Enhancing the work extraction of the entanglement engine via quantum coherence

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Abstract. Studying the relationship between quantum coherence and energy is an intriguing and vital subject in quantum thermodynamics. Quantum engine provides an excellent platform to emphasize it. By constructing a bipartite entanglement engine (BEE) model consisting of a single qubit as the target qubit and a two-qubit as the ancilla, we have investigated the influence of quantum coherence of ancilla on work extraction of BEE at length, and analytically calculated the extractable work of BEE for the ancilla with quantum coherence (QC). We found that the extractable work of BEE is not only dependent on coherence magnitude but also related to the coherence phase of ancilla. Moreover, under certain coherent parameters, the QC can boost the work extraction from BEE. The study reveals the role of QC acting as "fuel" of engine well which deepens the understanding of the energetic effect of QC.

1. Introduction

Quantum effects are unique phenomena in microscopic quantum systems. In recent years, exploring the connection between quantum effects and energy and understanding the physical mechanisms of quantum effects in quantum systems have received extensive attention. Researchers have extensively explored the thermodynamic characteristics of quantum effects based on different quantum system models, such as quantum battery [1-4], quantum engine [5-8], and quantum thermal manager [9-10]. However, we still lack a comprehensive and in-depth understanding of the relationship between quantum effects and energy, as well as the conversion mechanisms in quantum devices, which are worth further investigation.

Recently, Bresque et al. [11] constructed an entanglement engine model based on two detune qubits (a target qubit (TQ) and an auxiliary qubit). The auxiliary qubit is initially prepared in the excited state, while the TQ is in the ground state. They showed that entanglement can serve as the "fuel" for extracting work in the engine. Currently, a series of studies have shown that quantum effects in non-equilibrium environments have an impact on the thermodynamic properties of quantum systems [12-14]. Therefore, a natural question is what kind of impact quantum effects will have on the performance of the entanglement engine. For example, when are quantum effects involved [11]? What are the characteristics of the conversion between quantum effects and energy?

Based on the above questions, a bipartite entanglement engine (BEE) model consisting of an ancilla with a coherent two-qubit and a single TQ has been constructed. The characteristics of extracted work from BEE have been analyzed. It shows that the QC, under certain parameters, can enhance the work extraction of BEE, that is, the QC can serve as the "fuel" of BEE.

2. Model

Here, we model the BEE shown in Figure 1 which includes four steps: entanglement preparation, measurement, feedback control, and initial state resetting. The specifical implement in each step is given below.

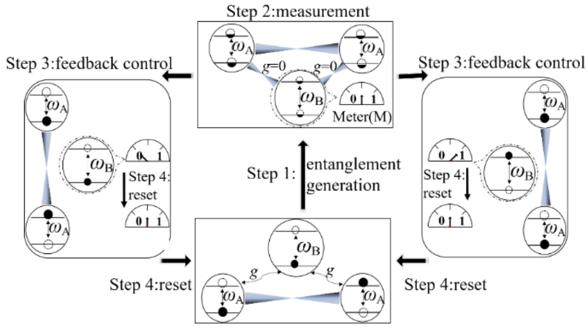

Figure 1. Diagram of bipartite entanglement engine (BEE) cycle.

Step 1: Entanglement preparation. We denote A as the ancilla composed of two identical qubits and B for the TQ, and ω_A and ω_B are the transition frequencies of A and B, respectively. We assume that the detune

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$\delta = \omega_{\mathrm{B}} - \omega_{\mathrm{A}} > 0$. The free Hamiltonian of the composite system formed by TQ and ancilla can be expressed as follows:

$$\boldsymbol{H}_0 = \hbar\omega_{\mathrm{B}}\boldsymbol{\sigma}_{\mathrm{B}}^{\dagger}\boldsymbol{\sigma}_{\mathrm{B}} + \hbar\omega_{\mathrm{A}}\sum_{j=1}^{2}\boldsymbol{\sigma}_j^{\dagger}\boldsymbol{\sigma}_j \tag{1}$$

where $\sigma_k^{\dagger} = |1_k\rangle\langle 0_k|$ ($\sigma_k = |0_k\rangle\langle 1_k|$), with $k = \{1, 2, \mathrm{B}\}$, represents the raising (lowering) operators of the qubit *k*, and $\hbar$ is the Planck constant. The initial state of the composite system is:

$$|\psi_0\rangle = |\psi_{\mathrm{A}}\rangle \otimes |0_{\mathrm{B}}\rangle \tag{2}$$

