is the Toeplitz matrix constructed using the vector $\breve{h}_e^{(k)}$ with the design shown in Eq. **Ошибка! Источник ссылки не найден.**.

2.2 Channel estimation of OFDM using a block precoding matrix

2.2.1 System model

This model suggests to precode data vectors using block matrices of the precoding matrix. This is accomplished by constructing a block matrix $A_b^{(k)}$ of precoding matrices by utilizing the following method:

Assuming that d_{2i} and d_{2i+1} are even and odd data vectors, respectively, we define the block version of the data vectors as $D_i = \begin{bmatrix} d_{2i} & d_{2i+1} \end{bmatrix}^T$, and the transmitted symbols are then stated as:

$$S_i^{(k)} = \begin{bmatrix} s_{2i}^{(k)} \\ s_{2i+1}^{(k)} \end{bmatrix} = \begin{bmatrix} A_{11}^{(k)} & A_{12}^{(k)} \\ A_{21}^{(k)} & A_{22}^{(k)} \end{bmatrix} \begin{bmatrix} d_{2i} \\ d_{2i+1} \end{bmatrix} = A_b^{(k)} D_i \tag{20}$$

where $S_i^{(k)} = \begin{bmatrix} s_{2i}^{(k)} & s_{2i+1}^{(k)} \end{bmatrix}^T$, $k = 1, 2, 3$ is a matrix of precoded blocks, and $A_b^{(k)}$ of size $2N \times 2N$ is a block precoding matrix of the matrices $(A_{11}^{(k)}, A_{12}^{(k)}, A_{21}^{(k)}, A_{22}^{(k)})$. The signal goes through the same processes as in the prior model. The final block vector is as follows:

$$Y_i^{(k)} = \begin{bmatrix} y_{2i}^{(k)} \\ y_{2i+1}^{(k)} \end{bmatrix} = \begin{bmatrix} HF^H & 0 \\ 0 & HF^H \end{bmatrix} \begin{bmatrix} A_{11}^{(k)} & A_{12}^{(k)} \\ A_{21}^{(k)} & A_{22}^{(k)} \end{bmatrix} \begin{bmatrix} d_{2i} \\ d_{2i+1} \end{bmatrix} + \begin{bmatrix} v_{2i}^{(k)} \\ v_{2i+1}^{(k)} \end{bmatrix} \tag{21}$$

$$Y_i^{(k)} = \bar{H} A_b^{(k)} D_i + V_i^{(k)}$$

where $\bar{H} = diag([HF^H \quad HF^H]^T)$ and H is the convolution matrix of the channel mentioned in Eq. **Ошибка! Источник ссылки не найден.**, F is the matrix of the discrete Fourier transform, $V_i^{(k)} = [v_{2i}^{(k)} \quad v_{2i+1}^{(k)}]^T$ is an additive noise vector. We assume that the noise is complex Gaussian, zero mean, with variance of $s_{v^{(k)}}^2$.

As this approach is dependent on the cross-correlation of each pair of neighboring symbols, the resulting symbol can be written as:

$$y_{2i}^{(k)} = HF^H \left(A_{11}^{(k)} d_{2i} + A_{12}^{(k)} d_{2i+1} \right) + v_{2i}^{(k)} \tag{22}$$

$$y_{2i+1}^{(k)} = HF^H \left(A_{21}^{(k)} d_{2i} + A_{22}^{(k)} d_{2i+1} \right) + v_{2i+1}^{(k)} \tag{23}$$

2.2.2 Blind channel estimation

As a result of the design of precoding system, the cross-correlation of two successive symbols develops a unique formation as seen below:

$$R^{(k)} = E \left[y_{2i}^{(k)} y_{2i+1}^{(k)H} \right] \tag{24}$$

$$R^{(k)} = E \left[\left(HF^H \left(A_{11}^{(k)} d_{2i} + A_{12}^{(k)} d_{2i+1} \right) + v_{2i}^{(k)} \right) \left(\left(d_{2i}^H A_{21}^{(k)H} + d_{2i+1}^H A_{22}^{(k)H} \right) FH^H + v_{2i+1}^{(k)H} \right) \right]$$

In this method the noise is been eliminated by using the feature of uncorrelation in noise. By the assumption of signal and noise, The matrix $R^{(k)}$ can be written as follows:

$$R^{(k)} = E \left[\left(HF^H \left(A_{11}^{(k)} d_{2i} + A_{12}^{(k)} d_{2i+1} \right) \left(d_{2i}^H A_{21}^{(k)H} + d_{2i+1}^H A_{22}^{(k)H} \right) FH^H \right) \right] + E \left[\left(v_{2i}^{(k)} v_{2i+1}^{(k)H} \right) \right] \tag{25}$$

$$R^{(k)} = HF^H \left(A_{11}^{(k)} A_{21}^{(k)H} + A_{12}^{(k)} A_{22}^{(k)H} \right) FH^H \tag{26}$$

$$R^{(k)} = HP^{(k)} H^H$$

where,

$$P^{(k)} = F^H \left(A_{11}^{(k)} A_{21}^{(k)H} + A_{12}^{(k)} A_{22}^{(k)H} \right) F \tag{27}$$

The matrix $P^{(k)}$ is diagonal as in the previous model. This is the basic assumption in this paper, by which we had proposed the design of each precoding matrix.

