Study on contact lines of rolls of two-roll modules

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Abstract. The results of solving the main problems of contact interaction in squeezing machines are presented, when the deformation of the contacting bodies is given by the Kelvin Voigt models. Analytical expressions are found that describe the shape of the contact line of the rolls, as well as the patterns of distribution of normal and tangential forces. It has been established that one of the main factors determining the roll contact lines is the ratio of the strain rates of the material being processed and the roll coating. It is revealed that the point where the shear stresses change their signs is located in the sticking area.

1. Introduction

One of the most economical and versatile types of machines used in many industries, including construction, when performing various technological processes, are roller machines.

There are quite a lot of works aimed at developing the scientific foundations for designing and improving roller machines used in many industries when performing various technological processes. At the same time, solving problems of contact interaction, taking into account the physical essence of technological processes, are relevant [1, 2].

The analytical description of the processes of the contact line of rolls having elastic coatings is hampered by significant deformations of both contacting bodies. The difference between the lines of contact from a straight line or an arc of a circle makes it impossible to apply the results of the contact theory of elasticity [3], which contains the most general and rigorous analysis of the interaction of bodies, to solve the problem under consideration.

Solutions to the problems of contact interaction in roller machines, where the work rolls have an elastic coating, depend on how fully it represents the laws of deformation of the contacting bodies [4, 5].

An analysis of the experimental data [6-20] has shown that, in general, the deformation of such contacting bodies of roller machines is described either by empirical formulas of the form $\sigma = A\varepsilon^n$, or by rheological models of the form $\sigma = E\varepsilon + \mu\dot{\varepsilon}$.

The problems of contact interaction in roller machines, in the case when the deformations of the contacting bodies are given by the formulas $\sigma = A\varepsilon^n$ are solved in [21-27]. This paper considers the issue of solving the Problem of contact interaction in roller machines, in the case when the deformations of the contacting bodies are given by models of the form $\sigma = E\varepsilon + \mu\dot{\varepsilon}$.

2. Resultative Methods

The deformation region of the rolls relative to the line of centers is divided into I and II zones (Fig. 1). We will assume that in zone I, there is a simultaneous compression of the interacting bodies, and in zone II, their deformation is recovered.

According to [28-32] on the line of contact AE, there are sections of slip lagging areas AB, no-slip areas BD and advance slip areas DE. In this case, part of the no-slip area (segment BC) refers to the compression zone, and another part (segment CD) refers to recovery, therefore, the deformation properties of the contacting bodies differ.

In this regard, we divide the no-slip area into two sections (2 and 3), corresponding to segments BC and CD, where $C-$ is the point at which the center line and the contact line intersect. The contact line AE has sections 1, 2, 3, and 4 corresponding to segment AB, BC, CD, and DE.

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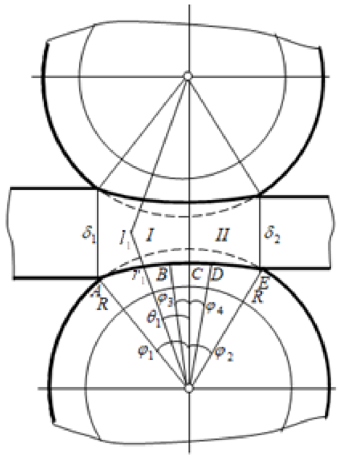

Fig. 1. Scheme of interaction of material with pairs of rolls

Let the deformation of contacting bodies in these areas be determined by the following rheological models:

$$\sigma_{i_i} = E_k \varepsilon_i \pm \mu_k \frac{d\varepsilon_i}{dt}, \qquad \sigma'_{i_i} = E'_k \varepsilon'_i \pm \mu'_k \frac{d\varepsilon'_i}{dt}, \;\; i = 1,2,3,4, \tag{1}$$

where $i-$ is the index indicating the number of sections; $k-$is the index indicating the number of zones; $\sigma_{i/,}\sigma'_i, \varepsilon_i, \varepsilon'_i, E_i, E'_i, \mu_i, \mu'_i -$are the stresses, strains, modulus of deformation, coefficient of viscosity of the roll coating and the processed material, respectively. Sign $(+)$ refers to k $k=1$, sign $(-)$ refers to k $k=2$, for $i=1,2$, $k=1$ and for $i=3,4$, $k=2$.

On the line of contact, we select an elementary sector corresponding to the angle θ_i. Having imagined the coatings of the roll and the material being processed, consisting of such sectors, let us consider the changes that occur with them during the transition from a position corresponding to the angle θ_i, to a new one, determined by the angle $\theta_i - d\theta_i$.

From Figure 1 it follows that

$$2(r_i \pm dr_i + l_i \pm dl_i)\cos(\theta_i \pm d\theta_i) = A, \tag{2}$$

where $r_i, \theta_i -$ are the polar coordinates of the $i-$th section of the roll; $A-$is the center distance of rolls.