where $|\psi_{\mathrm{A}}\rangle = \exp(-i\varphi)\sin(\alpha)|10\rangle + \cos(\alpha)|01\rangle$ with α and φ being the probability amplitude parameter and coherent phase, respectively, and $\alpha \in [0, \pi/2]$ and $\varphi \in [0, 2\pi]$. $|\psi_{\mathrm{A}}\rangle$ and $|0_{\mathrm{B}}\rangle$ represents a general pure state of ancilla with coherence and the ground state of TQ. The density matrix of the initial state $|\psi_{\mathrm{A}}\rangle$ can be expressed as:

$$\boldsymbol{\rho}_{\mathrm{A}} = |\psi_{\mathrm{A}}\rangle\langle\psi_{\mathrm{A}}| = P_1^{\mathrm{dia}}|01\rangle\langle 01| + P_2^{\mathrm{dia}}|10\rangle\langle 10| + P_1^{\mathrm{nd}}|01\rangle\langle 10| + P_2^{\mathrm{nd}}|10\rangle\langle 01|, \tag{3}$$

where $P_1^{\mathrm{dia}} = \cos^2(\alpha)$, $P_2^{\mathrm{dia}} = \sin^2(\alpha)$, $P_1^{\mathrm{nd}} = \exp(i\varphi)\sin(2\alpha)/2$, and $P_2^{\mathrm{nd}} = \exp(-i\varphi)\sin(2\alpha)/2$. According to the l_1-norm measure of quantum coherence [15], the coherence of an arbitrary quantum state $\boldsymbol{\rho}$ is equal to the sum of absolute values of all off-diagonal elements in its density matrix, i.e., $C(\boldsymbol{\rho}) = \sum_{i \neq j}|P_{ij}|$, where P_{ij} represents the element of the density matrix $\boldsymbol{\rho}$. The coherence of ancilla with the state $\boldsymbol{\rho}_{\mathrm{A}}$ is:

$$C(\boldsymbol{\rho}_{\mathrm{A}}) = |P_1^{\mathrm{nd}}| + |P_2^{\mathrm{nd}}| = |\sin(2\alpha)|. \tag{4}$$

We denote $C_{\mathrm{A}} = C(\boldsymbol{\rho}_{\mathrm{A}})$. According to Equation (4), the coherence of $\boldsymbol{\rho}_{\mathrm{A}}$ satisfies $0 \le C_{\mathrm{A}} \le 1$, where $C_{\mathrm{A}} = 0$ and $C_{\mathrm{A}} = 1$ correspond to $\boldsymbol{\rho}_{\mathrm{A}}$ being an incoherent state and a maximal coherent state, respectively. Here, we consider the entanglement generated by the energy-conserving coupling between the ancilla and TQ. The interaction is given as:

$$\boldsymbol{V} = \hbar g\sum_{j=1}^{2}\left(\boldsymbol{\sigma}_j^{\dagger}\boldsymbol{\sigma}_{\mathrm{B}} + \boldsymbol{\sigma}_j\boldsymbol{\sigma}_{\mathrm{B}}^{\dagger}\right) \tag{5}$$

where g represents the coupling strength between the TQ and each qubit of ancilla. The composite system undergoes unitary evolution with the evolution operator:

$$\boldsymbol{U}(t) = \exp\left[-\mathrm{i}\left(\boldsymbol{H}_0 + \boldsymbol{V}\right)t\right] \tag{6}$$

After a time t , the composite system reaches the state:

$$|\psi_t\rangle = \boldsymbol{U}(t)|\psi_0\rangle = |\psi_t^{\mathrm{A}}\rangle|0_{\mathrm{B}}\rangle + \gamma|001_{\mathrm{B}}\rangle \tag{7}$$

where $|\psi_t^{\mathrm{A}}\rangle = (\xi|10\rangle + \kappa|01\rangle)/2$ and $\gamma = -g[\exp(it\Omega) - 1][\exp(i\varphi)\cos(\alpha) + \sin(\alpha)]/\Omega$, $\xi = \cos(\alpha)[X + Y + \exp(it\delta/2)] + \exp(-i\varphi)\sin(\alpha)[X - Y + \exp(it\delta/2)]$, $\kappa = \cos(\alpha)[X + Y + \exp(it\delta/2)] + \exp(-i\varphi)\sin(\alpha)[X + Y - \exp(it\delta/2)]$, $X = [4\exp(it\Omega/2)g^2]/(\Omega^2 - \delta\Omega)$, $Y = [4\exp(-it\Omega/2)g^2]/(\Omega^2 + \delta\Omega)$, and $\Omega = \sqrt{8g^2 + \delta^2}$ is the Rabi frequency. Equation (7) shows that if the coefficient γ of the state vector $|001\rangle$ in $|\psi_t\rangle$ is not equal to 0 or 1, then the entanglement between TQ and ancilla occurs.