Eq. **Ошибка! Источник ссылки не найден.** has the same structure as the first term in Eq. **Ошибка! Источник ссылки не найден.**. We follow the same steps from Eq. **Ошибка! Источник ссылки не найден.** to Eq. **Ошибка! Источник ссылки не найден.** in order to calculate the final channel estimation vector in time domain.

2.2.3 Data detection

We use the following equation to estimate the data:

$$\breve{D}_i^{(k)} = \left(A_b^{(k)} \right)^{-1} F_d \left(\bar{H}_e^{(k)} \right)^{-1} Y_i^{(k)} \tag{28}$$

where, $\tilde{\boldsymbol{D}}_i^{(k)}$ is the estimated symbols block vector, $\bar{\boldsymbol{H}}_e^{(k)} = diag([\tilde{\boldsymbol{H}}_e^{(k)} \quad \tilde{\boldsymbol{H}}_e^{(k)}]^T)$ and $\boldsymbol{F}_d = diag([\boldsymbol{F} \quad \boldsymbol{F}]^T)$.

2.3 Design of precoding matrix

The precoder must satisfy some important conditions, such as:

C1- In order for the matrix $\boldsymbol{P}^{(k)}$ to be diagonal as in Eq. **Ошибка! Источник ссылки не найден.** and Eq. **Ошибка! Источник ссылки не найден.**. The matrix product $\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}$ should be circulant.

C2- The precoding matrix must have a full rank. This condition is necessary specially for data detection.

C3- The average symbol power must be preserved by the precoder.

2.3.1 Circulant precoder $\boldsymbol{A}^{(1)}$

Many papers [2,11] have used this design of precoding matrix to estimate the OFDM channel as in Eq. **Ошибка! Источник ссылки не найден.**:

$$\boldsymbol{A}^{(1)} = \begin{bmatrix} \sqrt{r} & \sqrt{\dfrac{1-r}{N-1}} & \cdots & \sqrt{\dfrac{1-r}{N-1}} \\ \sqrt{\dfrac{1-r}{N-1}} & \sqrt{r} & \cdots & \sqrt{\dfrac{1-r}{N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\dfrac{1-r}{N-1}} & \sqrt{\dfrac{1-r}{N-1}} & \cdots & \sqrt{r} \end{bmatrix} \tag{29}$$

where $0 < r \leq 1$. Eq. **Ошибка! Источник ссылки не найден.** represents the precoding matrix used in the first system. In the second system, the precoding matrix will have the following structure:

$$\boldsymbol{A}_b^{(1)} = \begin{bmatrix} \boldsymbol{A}_{11}^{(1)} & \boldsymbol{A}_{12}^{(1)} \\ \boldsymbol{A}_{21}^{(1)} & \boldsymbol{A}_{22}^{(1)} \end{bmatrix} \tag{30}$$

We choose the structure of the blocked matrix according to [2,11] as follows:

$$\boldsymbol{A}_{11}^{(1)} = \boldsymbol{A}_{22}^{(1)} = \frac{2}{3}\boldsymbol{A}^{(1)} \tag{31}$$

$$\boldsymbol{A}_{12}^{(1)} = \boldsymbol{A}_{21}^{(1)} = \frac{1}{3}\boldsymbol{A}^{(1)} \tag{32}$$

This matrix is implemented in the second system for the first case of precoding of the matrix.

2.3.2 Precoding matrices $A^{(2)}$ and $A^{(3)}$

We propose two precoding matrices $A^{(2)}$ and $A^{(3)}$ which are already been mentioned in [16] as follows:

1-   Generate two row vectors r_N, $r_{N/4}$ following the equation in Eq. **Ошибка! Источник ссылки не найден.**.

$$r_W(i) = \begin{cases} 2r\,e^{\left(-2r\frac{i-1}{N}\right)} & , r \geq 0 \quad for\ \frac{i-1}{W} \in \mathbf{Z} \\ \dfrac{1}{\sqrt{N}} & otherwise \end{cases} \tag{33}$$

where $i = 1,2,...,N$.

2-   Constructing the corresponding Toeplitz matrix $A_T^{(l)}$, as in Eq. **Ошибка! Источник ссылки не найден.**.

$$A_T^{(l)} = Toeplitz\left(\left[r_{W_l}(1)\quad flip\left(r_{W_l}(2:N)\right)\right], r_{W_l}^T\right) \tag{34}$$

where $l = 2,3$, $W_2 = N$, $W_3 = N/4$.

3-   Finally the precoding matrix is constructed using SVD as follows:

$$\left[U^{(l)}, S^{(l)}, V^{(l)}\right] = SVD\left(A_T^{(l)}\right) \tag{35}$$

$$A^{(l)} = U^{(l)}S^{(l)} \tag{36}$$

3 Channel order estimation

As shown in equations **Ошибка! Источник ссылки не найден.-Ошибка! Источник ссылки не найден.**), the structure of the dominant vector in Eq.**Ошибка! Источник ссылки не найден.** includes the channel coefficient h_i^* at column index $i+1$. This means that the existence of dominant vector at column index > 1 depends on the channel order and also the percentage of $P_0^{(k)}/P_j^{(k)}$, $j = 1,...,N-1$ which also depends on the type and structure of precoding matrix also the length of OFDM system N (as shown in Appendix A).

