# Modeling the contact interaction of leather with squeezing rollers

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**Abstract.** Mathematical models of the shape of roll contact curves and patterns of distribution of contact stresses in the roll module of a leather squeezing machine were developed. It was revealed that the shapes of the roll contact curves depend on the deformation and geometric parameters of leather and cloth, as well as the coefficient of friction between leather and rolls. It was established that normal stresses change from zero at the beginning and end of the roll contact zone to a maximum at the point of maximum deformation of leather located in the compression zone; shear stresses change their signs at the neutral point, which in the drive roll is located in the compression zone.

## 1 Introduction

The technology of processing various materials using roller machines is widely used in many industries. For example, the fleet of equipment for mechanical processing of semi-finished leather products consists mainly of roller machines. Among them are roller machines for squeezing out fluid from semi-finished leather products after tanning or dyeing [1]. These machines have an impact on the environmental safety of enterprises since the extraction process is directly related to the problem of wastewater disposal.

The fluid squeezing from the leather is conducted as a result of the interaction of the leather with the squeezing rollers. The possibility of the intensive impact of rollers on leather in the contact zone helps to increase the efficiency of the pressing process and the quality of the finished product. Therefore, the problems of improving existing and developing new roller squeezing machines are closely related to solving the problem of contact interaction between leather layers and squeezing rollers.

During the squeezing process, the phenomenon of leather contact interaction occurs simultaneously with the phenomenon of fluid filtration. Consequently, the problem of contact interaction of leather with squeezing rollers must be solved taking into account the phenomenon of water filtration in leather and in the coating of rollers (cloth).

The ability to simulate the contact interaction of leather with squeezing rollers is largely prepared by previously performed theoretical and experimental studies in various industries

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[2-18]. References [19, 20] are devoted to modeling the contact interaction of a semi-finished leather product after chrome tanning. These solutions are based on analytical dependencies that describe the deformation and filtration properties of the semi-finished leather product at this stage [21, 22].

This article is devoted to modeling the contact interaction of a semi-finished leather product after dyeing, taking into account the deformation properties of the semi-finished leather product at this stage and the phenomenon of fluid filtration in the roll contact zone.

## 2 Materials and methods

According to [1], roll modules of leather squeezing machines generally have a symmetrical appearance.

We consider a symmetrical roll module, in which a layer of semi-finished leather after dyeing with a thickness of $\delta_1$ interacts with drive rollers, having radius $R$ and the elastic coating made of technical cloth with a thickness of $H$. Figure 1 shows the upper part of the roll module relative to the line of symmetry.

The contact interaction of the processed material with a roll having an elastic coating can be considered by analogy with the rolling of an elastic wheel on deformable soil [23].

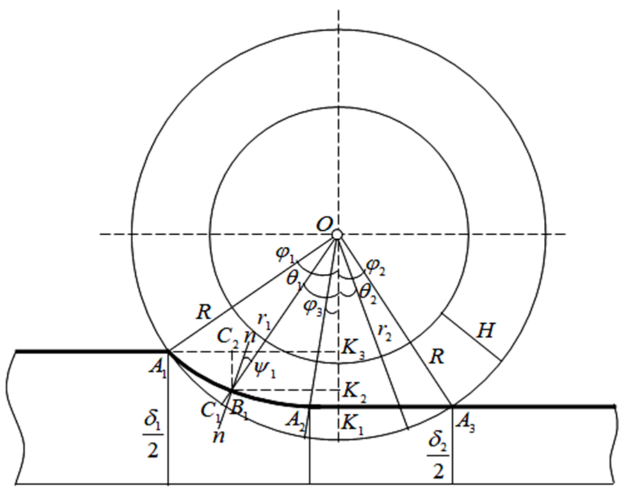

**Fig.1.** Scheme of contact interaction in the roll module of leather squeezing.

The semi-finished leather product after dyeing has a uniform and thin thickness. Therefore, it can be assumed that the deformation in the contact zone will not be recovered.

Based on this, as in the rolling of an elastic wheel on deformable soil, we believe that the roll contact curve consists of two sections - curvilinear and rectilinear. In the curvilinear section, the leather layer and cloth are compressed, therefore, it is in the front part of the contact zone.

During the squeezing process, due to the action of reactive forces, the point of maximum deformation of the leather will be shifted from the line of centers towards the entrance of leather into the contact zone, and this displacement increases with increasing roller velocity and decreasing roller radius [19]. Therefore, the rectilinear section is located in the middle and rear parts of the contact zone.

According to Figure 1

$$-\varphi_1 \le \theta_1 \le -\varphi_3, \quad -\varphi_3 \le \theta_2 \le \varphi_2,$$

where $\varphi_1, \varphi_2 -$ are the contact angles, $\varphi_3 -$ is the angle separating sections 1 and 2.

