Application of Variational Methods for the Development of Optimal Multiparametric Models of Durability of Composite Materials and Structures under the Influence of Extreme Environmental Factors

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Abstract. The increasing use of composites in such modern fields as aviation and space technology, shipbuilding, oil and gas industry, etc., determines the importance and relevance of developing effective high-precision methods for long-term forecasting of the defining characteristics of composite materials and structures made of them (residual life, strength, reliability, durability). A study of the promising possibilities of applying modern provisions of the kinetic theory of strength to develop optimal models of durability of composite materials and structures made of them under the influence of extreme environmental factors has been carried out. Based on the application and development of mathematical and computer modelling methods, effective methods for predicting the defining characteristics (residual life, strength) have been developed.

1 Introduction

Polymer composite materials (PCM) and structures made of them in recent decades have been an integral part of modern technology in many areas that determine scientific and technological progress [1 – 5]. In recent decades, when developing samples of modern technology, including high-speed transport, considerable attention has been paid to the problem of creating reliable methods for quantifying the operability of structures made of polymer and composite materials (CM). The importance of developing effective high-precision methods for assessing the durability of CM under the influence of extreme

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environmental factors is due to the fact that CM and structures made of them are beginning to be widely used in critical products of modern aviation and space technology, shipbuilding, oil and gas industry, etc. [6 – 9]. The preservation of the working capacity of the material is determined by the long-term ability to resist aging, which is associated with a combination of reversible and irreversible physical and chemical transformations under the influence of extreme environmental factors [10 – 15] . The impact of extreme environmental factors can be associated with temperature factors, humidity, exposure to solar and penetrating radiation, mechanical stresses and other factors, both individually and in various combinations [1-3, 8, 10].

The widespread use of composite materials in modern technology determines the importance and relevance of developing effective high-precision methods for long-term forecasting of the defining characteristics of composite materials and structures made of them (residual life, strength, reliability, durability) [14-16] . When deciding on the use of the material in modern technology, from a large number of possible sample options, it is necessary to choose a sample with such a composition that can provide the optimal combination of the necessary indicators. Under the influence of extreme environmental factors in various combinations, the properties of composites change over time, which, as a rule, affects their performance. At the same time, the speed and nature of changes occurring in composites at both the micro and macro levels will also depend on the number of factors affecting the material and the intensity of these influences.

The basis for making a decision on the use of polymer composite materials in modern technology is the totality of its required indicators at an acceptable cost and the ability to resist aging . In modern technology, the dominant position is occupied by materials that can function without replacement for 20-30, and in necessary cases for more than 50 years. When using materials in aviation and space technology, penetrating radiation is an effective activator of aging, which, unlike light, is able to initiate transformations in the entire volume of the material. Therefore, the use of polymer composite materials in aviation and space technology makes it even more relevant to study the resistance of polymers to radiation [3,14-18].

The mechanism of polymer aging has not been sufficiently studied yet. There is no established quantitative relationship between the influence of various types of external influences on the rate of the prevailing aging process. For a satisfactory prediction, it is necessary to have simple and reliable relationships between the kinetic parameters of physical-chemical processes and the macro properties of materials that determine their operational suitability. These, as well as other issues, constitute an important scientific problem that stands in the way of creating scientifically based approaches to predicting changes in the properties of PCM during their storage and operation. The importance and relevance of developing effective methods for predicting changes in the properties of PCM is explained by the fact that without data on the nature and speed of changes in the mechanical, electrical, thermophysical properties of polymers in the operating conditions of products, it is impossible to ensure a rational choice of materials and thereby increase the reliability of structures made of polymer and composite materials.

Based on the current provisions of the kinetic theory of strength, the possibility of constructing optimal models of durability of composites of optimal structure and complexity under the influence of extreme environmental factors has been investigated.

2 Construction of generalized multiparametric models of durability based on modern provisions of the kinetic theory of strength

In general, polymer, composite materials, composite structures can be affected in various combinations simultaneously by several different factors associated with hardening processes, exposure to solar radiation, moisture saturation, exposure to ultraviolet radiation, exposure to extreme climatic factors, exposure to operational loads of both cyclic and non-cyclic nature, etc. Each of these factors has an independent effect the degree of damage to the polymer, composite material, composite structure.