We denote the ratio of the strain rate of the processed material $\frac{dl_k}{dt}$ under compression $(k=1)$ and recovery $(k=2)$ to the strain rate of the roll coating $\frac{dr_k}{dt}$ by m_k, ignore the values of the highest order of smallness, and, as a result, transform (2) into the following differential equation:

$$(1+m_k)dr_i - \frac{A}{2}\frac{\sin\theta_i}{\cos^2\theta_i}d\theta_i = 0. \tag{3}$$

The solution to the differential equation (3) is found with the initial conditions for $\theta_1 = -\varphi_1$, $r_1 = R$ and for $\theta_4 = \varphi_2$, $r_2 = R$.

As a result, we obtain the equation of the roll contact line:

$$r_i = R - \frac{A}{2(1+m_k)}\left(\frac{1}{\cos\varphi_k} - \frac{1}{\cos\theta_i}\right), \;\; i = 1,2,3,4, \;\; k = 1,2, \tag{4}$$

where $-\varphi_1 \le \theta_1 \le -\varphi_3$, $-\varphi_3 \le \theta_2 \le 0$, $0 \le \theta_3 \le \varphi_4$, $\varphi_4 \le \theta_4 \le \varphi_2$; $\varphi_1, \varphi_2, \varphi_3, \varphi_4 -$ are the angles between the rays OC and OA, OE, OB, OD, respectively; $R-$is the roll radius.

Equations (4) reflect the relationship between the geometric, kinematic and deformation parameters of the contacting bodies.

Among these parameters, the main ones are the strain rate ratios m_k. We define parameter m_k.
For any time point t in the contact curves, the following relations are valid:

$$\Delta r_i + \Delta l_i = \Delta h_i, \tag{5}$$

$$\frac{dr_i}{dt} + \frac{dl_i}{di} = \frac{dh_i}{dt}, \tag{6}$$

$$E_k \varepsilon_i + \mu_k \frac{d\varepsilon_i}{dt} = E'_k \varepsilon'_i + \mu'_k \frac{d\varepsilon'_i}{dt}, \tag{7}$$

where $h_i = r_i + l_i$, $\varepsilon_i = \dfrac{R - r_i}{\lambda}$, $\varepsilon'_i = \dfrac{l_{k0} - l_i}{l_{k0}}$, $l_{k0} = \dfrac{\delta_k}{2\cos\varphi_k}$, here $\lambda -$ is the thickness of elastic coating of the roll; δ_1, δ_2 – are the initial and final thicknesses of the processed material.
Equation (7) is transformed into the following form:

$$\frac{\Delta r_i}{\lambda} E_k \pm \mu_k \frac{1}{\lambda}\frac{dr_i}{dt} = \frac{\Delta l_i}{l_{k0}} E_k \pm \mu'_k \frac{1}{l_{k0}}\frac{dl_i}{dt}, \tag{8}$$

where $\Delta r_i = R - r_i$, $\Delta l_i = l_{i0} - l_i$.
Taking into account expression $m_k = \dfrac{dl_i}{dr_i} = \dfrac{\Delta l_i}{\Delta r_i} = \dfrac{dl_i/dt}{dr_i/dt}$, from relation (6) we obtain

$$\Delta r_i = \frac{\Delta h_i}{1+m_k}, \quad \Delta l_i = \frac{m_k \Delta h_i}{1+m_k}, \quad \frac{dr_i}{dt} = \frac{1}{1+m_k}\frac{dh_i}{dt}, \quad \frac{dl_i}{dt} = \frac{m_k}{1+m_k}\frac{dh_i}{dt}.$$

Taking into account the latter, from equation (8) we obtain

$$\frac{\Delta h_i}{\lambda} E_k \pm \mu_k \frac{1}{\lambda}\frac{dh_i}{dt} = m_k \frac{\Delta h_i}{l_{k0}} E_k \pm m_k \mu'_k \frac{1}{l_{k0}}\frac{dh_i}{dt}.$$

hence

$$m_k = \frac{\delta_k}{2\lambda\cos\varphi_k}\left(\frac{E_k \Delta h_i \pm \mu_k dh_i/dt}{E'_k \Delta h_i \pm \mu'_k dh_i/dt}\right)$$

or replacing m_k by the average value of m_{kc}

$$m_{kc} = \frac{\delta_k}{2\lambda\cos\varphi_k}\left(\frac{E_k \Delta h_{ic} \pm \mu_k (dh_i/dt)_c}{E'_k \Delta h_{ic} \pm \mu'_k (dh_i/dt)_c}\right).$$