Because the channel estimation process depends on the correlation matrix that has already been computed, we propose a simple method that can be employed at stage. The algorithm is as follows:

1.   Create new vector $Nv^{(k)}(i)$ that contains the norm values of vectors $\bar{h}^{(i,k)}$.

$$Nv^{(k)}(i) = \left\|\bar{h}^{(i,k)}\right\|,\ i = 1,...,L+1 \tag{37}$$

2.   Calculate the mean value $m^{(k)}$ of the $Nv^{(k)}(i)$.

3.   Find the indexes of i such that $Nv^{(k)}(i) > m^{(k)}$ and choose the largest index to be $\check{L}m^{(k)}$. Then the estimated channel order is as follows:

$$\breve{L}^{(k)} = \breve{L}m^{(k)} - 1 \tag{38}$$

4 Results and discussion

We examine the performance of the proposed estimator under various scenarios. We use the channel model with the exponential power delay profile mentioned in [16]. The parameter for channel model are $t_{rms}/T_s = 10$, $n = 40$ and $L_c = 2$ (3-tapped channel). The other parameters of the model is as follows : modulation type 16QAM with $N = 64$, length of CP (L) = 16. The simulation carried out with different numbers of OFDM symbols and with Monte-Carlo run $n_w = 160$.

The normalized estimation mean square errors (NMSE) is defined as:

$$NMSE(k) = \frac{1}{n \times n_w} \sum_{i}^{n_w} \sum_{j}^{n} \frac{var\left(\boldsymbol{h}_{(j)} - (\tilde{\boldsymbol{h}}_e^{(k)})_{(i,j)}\right)}{var\left(\boldsymbol{h}_{(j)}\right)} \tag{39}$$

where *j* is the index of different channel realizations and *i* is the index of Monte-Carlo runs.

4.1 Example 1

The effects of utilizing the exponential decaying precoder, specifically at $r = 0.1$, are shown in the NMSE results in Fig. 2. When comparing the results of the same r value, we can observe that using lower r values produces better results.

Depending on the SNR value, each curve exhibits a linear relationship till specific limit. For SNR < 20 dB and employing $A^{(2)}$ at $r = 0.1$, 0.3 and $A^{(3)}$ at $r = 0.1$ yields the best performance, and for greater SNR values, the results that surpass all other results.
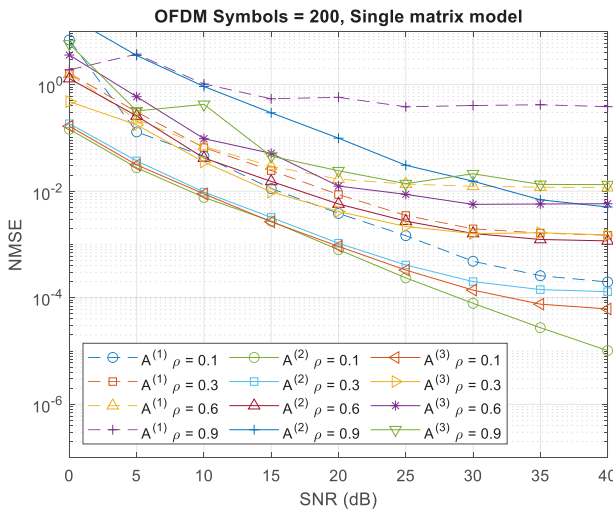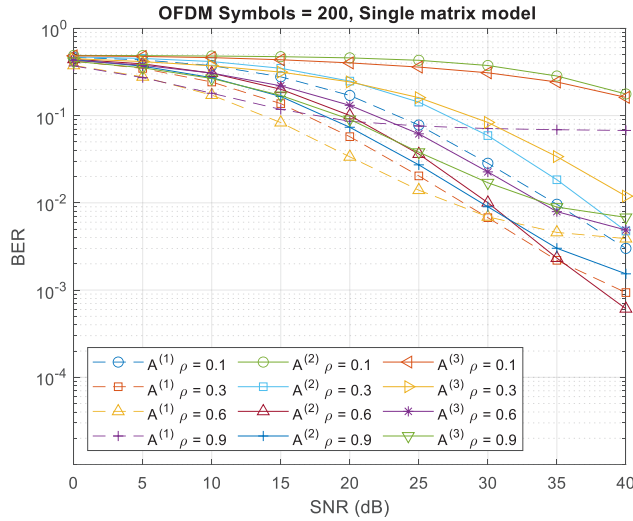

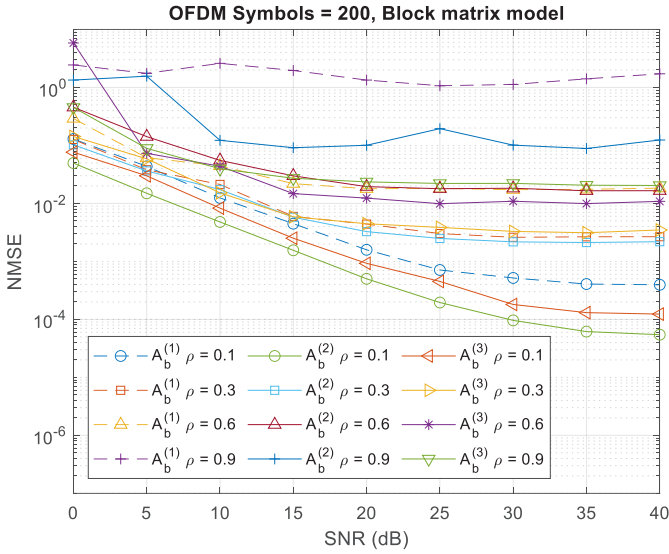**Fig 2.** NMSE for the first system with lower OFDM symbols.