## 3 Results

The main tasks of the contact interaction of leather with the squeezing rollers are mathematical modeling of the shape of the roll contact curves and mathematical modeling of the distribution of normal and shear stresses along the roll contact curves.

Solutions to two problems of contact interaction between leather and squeezing rollers, in polar coordinates with the pole at the center of the roller, can be expressed using three equations:

$$r_i = r_i(\theta_i), \quad n_i = n_i(\theta_i), \quad t_i = t_i(\theta_i), \quad i = 1,2,$$

where $i -$ is the index indicating the number of the section; $r_i$, $\theta_i -$ are the radius vector and polar angle; $n_i, t_i -$ are the normal and shear stresses at the point of the $i-$th section.

Equations $r_i = r_i(\theta_i)$ that determine the shape of the roll contact curves primarily depend on the deformation of leather. The shape of the roll contact curve also depends on the deformation of the coating cloth. Accordingly, the solution to the first problem is determined by the deformation properties of leather and cloth.

According to [24, 25], leather and cloth are formalized as a continuous media with the properties of elasticity, viscosity, and plasticity and are described by Kelvin–Voigt rheological models:

$$\sigma_1^* = E_1^* \varepsilon_1^* + \mu_1^* \frac{d\varepsilon_1^*}{dt}, \qquad \sigma_1 = E_1 \varepsilon_{1.} + \mu_1 \frac{d\varepsilon_1}{dt}, \tag{1}$$

where $\sigma_1^*, \varepsilon_1^*, E_1^*, \mu_1^* -$ are the stresses, strains, moduli of elasticity and viscosity of leather under compression, $\sigma_1, \varepsilon_1, E_1, \mu_1 -$are the stresses, strains, moduli of elasticity and viscosity of the cloth under compression.

In the theory of wheel rolling, the analytical determination of the contact line is related to the ratio of the deformation rates of the contacting bodies. In most studies, this ratio is considered constant [26, 27].

Considering the ratio of the rates of relative deformations of cloth and leather under compression to be constant (as in the theory of a wheel), for section 1 of the roll contact zone, we have:

$$\frac{\varepsilon_1}{\varepsilon_1^*} = \gamma_1 . \tag{2}$$

From Figure 1, it follows that

$$\varepsilon_1 = \frac{R - r_1,}{H} \qquad \varepsilon_1^* = \frac{2\left(r_1 - R\dfrac{\cos\varphi_1}{\cos\theta_1}\right)}{\delta_1}. \tag{3}$$

Then from equality (2) taking into account expression (3), we have

$$R - r_1 = \frac{2H\gamma_1}{\delta_1}\left(r_1 - R\frac{\cos\varphi_1}{\cos\theta_1}\right). \tag{4}$$

Having solved equalities (3) with respect to $r_1$, we obtain the equation of the roll contact curve in section 1:

$$r_1 = \frac{R\delta_1}{\delta_1 + 2H\gamma_1}\left(1 + \frac{2H\gamma_1}{\delta_1}\frac{\cos\varphi_1}{\cos\theta_1}\right), \quad -\varphi_1 \le \theta_1 \le -\varphi_3. \tag{5}$$

According to Figure 1, for section 2, we get $r_2 = \frac{OK_1}{\cos\theta_2}$ or

$$r_2 = R\frac{\cos\varphi_2}{\cos\theta_2}, \quad -\varphi_3 \le \theta_2 \le \varphi_2. \tag{6}$$

The resulting equations (4) and (5) can be transformed as:

$$r_1 = \frac{R\delta_1}{\delta_1 + 2H\gamma_1}\left(1 + \frac{2H\gamma_1}{\delta_1}\frac{\cos(-\varphi_1+\varphi_3)}{\cos(\theta_1+\varphi_3)}\right), \quad -\varphi_1+\varphi_3 \le \theta_1+\varphi_3 \le 0, \tag{7}$$

$$r_2 = R\frac{\cos(\varphi_2+\varphi_3)}{\cos(\theta_2+\varphi_3)}, \quad 0 \le \theta_2+\varphi_3 \le \varphi_2+\varphi_3. \tag{8}$$

It is known [28], that

$$\gamma_1 = \frac{E_1\,\Delta l_1 + \mu_1\,\frac{dl_1}{dt}}{E_1^*\Delta_1 + \mu_1^*\,\frac{dl_{11}}{dt}},$$

where $\Delta l_1$ and $\frac{dl_1}{dt}$ are determined by equality $\Delta l_1 = R\cdot\left(1 - \frac{\cos\varphi_1}{\cos\theta_1}\right)$.