The mean value of Δh_{ic} and $(dh_i/dt)_c$ is found using the theorem of the mean value of integral calculus:

$$\Delta h_{ic} = \frac{1}{\varphi_k}\int_0^{\varphi_k} h_{k0}\left(1 - \frac{\cos\varphi_k}{\cos\theta_i}\right)d\theta_i, \quad (dh_i/dt)_c = \frac{1}{\varphi_k}\int_0^{\varphi_k} \omega h_{k0}\cos\varphi_k \frac{\sin\theta_i}{\cos^2\theta_i}d\theta_i$$

or after integration and transformation:

$$\Delta h_{ic} = h_{k0}\left(1 - \frac{\sin 2\varphi_k}{2\varphi_k}\right), \quad (dh_i/dt)_c = \frac{\omega h_{k0}}{\varphi_k}(1 - \cos\varphi_k),$$

where $h_{k0} = R + \dfrac{\delta_k}{2\cos\varphi_k}$; $\omega -$ is the angular velocity of the roll.
With $r_2 = r_3$ for $\theta_2 = \theta_3 = 0$, from expression (3), we have:

$$\left(\frac{\sin^2\frac{\varphi_1}{2}}{\sin^2\frac{\varphi_2}{2}}\right)\frac{\cos\varphi_2}{\cos\varphi_2} = \frac{1+m_1}{1+m_2}.$$

Using the relationship between the contact angles, it is possible to find restrictions on the choice of m_1 and m_2. Considering that $\varphi_1 \geq \varphi_2$, this restriction is presented in the form $m_1 \geq m_2$.
On the basis of the mathematical models of the roll contact line obtained, we determine the patterns of distribution of contact stresses.

According to [18], at each point of the roll contact line, the following conditions are met:
$n_i = \sigma_i$ or taking into account expression (1):

$$n_i = E_k \varepsilon_i \pm \mu_k \frac{d\varepsilon_i}{dt}, \tag{9}$$

where $n-$ are the normal stresses distributed along the roll contact line.
It follows from equality (4) that

$$r_i' = \frac{A}{2(1+m_k)} \frac{\sin\theta_i}{\cos^2\theta_i}. \tag{10}$$

Taking into account expressions (4) and (10), we obtain:

$$\varepsilon_i = \frac{A}{2\lambda(1+m_k)}\left(\frac{1}{\cos\varphi_k} - \frac{1}{\cos\theta_1}\right), \quad \frac{d\varepsilon_i}{dt} = \frac{A}{2\lambda(1+m_k)} \frac{\sin\theta_i}{\cos^2\theta_i}. \tag{11}$$

Then from equality (9), we find the patterns of distribution of normal stresses:

$$n_i = \frac{A}{2\lambda(1+m_k)}\left(\frac{E_k}{\cos\varphi_k} - \frac{E_k \mp \mu_k tg\theta_i}{\cos\theta_1}\right). \tag{12}$$

The main factor in determining the patterns of distribution is the friction stress model that defines the relationship between normal and shear stresses.
According to studies in [18, 29], in slip zones, it is possible to use the Amontons law, that is, a model of the following form

$$\tau = \pm fn, \tag{13}$$

and for the sticking zone-the friction stress model of the form:

$$\tau = tg(\theta + \xi)n, \tag{14}$$

where $\xi = arctg\frac{F}{Q}$, $F-$is the projection of the horizontal response of the roll onto the Ox-axis, $Q-$is the projection of the pressure of the pressing device and the gravity force of the roll onto the Oy-axis.
Thus, the patterns of distribution of shear stresses in the zone of the roll contact are described by the following equations:

$$\tau_1 = \frac{Af}{2\lambda(1+m_1)}\left(\frac{E_1}{\cos\varphi_1} - \frac{E_1 - \mu_1 tg\theta_1}{\cos\theta_1}\right), \; -\varphi_1 \leq \theta_1 \leq -\varphi_3, \tag{15}$$

$$\tau_2 = \frac{A}{2\lambda(1+m_1)}\left(\frac{E_1}{\cos\varphi_1} - \frac{E_1 - \mu_1 tg\theta_2}{\cos\theta_2}\right) tg(\theta_2 + \xi), \; -\varphi_3 \leq \theta_2 \leq 0, \tag{16}$$

$$\tau_3 = \frac{A}{2\lambda(1+m_2)}\left(\frac{E_2}{\cos\varphi_2} - \frac{E_2 + \mu_2 tg\theta_3}{\cos\theta_3}\right) tg(\theta_3 + \xi), \; 0 \leq \theta_3 \leq \varphi_4, \tag{17}$$

$$\tau_4 = -\frac{Af}{2\lambda(1+m_2)}\left(\frac{E_2}{\cos\varphi_2} - \frac{E_2 + \mu_2 tg\theta_3}{\cos\theta_3}\right), \; \varphi_4 \leq \theta_4 \leq \varphi_2. \tag{18}$$

3. Conclusions

a) Mathematical models have been developed for the shape of the contact line of rolls and the patterns of distribution of contact stresses in roll machines, in the case when the deformations of the interacting bodies are given by models of the form $\sigma = E\varepsilon + \mu\dot{\varepsilon}$.
b) An expression was found to determine the ratio of the strain rates of the processed material and the roll coating.
c) Based on the analysis of the mathematical models obtained, it was revealed that one of the main factors determining the roll contact lines is the ratio of strain rates of the processed material and the roll coating.

d) An analysis of the patterns of distribution of contact stresses showed that the neutral point, that is, the point where the shear stresses change their signs, is located in the no-slip area.

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