In Fig. 3, an overview of the results of the probability of a bit error. For SNR < 35 dB, using $A^{(1)}$ at various r values (at SNR ≤ 10 with $r = 0.6$ and $r = 0.9$, at $10 <$ SNR ≤ 30 with $r = 0.6$ and at $30 <$ SNR ≤ 35 $r = 0.3$) generally results in the best performance. The matrix $A^{(2)}$ at $r = 0.6$ is superior to the other values with higher SNR. At the same value of r, using matrix $A^{(1)}$ shows an impact in BER over the other cases at $r = 0.1$, $r = 0.3$. For other values of r the performance is related to SNR values.

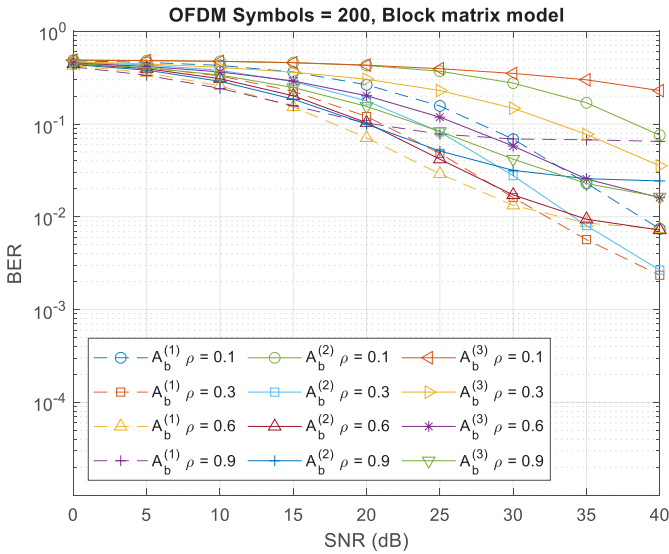**Fig 3.** BER for the first system with lower OFDM symbols.

4.2 Example 2

In Fig. 4 and Fig. 5, we examined the proposed matrices in the second system. The NMSE results in Fig. 4 shows that generally the best performance using $A_b^{(2)}$ at $r = 0.1$. Same conclusions of the previous model when comparing results at the same r value. In this system, all curves lose their dependence on SNR at different values of SNR with no exceptions specially for SNR > 35dB.

**Fig. 4.** NMSE for the second system with lower OFDM symbols.

In Fig.5, at the same value of r, the matrix $A_b^{(3)}$ shows worse performance compared to other matrices. As in the previous scenario, the matrix $A_b^{(1)}$ shows the best BER performance at different r value depending on SNR (at SNR ≤ 15 with $r = 0.6$ and $r = 0.9$, at $15 <$ SNR ≤ 32 with $r = 0.6$ and at SNR > 32 $r = 0.3$)

**Fig. 5.** BER for the second system with lower OFDM symbols.

4.3 Example 3

In this Example, we compare the results of both systems. In Fig. 6, we consider a comparison of curves with the best NMSE results of both systems for each matrix, in which we can see the advantage of using precoding matrix $A^{(2)}$ for SNR < 27 dB and $A_b^{(2)}$ for higher SNR values.
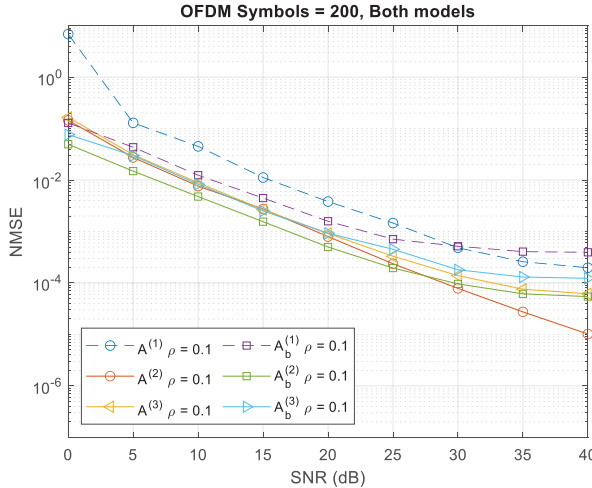
Fig 6. NMSE comparison of both systems using curves with the best performance of each precoding matrix.