We denote the defining property of the material by R (residual life, durability, reliability, etc.), and its initial value by R_0. Denote the total number of factors affecting the material, and leading to its damage and premature aging through p. We denote

$F_j(u_{j,1},u_{j,2},\ldots,u_{j,l_j};t),(j=1,2,...,p)$ by the model describing the effect of the j-th factor on the defining property of the material, $u_{j,1},u_{j,2},\ldots,u_{j,l_j}$ - the system of undefined parameters of the model, l_j – the number of undefined parameters of the model. We will assume that various physical factors have an influence on the composite, independent of the influence of other factors. And we will also assume that the changes caused by the influencing factors are summed up. Then, when introducing these assumptions, it can be assumed that a generalized durability model describing the simultaneous impact of several factors on the material can be represented as:

$$R = R_0 + \sum_{j=1}^{p} F_j\left(u_{j,1},u_{j,2},\ldots,u_{j,l_j};t\right) \tag{1}$$

In these notations; $u_{j,1},u_{j,2},\ldots,u_{j,l_j}$ - parameters describing the nature of the impact of the j-th factor on the composite, t- time. In accordance with the modern provisions of the kinetic theory of strength, it was assumed that the effect of each of the factors Fj on the material will activate at the micro level a set of destructive elementary processes, the chemical reactions occurring in which can be described in the form of a superposition of Arrhenius equations.

Each of the functions Fj, describing the effect of the j-th factor on a composite composite, can be represented as a decomposition in a series according to some system of basic functions $\psi_{kj}\left(\beta_{kj};t\right),(k=1,2,3,...)$, that most fully characterize the features of the process of increasing the damage of the material under the influence of extreme environmental factors. Choosing as a system of reference functions when presenting descriptions of factors affecting PM, CM Fj , in the form of series, a system of exponential functions describing the nature of processes occurring at the molecular level, in accordance with the Arrhenius equation, it is possible to construct the following universal forecasting model describing at the physical level the processes occurring in composite materials, structures under the simultaneous influence of several destabilizing physical factors:

$$R(;t) = R_0 + \sum_{j=1}^{p}\sum_{k=1}^{N_j} \alpha_{kj}\left(u_{j,1},\ldots,u_{j,l_j}\right)\psi_{kj}\left[\beta_{kj}\left(u_{j,1},\ldots,u_{j,l_j}\right);t\right] \tag{2}$$

In these notations: $\alpha_{kj}\left(u_{j,1},\ldots,u_{j,l_j}\right),\beta_{kj}\left(u_{j,1},\ldots,u_{j,l_j}\right),(j-1,2...,p;k=0,1,2,...)$ undefined parameters of the model describing the impact of the j-th factor. The value of Nj corresponds to the number of terms when decomposing the j-th factor into a series according to the system of basic functions, in particular according to the system of exponential functions, if the corresponding physical factor Fj describes the aging processes.

Within the framework of the developed models, it is possible to solve the problems of predicting the residual resource both under the influence of an indefinite number of extreme factors, and the problems of predicting the residual resource under the influence of strictly defined pre-known factors, for example, under the influence of a factor associated with climatic effects and a factor associated with hardening processes.

The principle of multiplicity of forecasting models is formulated, which was the basis for the development of optimal forecasting models of optimal structure and complexity. In accordance with the principle of multiplicity of forecasting models, the model most adequate to the real predicted time dependence of the defining property of a composite composite R* (t) is a model of optimal structure and complexity. A model of optimal structure and complexity is understood to be a model containing the optimal number of terms that allows solving the forecasting problem with the required accuracy.

An extreme problem, from the solution of which a model of optimal structure and complexity can be determined, can be formulated as follows.

$$R^*_{n^*}\left(\left(u^{n^*}\right)^*;t\right)=\arg\min_{\substack{\{R_n(u^n;t)\}\\ 1\le n<\infty}} J\left(R_n\left(u^n;t\right)\right) \quad (3)$$

In these notations: u^n - vector of parameters of the prediction model of the parametric family corresponding to the value of parameter n: $u^n=(u_1^n,u_2^n,...,u_n^n);J(R_n(u^n;t))$; - estimation of the degree of deviation of this model $R_n(u^n;t)$, the multiparametric family corresponding to the value of parameter n ($1\le n<\infty$), from the real time dependence of the defining property

$R^*;R^*_{n^*}\left(\left(u^{n^*}\right)^*;t\right)$- prediction model of the multiparametric family of optimal structure and complexity; $\left(u^{n^*}\right)^*$ - vector parameters of the prediction model of the optimal structure and complexity corresponding to the optimal number of parameters equal to n*.