Step 2: Measurement. When the evolution time reaches $t_0 = \pi/\Omega$, the entanglement of the composite system becomes the maximum, and the corresponding state is:

$$|\psi_{t_0}\rangle = |\psi_{t_0}^{\mathrm{A}}\rangle|0_{\mathrm{B}}\rangle + \gamma(t_0)|001_{\mathrm{B}}\rangle \tag{8}$$

with $\gamma(t_0) = 2g[\cos(\alpha) + \exp(-i\varphi)\sin(\alpha)]/\Omega$ and $|\psi_{t_0}^{\mathrm{A}}\rangle = [\xi(t_0)|10\rangle + \kappa(t_0)|01\rangle]/2$, $\xi(t_0) = \exp[i\pi\delta/(2\Omega)][-\exp(i\varphi)\cos(\alpha) + \sin(\alpha)] + i[\exp(i\varphi)\delta\cos(\alpha)/\Omega + \sin(\alpha)]$, $\kappa(t_0) = \exp[i\pi\delta/(2\Omega)][\exp(i\varphi)\cos(\alpha) - \sin(\alpha)] + i\delta[\exp(i\varphi)\cos(\alpha) + \sin(\alpha)]/\Omega$. The corresponding density matrix becomes:

$$\boldsymbol{\rho}_{\mathrm{AB}}(t_0) = |\gamma(t_0)|^2|001_{\mathrm{B}}\rangle\langle 001_{\mathrm{B}}| + |\psi_{t_0}^{\mathrm{A}}0_{\mathrm{B}}\rangle\langle\psi_{t_0}^{\mathrm{A}}0_{\mathrm{B}}| + \gamma^*(t_0)|\psi_{t_0}^{\mathrm{A}}0_{\mathrm{B}}\rangle\langle 001_{\mathrm{B}}| + \gamma(t_0)|001_{\mathrm{B}}\rangle\langle\psi_{t_0}^{\mathrm{A}}0_{\mathrm{B}}| \tag{9}$$

At the moment, we first turn off the coupling channel ($g = 0$), and then implement a positive operator valued measure (POVM) on TQ via a measurement device M. After the measurement, the coherence in TQ, i.e., the supposition between the basis $|0_{\mathrm{B}}\rangle$ and $|1_{\mathrm{B}}\rangle$, can be destroyed or disappears. And, the state of the entire system (ABM) can be expressed by:

$$\boldsymbol{\rho}_{\mathrm{ABM}} = P_{\mathrm{B}}^{1}|00\rangle\langle 00| \otimes |1_{\mathrm{B}}1_{\mathrm{m}}\rangle\langle 1_{\mathrm{B}}1_{\mathrm{m}}| + |\psi_{t_0}^{\mathrm{A}}\rangle\langle\psi_{t_0}^{\mathrm{A}}| \otimes |0_{\mathrm{B}}0_{\mathrm{m}}\rangle\langle 0_{\mathrm{B}}0_{\mathrm{m}}| \tag{10}$$

where we have assumed that when the TQ collapses to the excited (ground) state $|1\rangle$ ($|0\rangle$), the pointer of the device M points to 1 (0) corresponding to $|1_M\rangle$ ($|0_M\rangle$), and:

$$P_B^1 = \frac{4g^2}{\Omega^2}\left[1+\cos(\varphi)\sin(2\alpha)\right] \quad (11)$$

Step 3: Feedback Control. Based on the information obtained from measurement device M, different control protocols are implemented on the system. When the device M remains in $|1_M\rangle$, implying that the TQ is in excited state $|1_B\rangle$, one can perform a π pulse on TQ to flap TQ's state to the ground state $|0_B\rangle$, and the extracted work is:

$$\Delta E = \hbar(\omega_B - \omega_A) = \hbar\delta . \quad (12)$$

When the device M remains in $|0_M\rangle$, no operation is carried out. And, the working medium returns to Step 1 to implement the entanglement evolution. Therefore, the average work in a single cycle can be given as:

$$W = P_B^1 \Delta E = \frac{4g^2}{\Omega^2}\left[1+\cos(\varphi)\sin(2\alpha)\right]\hbar\delta . \quad (13)$$

Step 4: Initial state resetting. After the feedback control operations, TQ has returned to its initial state. One needs to reset the ancilla and the device to their initial states where the cost of resetting operation is not the interest of this paper and cannot be considered further. After the resetting process, the BEE repeats the above four cyclic steps to enter the next cycle.