In Fig. 7 we investigate the variation of NMSE curves with the best performance at different numbers of OFDM symbols at lower and higher SNR values.
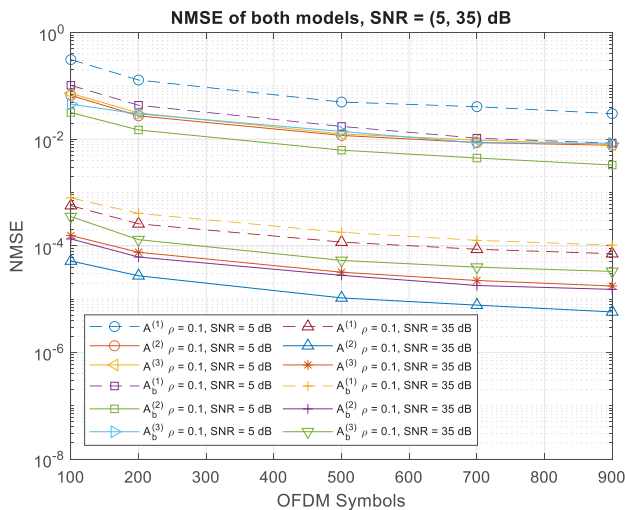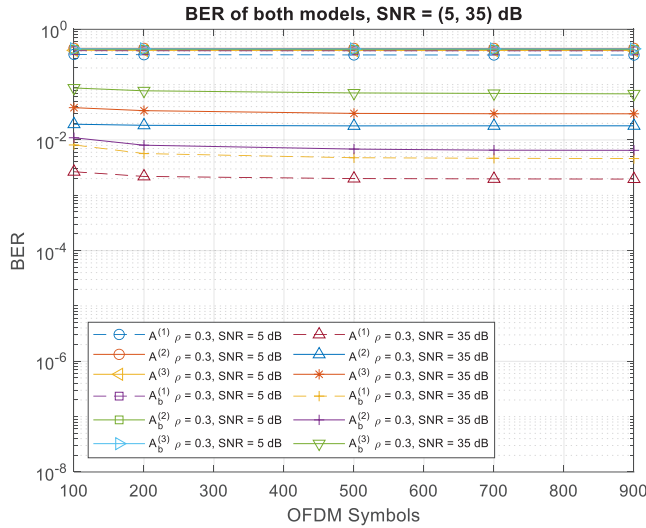

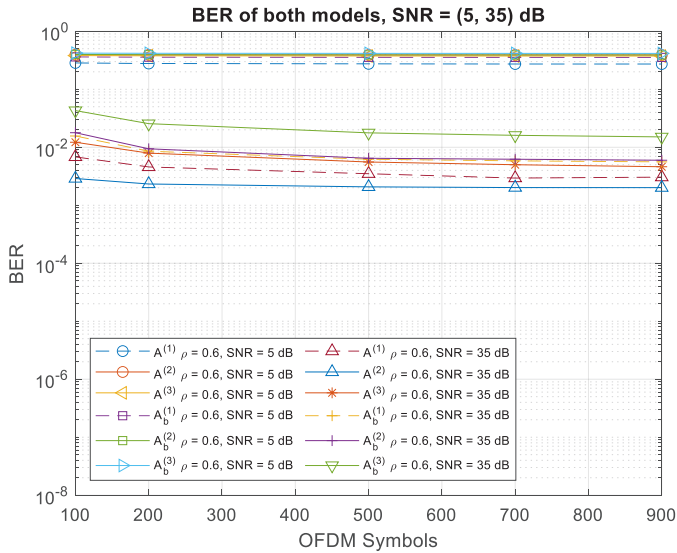**Fig. 7.** Comparison of NMSE of both systems with the number of OFDM symbols.

At higher SNR, the best performance is achieved by implementing the proposed matrices $A^{(2)}$, $A_b^{(2)}$ with the advantage of the first model over all the results. Since the number of multiplications required to calculate $R^{(k)}$ in the second model is half that of the first model, the results of NMSE using $A^{(2)}$ as a comparison with the rest matrices shows an impact even

with the remainder matrices of the first model. This makes it a good option for practical systems.

In Fig. 8, Fig. 9 and Fig. 10 we consider two different cases of SNR. Even with different numbers of OFDM symbols, the behavior of all systems is similar at lower SNR levels. At varying values of OFDM symbols and at higher SNR, we can see that matrix $A^{(2)}$ performs better in terms of the overall findings of the NMSE.

Fig. 8. Comparison of BER in both systems with the number of OFDM symbols at $r = 0.3$.

**Fig. 9.** Comparison of BER in both systems with the number of OFDM characters at $r = 0.6$.
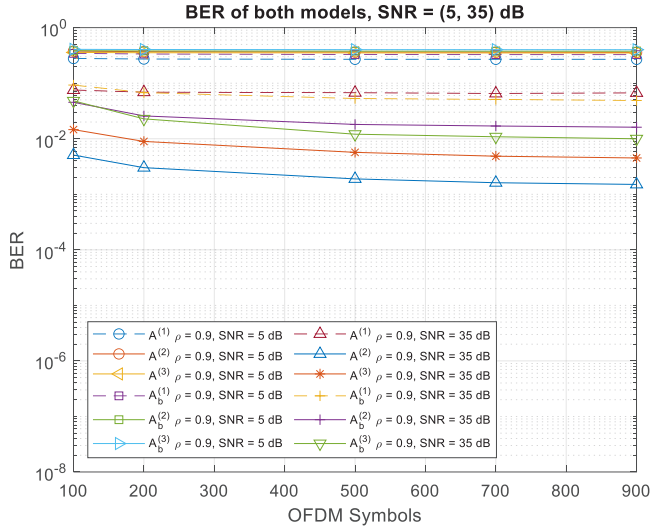
Fig. 10. Comparison of BER in both systems with the number of OFDM symbols at $r = 0.9$.