3 Application of optimal generalized durability models for the study of promising opportunities to increase the accuracy of forecasting the defining characteristics of composites

A study of the possibility of using the methodology being developed to study the promising possibilities of increasing the accuracy of predicting the determining characteristics of polymer composite materials (PCM) under extreme conditions has been conducted [13-16] . The application of optimal generalized models of durability is investigated to identify the determining factors that have a significant impact on the stability of the estimation of the accuracy of the forecast in the predicted time interval $[t_{\min},t_{\max}]$. On the basis of the developed approach based on the principle of multiplicity of forecasting models and the concept of models of optimal structure and complexity, qualitative patterns of dependence of optimal models of durability of optimal structure and complexity are investigated $R_\tau^*(t)=R\left(u_\tau^*\,;\,t\right)$ $\left(0\le\tau\le t_{\max},0\le t\le t_{\max}\right)$. A parametric family of forecasting problems was considered, in which the parameter is the current size of the retrospection interval $\tau\left(0\le\tau\le t_{\max}\right)$. In these notations, u_τ^* - the optimal vector of indeterminate parameters is defined, which determines the optimal model of the durability of the optimal structure and complexity $R^*_\tau(t)$, which corresponds to the value of the parameter τ. Qualitative analysis allowed us to establish important patterns of the structure of optimal models of the durability of the optimal structure and complexity when changing the size of the retrospection interval τ. The analysis made it possible to identify areas of stability and instability in the retrospection interval. A qualitative comparative characteristic of these areas was carried out, a qualitative analysis of the structural features of the areas of stability and instability was carried out. A study of the behavior of optimal durability models in these areas has been conducted, a comparative assessment of the boundaries of these areas, as well as their mutual location, has been carried out. Fig. 1 shows the results of computational experiments on predicting the residual life of polymer fiber composites (PFC) based on the developed methodology. The combined effect of the hardening factor and the aging factor on the composite was considered.

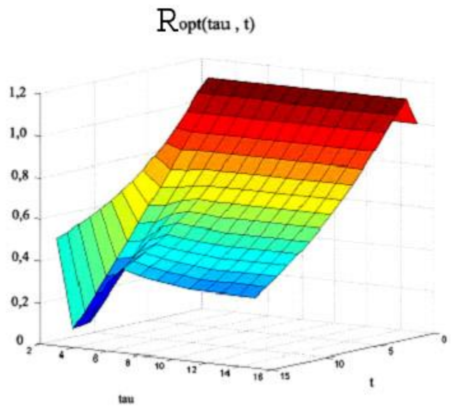

Fig. 1. Three-dimensional graph of the dependence of the parametric family of optimal models on the durability of the optimal structure and complexity $R^*\left(\tau_r ; t\right)$ depending on the parameter τ_r - the size of the time interval of retrospection (0≤τr ≤tmax). The strength measured in Mpa is considered as the residual resource R of the investigated PFC type CM. The studied interval for studying changes in the residual resource of the PCM is tmax = 15 years. Notation: τ_r - the size of the time interval of retrospection in years (0≤τr ≤tmax); t – time.

To construct optimal generalized models of the durability of the optimal structure and complexity based on the developed methodology, the results of physical experiments conducted to measure the residual life of PFC were used as initial data [19]. . The strength of PFC – R was considered as a residual resource. Here R_0 is the initial strength value. Measurements of the residual resource were carried out at intervals of 1 year. We introduce a generalized persistence indicator $k_R(t)$, which is the ratio of the residual resource of PFC at time t to the initial value of the residual resource $R_0 : k_R\left(t\right) = R(t) / R_0$.

As a result of the computational experiments carried out according to the developed methodology, optimal multiparametric generalized models of the durability of the optimal structure and complexity were constructed with a sequential increase in the size of the time interval of retrospection τ_r. The upper bound $t_{\min}$ of the retrospection interval $\left[0, t_{\min}\right]$ during multivariate computational experiments was taken as a parameter of the problem. According to the developed methodology, for each current value of the parameter - the upper limit of the retrospection interval $t_{\min}$, the task of constructing an optimal model of the durability of the optimal structure and complexity was solved $R^{n^*}\left((u^{n^*})^*;\, t\right)$ (3). For each current value of the parameter, based on the constructed optimal model of the durability of the optimal structure and complexity, the problem of constructing the predicted dependence of the residual resource of the PCM on the predicted time interval was then solved $[t_{\min}, t_{\max}]$ [13-16] . In accordance with the fact that the introduced multiparametric family of durability models $\left\{R^n\left(u^n;\, t\right)\right\}_{n=1}^{\infty}$ is presented in the form of a superposition of Arrhenius equations, which in the resulting form has the form of series (2), and reflects the nature of destructive processes described at the micro level, and the optimal durability model of optimal structure and complexity $R^{n^*}\left((u^{n^*})^*;\, t\right)$ belongs to this multi-parametric family, then, in accordance with Based on the results obtained in [14-16] , it should be expected that this model in the introduced parametric family will most adequately describe the real time dependence of the residual resource on the prediction interval $[t_{\min}, t_{\max}]$. The conducted computational experiments confirm these conclusions.