To calculate $\gamma_1$, we simplify expressions $\Delta l_1$ and $\frac{dl_1}{dt}$ by replacing the average value:

$$(\Delta l_1)_{cp} = \frac{1}{\varphi_1-\varphi_3}\int_{-\varphi_1+\varphi_3}^{0} R\cdot\left(1 - \frac{\cos(-\varphi_1+\varphi_3)}{\cos(\theta_1+\varphi_3)}\right)d(\theta_1+\varphi_3),$$

$$\left(\frac{dl_1}{dt}\right)_{cp} = \frac{R\omega}{\varphi_1-\varphi_3}\cdot\int_{-\varphi_1+\varphi_3}^{0}\left(-\frac{\cos(-\varphi_1+\varphi_3)\sin(\theta_1+\varphi_3)}{\cos^2(\theta_1+\varphi_3)}\right)d(\theta_1+\varphi_3)$$

or after integration

$$(\Delta l_1)_{cp} = R\cdot\left(1 - \frac{\cos(\varphi_1-\varphi_3)}{2(\varphi_1-\varphi_3)}\ln\left|\frac{1+\sin(\varphi_1-\varphi_3)}{1-\sin(\varphi_1-\varphi_3)}\right|\right).,$$

$$\left(\frac{dl_1}{dt}\right)_{cp} = \frac{R\omega}{\varphi_1-\varphi_3}(1-\cos(\varphi_1-\varphi_3))\cdot$$

Expanding the logarithmic function in a series, we have

$$(\Delta l_1)_{cp} = \frac{R(\varphi_1-\varphi_3)^2}{2}, \quad \left(\frac{dl_1}{dt}\right)_{cp} = \frac{R\omega(\varphi_1-\varphi_3)}{2}$$

Thus, the ratio of the deformation rates of the lower roll coating and the layer of material in the compression zone is determined by the following formula:

$$\gamma_1 = \frac{E_1\,(\varphi_1-\varphi_3) + \mu_1\omega}{E_1^*(\varphi_1-\varphi_3) + \mu_1\omega}, \tag{9}$$

where $\omega -$ is the angular velocity of the roll.

For $\theta_1+\varphi_3=\theta_2+\varphi_3=0$, the following equality holds (Figure 1)

$$r_1(0)=\frac{R\delta_1}{\delta_1+2H\gamma_1}(1+\frac{2H\gamma_1}{\delta_1}\cos(-\varphi_1+\varphi_3))=R\cos(\varphi_2+\varphi_3)=r_2(0),$$

Transforming these equalities, we find the expression that allows us to determine the value of $\varphi_3$:

$$\varphi_3=\frac{2H\gamma_1(\varphi_1^2-\varphi_2^2)-\varphi_2^2}{2(\delta_1\varphi_2+2H\gamma_1(\varphi_1+\varphi_2))}. \tag{10}$$

From here, taking into account condition $\varphi_3\geq 0$, we can find the condition for the angle $\varphi_2$:

$$\varphi_2\leq\sqrt{\frac{2H\gamma_1}{\delta_1+2H\gamma_1}}\varphi_1, \tag{11}$$

where the nip angle $\varphi_1$ is determined by formula $\varphi_1=arctgf$ [3], where $f$ – is the coefficient of friction of leather against the surface of the cloth.

Equations (6) and (7) describe the shape of roll contact curves in the roll module of the leather squeezing machine. It follows that the main indices of mathematical models of the shape of roll contact curves are $\varphi_1$, $\varphi_2$, $\varphi_3$ and $\gamma_1$. From expressions (8)-(10), it follows that these values, in turn, depend on the deformation and geometric parameters of leather, cloth, and roll, as well as coefficients of friction between leather and cloth.

We proceed to solve the second problem of contact interaction of leather with the squeezing rollers, that is, to determine the patterns of distribution of contact stresses along the roll contact curves $n_i=n_i(\theta_i)$ and $t_i=t_i(\theta_i)$.

According to previously performed studies, we assume that in section 1 the normal stress increases according to a parabolic law from 0 to $n_{\max}$ [28, 29]; in section 2, it decreases according to an elliptical law from $n_{\max}$ to 0 [27, 29].

The position of the application point $n_{\max}$ corresponds to the point of maximum deformation of leather, which is determined by the angle $(-\varphi_3)$ (or for $\theta_1=\theta_2=0$).