Probability of correct channel order estimation (*Pe*) is investigated using both models. The channel impulse response used in this method is $h = \begin{bmatrix} -0.4 - 0.17i, \ 0.11 + 0.06i, -0.1 + 0.12i, \ 0.66 - 0.5i, -0.24 + 0.16i \end{bmatrix}^T$ of order $L_c = 4$ and at Monte Carlo runs of 5000.

The formula for *Pe* is as follows:

$$Pe = \frac{n_{\tilde{L}^{(k)} = L_c}}{n_w} \tag{40}$$

where $n_{\tilde{L}^{(k)} = L_c}$ is the number of times that $\tilde{L}^{(k)} = L_c$ and n_w is the Monte Carlo runs.
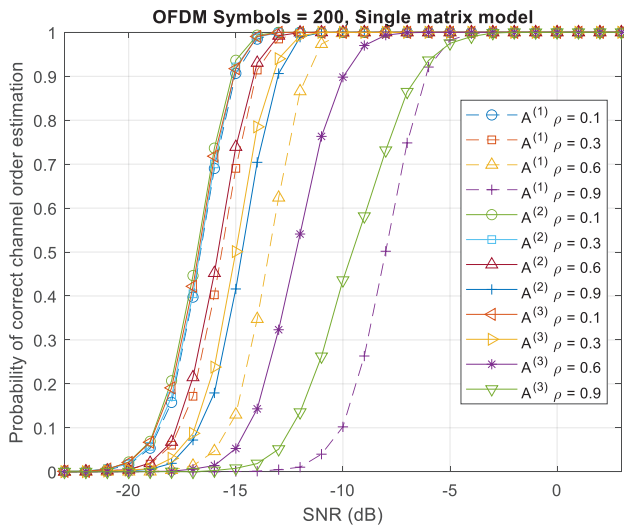
Fig. 11. Probability of correct channel order estimation in the first model.

In Fig. 11 Probability of correct estimation shows a similar results when using any precoding matrix at $r = 0.1$ and using $A^{(2)}$ at $r = 0.3$, by which the channel order is fully estimated for SNR > -13dB. Clearly shown for lower r is the better results of estimation. It is worth to mention is that for SNR > -2dB the channel order is fully estimated with all options of precoding matrices and r values.

In Fig. 12 we see similar pattern as in the previous results but with some differences. The best performance of order estimation achieved at $r = 0.1$ and for all precoding matrices. The channel order in this model is fully estimated at SNR > 2dB for all scenarios.
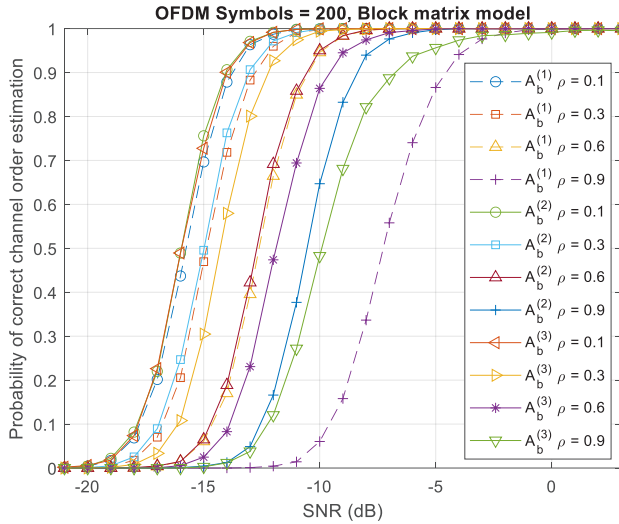
Fig. 12. Probability of correct channel order estimation in the second model.

5 Conclusion

This article demonstrates a new precoding blind channel estimation method for OFDM systems with cyclic padding. This algorithm appends low complexity to the system as compared with the traditional precoding techniques used for channel estimation. NMSE results and bit error rates were investigated for all scenarios and compared under different conditions of SNR, number of OFDM symbols and r values with the same channel condition. Channel order estimation is also investigated using proposed algorithm of norms mean as a threshold. This algorithm does not add affective complexity to the overall system of the receiver since it relies on the correlation matrix which already been calculate for channel estimation. A mathematical proof is mentioned which shows how the use of all matrices is robust to channel order estimation. The results of channel estimation and channel order estimation show how the precoding matrix can improve the system performance. The design of precoding matrix should be investigated in future works to obtain a general optimal structure for the precoding matrix and also new algorithm for data detection that exhibits this design for better bit error rate performance. The mathematical proof shows the effect of the number of transmitted OFDM symbol and the length of subcarriers in the system *N* over the performance of channel order estimation is also a case of future study.

6 Appendix A

In this appendix we derive the expression of $P_0^{(k)}$ and $P_j^{(k)}$ for $1 \le j \le N-1$. First we prove the diagonality property of $\boldsymbol{P}^{(k)}$ as follows:

1-   At $k=1$ it is trivial to prove that $\boldsymbol{P}^{(1)}$ is diagonal since $\boldsymbol{A}^{(1)}\boldsymbol{A}^{(1)H}$ is circulant.