3. The impact of quantum coherence on work extraction.

Here, we mainly focus on the influence of QC on the output work of BEE. Combing with Equations (4) and (13), the average output work of BEE can be further expressed as:

$$W = \frac{4g^2}{\Omega^2}\left[1 \pm \cos(\varphi) C_A\right]\hbar\delta = \frac{1}{2+\Delta^2/4}\left[1 \pm \cos(\varphi) C_A\right]\hbar\delta \quad (14)$$

where $\Delta = \delta/g$. The signs + and − in Equation (14) correspond to $\sin(2\alpha) > 0$ and $\sin(2\alpha) < 0$, respectively. Equation (14) is the main result in this paper. When the coupling strength g and the detune δ are fixed, work extracted from BEE are determined by the coherence C_A and the coherent phase φ . Meanwhile, the extractable work of BEE is bounded by:

$$\frac{1}{2+\Delta^2/4}\left[1-|\cos(\varphi)| C_A\right]\hbar\delta \le W \le \frac{1}{2+\Delta^2/4}\left[1+|\cos(\varphi)| C_A\right]\hbar\delta . \quad (15)$$

Equation (15) shows that QC can not only boost the extractable work but also suppress the work extraction. Since $0 \le |\cos(\varphi)| C_A \le 1$ the maximum and minimum values of the extractable work W are given as:

$$W_{max} = 2W^{dia} = \frac{\hbar\delta}{1+\Delta^2/8} \quad \text{and} \quad W_{min} = 0 \quad (16)$$

where the W_{max} and W_{min} correspond to $\cos(\varphi)\sin(2\alpha) = 1$ and $\cos(\varphi)\sin(2\alpha) = -1$. The ancilla's state for W_{max} can be obtained easily, such as $|\psi_A\rangle = (|10\rangle + |01\rangle)/\sqrt{2}$, which can be regarded as the optimal QC state.

In the similar entanglement engine model in [11], where the ancilla with a single excitation and no coherence is considered, the extractable work is obtained as $W^{single} = \hbar\delta/(1+\Delta^2)$. Therefore, comparing both models of ours with coherence and the one without coherence in [11], the ratio of the maximum extractable work can be expressed as:

$$R = \frac{W_{max}}{W^{single}} = \frac{1+\Delta^2}{2+\Delta^2/4}\left[1+|\cos(\varphi)| C_A\right] \quad (17)$$

which satisfies:

$$\frac{1+\Delta^2}{2+\Delta^2/4} \le R \le \frac{1+\Delta^2}{1+\Delta^2/8} . \quad (18)$$

From Equation (18), we note that the upper bound of ratio R (denoted as R_{up}) is always twice its lower bound (denoted as R_{low}), i.e., $R_{up} = 2R_{low} = (1+\Delta^2)/(8+\Delta^2)$. Moreover, R_{up} is a monotonically increasing function of the parameter Δ, as shown in Figure 2. It also means that the more obvious the advantage of the coherent BEE is the weaker the coupling strength ($g \propto 1/\Delta$) of TQ-ancilla is. Meanwhile, $R_{up} > 1$ always holds for $\Delta > 0$, which suggests that the maximum extractable work W_{max} , under the QC ancilla with optimal coherence parameters ($C_A = 1$ and $\varphi = 0$) is always greater than that under the incoherent single-qubit ancilla with equal energy. In the limit $\Delta \to \infty$ (or weak-coupling limit $g \ll \delta$), R_{up} has the maximum value:

$$R_{up}^{max} = \lim_{\Delta\to\infty} \frac{1+\Delta^2}{1+\Delta^2/8} = 8 \quad (19)$$

which demonstrates that the maximal extractable work of coherent BEE can reach eight times that of incoherent engine in [11].

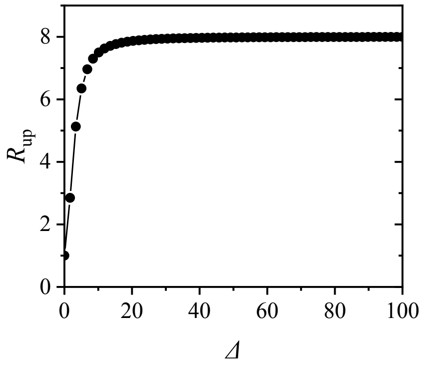

Figure 2. Curve of R_{up} varying with parameter Δ ($\Delta = \delta/g$).

4. Conclusion

In this paper, we have constructed a BEE model with a two-qubit ancilla and investigated the influence of QC of the ancilla on extractable work from the engine. We have analytically derived the extractable work of the BEE with the single-excitation general QC state of ancilla, analyzed the range of the extractable work, and obtained the upper and lower bounds of extractable work. Compared with the incoherent entanglement engine, QC can act as the work resource of the BEE to effectively increase the extractable work of the engine. This study enriches our understanding of the relationship between quantum coherence and energy, providing useful insights for improving the performance of related quantum devices.

Acknowledgments

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