Then the patterns of distribution of normal stresses along the roll contact curves in the roll module of the machine for leather squeezing are described by the following equations:

$$n_1=\frac{n_{\max}}{(\varphi_1-\varphi_3)^2}((\varphi_1-\varphi_3)^2-(\theta_1+\varphi_3)^2),\ \ -\varphi_1+\varphi_3\leq\theta_1+\varphi_3\leq 0, \tag{12}$$

$$n_2=\frac{n_{\max}}{\varphi_2+\varphi_3}\sqrt{(\varphi_2+\varphi_3)^2-(\theta_2+\varphi_3)^2},\quad 0\leq\theta_2+\varphi_3\leq\varphi_2+\varphi_3. \tag{13}$$

In modeling the patterns of distribution of shear stresses, the main factor is a mathematical model of friction stress, which determines the relationships between normal and shear stresses.

For mathematical models of friction stresses, we use the models developed in [3].

For the drive roll, they have the following form:

$$\begin{cases} t_1=tg(\theta_1+\varphi_3-\psi_1+\xi)n_1, & -\varphi_1+\varphi_3\leq\theta_1+\varphi_3\leq 0,\\ t_2=tg(\theta_2+\varphi_3-\psi_2+\xi)n_2, & 0\leq\theta_2+\varphi_3\leq\varphi_2+\varphi_3, \end{cases} \tag{14}$$

where $\psi_1=arctg\frac{r_1'}{r_1}$, $\xi=arctg\frac{F}{Q}$; $Q, F$ – are the pressure force of the clamping devices and the horizontal reaction of the roll supports.

Then, from the system of equations (13) taking into account expressions (11) and (12), we find the patterns of distribution of shear stresses along the roll contact curves in the roll module of the leather squeezing machine.

They have the following form:

$$t_1 = \frac{n_{\max}}{(\varphi_1 - \varphi_3)^2}\left((\varphi_1 - \varphi_3)^2 - (\theta_1 + \varphi_3)^2\right) tg(\theta_1 + \varphi_3 - \psi_1 + \xi),\ -\varphi_1 + \varphi_3 \le \theta_1 + \varphi_3 \le 0, \quad (15)$$

$$t_2 = \frac{n_{\max}}{\varphi_2 + \varphi_3}\sqrt{(\varphi_2 + \varphi_3)^2 - (\theta_2 + \varphi_3)^2}\ tg(\theta_1 + \varphi_3 - \psi_1 + \xi),\ 0 \le \theta_2 + \varphi_3 \le \varphi_2 + \varphi_3. \quad (16)$$

From equalities (6) and (7), we obtain

$$r_1' = \frac{2RH\gamma_1 \cos(-\varphi_1 + \varphi_3)}{\delta_1 + 2H\gamma_1}\frac{\sin(\theta_1 + \varphi_3)}{\cos^2(\theta_1 + \varphi_3)}, \qquad r_2' = R\frac{\sin(\theta_2 + \varphi_3)}{\cos^2(\theta_2 + \varphi_3)}.$$

Considering these expressions, we have

$$\psi_1 = arctg\left(\frac{2H\gamma_1 \cos(-\varphi_1 + \varphi_3)}{\delta_1 \cos(\theta_1 + \varphi_3) + 2H\gamma_1} tg(\theta_1 + \varphi_3)\right), \quad \psi_2 = \theta_2 + \varphi_3. \quad (17)$$

To analyze the pattern of shear stress distribution, the neutral point at which the shear stress is zero is of particular importance. The neutral point in the drive roll is located in the compression zone [3].

The angle that determines the position of the neutral point is called the neutral angle. Let the neutral point be determined by angle $(-\varphi_4)$.

Then from the first equation of system (14), we have

$$-\varphi_4 - \psi_1(-\varphi_4) + \xi = 0$$

or considering expression (16) and assuming that $\cos\varphi_4 \approx 1$, $\sin\varphi_4 \approx \varphi_4$, we obtain

$$\varphi_4 = \frac{\delta_1 + 2H\cos(-\varphi_1 + \varphi_3)}{\delta_1} arctg\frac{F}{Q}. \quad (18)$$

Under static interaction of leather with the roller, the neutral point is located on the line of centers since, in this case, $F_i = 0$, therefore $\varphi_4 = 0$.

## 4 Conclusions

Mathematical models of the roll contact curves and the patterns of distribution of contact stresses in the roll module of a leather squeezing machine were developed. The developed mathematical models make it possible to calculate, in a manner acceptable for practical purposes, the shapes of the roll contact curves and diagrams of the distribution of normal and shear stresses along the roll contact curves.

Analysis of the developed mathematical models showed that normal and shear contact stresses were non-uniformly distributed along the roll contact lines:

- normal stresses changed from zero at the beginning and end of the roll contact zone to a maximum at the point of maximum deformation of leather, the position of which was determined by formula (9);
- shear stresses changed their signs at the neutral point, located in the compression area and the neutral angle was expressed by formula (17);
- under static interaction of the semi-finished leather product with the rollers, the neutral point was located on the line of centers.

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