2-   At $k=2,3$ and from Eq. **Ошибка! Источник ссылки не найден.** and Eq. **Ошибка! Источник ссылки не найден.**:

$$\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H} = \boldsymbol{U}^{(k)}\boldsymbol{S}^{(k)}\boldsymbol{S}^{(k)H}\boldsymbol{U}^{(k)H} = \boldsymbol{U}^{(k)}\boldsymbol{S}^{(k)}\boldsymbol{V}^{(k)}\boldsymbol{V}^{(k)H}\boldsymbol{S}^{(k)H}\boldsymbol{U}^{(k)H} \tag{42}$$
$$\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H} = \boldsymbol{A}_T^{(k)}\boldsymbol{A}_T^{(k)H}$$

This proves that the general structure of $\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}$ is also circulant. Next we derive an expression of P_0 and P_j upon the previous conclusions:

1-   At $k=1,2$ we notice that $\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H}$ has the same structure in both cases which can be defined as follows:

$$\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H} = \begin{bmatrix} x_k^2 + \sum_{m=1}^{N-1} y_k^2 & 2x_k y_k + \sum_{m=1}^{N-2} y_k^2 & \cdots & 2x_k y_k + \sum_{m=1}^{N-2} y_k^2 \\ 2x_k y_k + \sum_{m=1}^{N-2} y^2 & x_k^2 + \sum_{m=1}^{N-1} y_k^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 2x_k y_k + \sum_{m=1}^{N-2} y_k^2 & \cdots & \cdots & x_k^2 + \sum_{m=1}^{N-1} y_k^2 \end{bmatrix}$$
$$(43)$$

$$\boldsymbol{A}^{(k)}\boldsymbol{A}^{(k)H} = \begin{bmatrix} x_k^2 + (N-1)y_k^2 & 2x_k y_k + (N-2)y_k^2 & \cdots & 2x_k y_k + (N-2)y_k^2 \\ 2x_k y_k + (N-2)y_k^2 & x_k^2 + (N-1)y_k^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 2x_k y_k + (N-2)y_k^2 & \cdots & \cdots & x_k^2 + (N-1)y_k^2 \end{bmatrix} \tag{44}$$

where x_k is the diagonal element in $\boldsymbol{A}^{(1)}$ at $k=1$ and $\boldsymbol{A}_T^{(2)}$ at $k=2$, y_k is the off diagonal elements of the mentioned matrixes. Mathematical equation for x_k, y_k can be found in [16].

Then $P_0^{(k)}$ can be defined as follows:

$$P_0^{(k)} = x_k^2 + (N-1)y_k^2 + 2x_k y_k (N-1) + (N-2)(N-1)y_k^2 \tag{45}$$

$$P_0^{(k)} = x_k^2 + 2x_k y_k (N-1) + (N-1)^2 y_k^2 \tag{46}$$

The rest elements $P_j^{(k)}$ of $\boldsymbol{P}^{(k)}$ are viewed as follows:

$$P_j^{(k)} = x_k^2 + (N-1)y_k^2 + \left(2x_k y_k + (N-2)y_k^2\right)\sum_{i=1}^{N-1} W_N^{ji} \qquad, 1 \le j \le N-1 \tag{47}$$

$$\sum_{i=1}^{N-1} W_N^{ji} = -1 \quad 1 \le j \le N-1 \tag{48}$$

Substituting Eq. **Ошибка! Источник ссылки не найден.** in Eq. **Ошибка! Источник ссылки не найден.**:

$$P_j^{(k)} = x_k^2 + (N-1)y_k^2 - \left(2x_k y_k + (N-2)y_k^2\right) = x_k^2 - 2x_k y_k + y_k^2 \tag{49}$$

3-   At $k=3$ we see the following structure:

$$A_T^{(3)} A_T^{(3)H} = \begin{bmatrix} a_1 & b & \cdots & a_2 & b & \cdots & a_3 & b & \cdots & a_4 & b & \cdots \\ b & \ddots & \ddots & b & \ddots & \ddots & b & \ddots & \ddots & b & \ddots & \ddots \\ \vdots & \ddots & a_1 & \ddots & \ddots & a_2 & \ddots & \ddots & a_3 & \ddots & \ddots & a_4 \\ a_2 & b & \ddots & a_1 & b & \ddots & a_2 & b & \ddots & a_3 & b & \ddots \\ b & \ddots & \ddots & b & \ddots & \ddots & b & \ddots & \ddots & b & \ddots & \ddots \\ \vdots & \ddots & a_2 & \ddots & \ddots & a_1 & \ddots & \ddots & a_2 & \ddots & \ddots & a_3 \\ a_3 & b & \ddots & a_2 & b & \ddots & a_1 & b & \ddots & a_2 & b & \ddots \\ b & \ddots & \ddots & b & \ddots & \ddots & \ddots & \ddots & \ddots & b & \ddots & \ddots \\ \vdots & \ddots & a_3 & \ddots & \ddots & a_2 & \ddots & \ddots & a_1 & \ddots & \ddots & a_2 \\ a_4 & b & \ddots & a_3 & b & \ddots & a_2 & \ddots & \ddots & a_1 & b & \ddots \\ b & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \cdots & a_4 & b & \cdots & a_3 & \cdots & \cdots & a_2 & \cdots & \cdots & a_1 \end{bmatrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\}\dfrac{N}{4} \left.\begin{matrix} \\ \\ \end{matrix}\right\}\dfrac{N}{4} \left.\begin{matrix} \\ \\ \end{matrix}\right\}\dfrac{N}{4} \left.\begin{matrix} \\ \\ \end{matrix}\right\}\dfrac{N}{4} \tag{50}$$

where,

$$a_1 = \sum_{i=1}^{4} (\bar{x}_i)^2 + (N-4)y^2 \tag{51}$$

$$a_{2,4} = \bar{x}_1\bar{x}_2 + \bar{x}_2\bar{x}_3 + \bar{x}_3\bar{x}_4 + \bar{x}_4\bar{x}_1 + (N-4)y^2 \tag{52}$$

$$a_3 = \bar{x}_1\bar{x}_3 + \bar{x}_2\bar{x}_4 + \bar{x}_3\bar{x}_1 + \bar{x}_4\bar{x}_2 + (N-4)y^2 \tag{53}$$

$$b = 2y\sum_{i=1}^{4}\bar{x}_i + (N-8)y^2 \tag{54}$$

where $\bar{x}_i, y$ are defined in [16]. The elements of $P^{(3)}$ can be expressed as follows:

$$P_0^{(3)} = \sum_{j=1}^{4} a_j + (N-4)b \tag{55}$$

$$P_j^{(3)} = \sum_{i=1}^{4} a_i W_N^{j(z_i)} + b \sum_{i=1, i \notin z}^{N} W_N^{ji} \tag{56}$$

where $z = \{z_1, z_2, z_3, z_4\}$, z_i for $i = 1,...,4$ is defined as:

$$z_i = \frac{(i-1)N}{4} \tag{57}$$

Examining the second term in Eq. **Ошибка! Источник ссылки не найден.**, We can rewrite it as follows:

$$b \sum_{i=1, i \notin z}^{N} W_N^{ji} = -b \sum_{i=1, i \in z}^{4} W_N^{ji} \tag{58}$$

Substituting **Ошибка! Источник ссылки не найден.** in **Ошибка! Источник ссылки не найден.**:

$$P_j^{(3)} = \sum_{i=1}^{4} (a_i - b) W_N^{j(z_i)} \tag{59}$$

References

1. W. Feng, W. P. Zhu, M. N. S. Swamy, IEEE Trans. Signal Process **56(7)**, 2821 (2008)
2. B. K. Engiz, C. Kurnaz, H. Sezgin, IET Communications **9(15)**, 1915 (2015)
3. B. Muquet, M. Courville, P. Duhamel, IEEE Transactions on Signal Processing **50(7)**, (2002)
4. W. Fan, F. Gao, Y. Xue, *A New Blind Channel Estimation Algorithm for OFDM Systems*, *International Congress on Image and Signal Processing* (IEEE, 2009)
5. H. Xuejun, B. Houjie, Yu. Songyu, *A new subspace algorithm of blind channel estimation for OFDM systems, IEEE Proceedings on Personal, Indoor and Mobile Radio Communications, 2003. PIMRC 2003* (IEEE, 2003)
6. Y. Zeng, T. S. Ng, IEEE Signal Processing Letters **11(9)**, (2004)
7. S. H. Fang, J. Y. Chen, J. S. Lin, M. D. Shieh, J. Y. Hsu, *Blind Channel Estimation for CP/CP-Free OFDM Systems Using Subspace Approach, IEEE 81st Vehicular Technology Conference (VTC Spring)*, (IEEE, 2015)
8. X. Yue, X. Zhou, *Blind channel estimation for CP-free OFDM communications systems, Eighth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD)*, (IEEE, 2011)
9. A. Petropulu, R. Zhang, *Blind channel estimation for OFDM systems, Proceedings of 2002 IEEE 10th Digital Signal Processing Workshop, 2002 and the 2nd Signal Processing Education Workshop* (IEEE, 2002)
10. R. Zhang, *Blind Channel Estimation for Precoded OFDM System, IEEE International Conference on Acoustics, Speech, and Signal Processing* (IEEE, 2005)
11. L. Yongming, L. Hanwen, W. Yadong, H. Jianguo, Journal of Systems Engineering and Electronics **18(4),** (2007)
12. J. Rissanen, Automatica **14(5)**, 465 (1978)
13. H. Akaike, IEEE Trans. Automat. Contr. **19(6)**, 716 (1974)
14. A. P. Liavas, P. A. Regalia, J. P. Delmas, IEEE Trans. Signal Process. **47(12)**, 3336 (1999)
15. L. Tong, Q. Zhao, IEEE Trans. Signal Process. **47(9)**, 2345 (1999)
16. O. Al-Qadhi, E. Semenov, J. Talib, S. Al-Barrak, M. Al-Ghazali, Blind Channel Estimation in ZP-OFDM System Using New Precoding Matrix over Channels with

Memory, International Scientific Forum on Computer and Energy Sciences (WFCES 2023), (E3S Web of Conferences, 2023)