Based on the conducted computational experiments, it was found that at the retrospection intervals, the size of which τ_r is insufficient to identify the fundamental laws of the micro- and macrostructure of CM, determining the nature of the change in the residual resource of CM for a long period, the dependence of the parametric family of optimal models for predicting optimal structure and complexity $R^*\left(\tau_r;t\right)$ depending on the parameter τ_r has an unstable oscillatory character, while the amplitude of the oscillations decreases as the parameter τ_r increases. For the retrospection intervals, the size of which τ_r is already sufficient to identify the fundamental laws of the micro- and macrostructure of the PCM, which determine the nature of the change in the residual resource of the CM for a long period, the dependence of the parametric family of optimal models for predicting optimal structure and complexity $R^*\left(\tau_r;t\right)$ on the parameter τ_r acquires a monotonous stable character and smoothly approaches the real dependence of the residual resource as the parameter τ_r increases. At the same time, it was found that for the studied CM, after seven years of observations (τ_r =7), the dependence of the parametric family of optimal forecasting models of optimal complexity $R^*\left(\tau_r;t\right)$, depending on the parameter τ_r , acquires a monotonous stable character and differs insignificantly from the real dependence of the residual resource.

Thus, depending on how adequately it is possible to take into account in the durability models the essential fundamental laws inherent in the physical–chemical processes occurring at the micro and macro levels, which have a decisive influence on the change in the residual resource of materials, the success of the effective solution of forecasting problems largely depends. Thus, an objective assessment of the size of the retrospection intervals τ_r, at which these fundamental patterns are already fully manifested, becomes important for solving the problems of effective forecasting.

The conducted computational experiments allowed us to establish that the dependence of the parametric family of optimal forecasting models of optimal structure and complexity $\left\{R^*\left(\tau_r;t\right)\right\}$, depending on the parameter τ_r characterizing the size of the retrospection interval, is oscillatory. The nature of the oscillatory process largely depends on the extent to which the fundamental patterns that characterize the processes occurring at the micro and macro levels and have a decisive influence on the change in the residual life of the composite are manifested in the studied time period. The study of various types of stable and unstable oscillatory processes, which lead to parametric families of optimal models of the durability of the optimal structure and complexity $\left\{R^*\left(\tau_r;t\right)\right\}$ when the parameter τ_r changes, is given.

4 Conclusion

Based on the application and development of mathematical and computer modeling methods, effective methods for predicting the defining characteristics (residual life, strength, reliability, durability) of composites in extreme environmental conditions have been developed. A study of the promising possibilities of applying modern provisions of the kinetic theory of strength to develop optimal models of durability of composite materials and structures made of them under the influence of extreme environmental factors has been carried out.

The possibility of evaluating on the basis of physical experiments the determining parameters of physical-chemical processes occurring at the microlevel can allow in a single form to investigate the essential features of the influence of various types of external influences on the speed of the prevailing aging process, as well as to develop functional relationships characterizing the corresponding quantitative relationships describing the influence of various types of external influences on the speed of the prevailing aging process.

It is shown that the application of the principle of multiplicity of forecasting models and multiparametric models of optimal complexity in the framework of refined formulations of inverse problems of forecasting the residual resource in the variational formulation can significantly increase the accuracy of solving forecasting problems.

References

1. Bakulin VN, Gribanov VM et al. (2008) Methods of optimal design and calculation of composite structures. In 2 vols. V.2. Mechanical action of X-ray radiation on thin-walled composite structures. Nauka, Moscow
2. Bakulin VN, Ostrik AV (2015) Complex effect of radiation and particles on thin-walled structures with heterogeneous coatings. Fizmatlit, Moscow.
3. Gusev EL, Bakulin VN (2022) Application of Modern Provisions of the Kinetic Theory of Strength for the Development of Generalized Models of Durability of Composites. Springer Aerospace Technology book series (SAT). Recent Developments in High-Speed Transport. P.219-227.

4. Smerdov AA, Son TP (2016) Comparative analysis of optimal composite multiwalled and sandwich shells for modules of launch vehicles and upper stages. Composite materials constructions 143: 58-65
5. Lopatin AV, Khakhlenkova AA (2018) Finite Element Modeling of a Mesh Composite Coupling Section of a Spacecraft. Mech. Solids 53 : 435-439
6. Bakulin VN (2020) Investigation of the Influence of the Cutout Dimensions on the Stress-strain State of Three-layer Shells with Load-bearing Layers of Composite Materials. Journal of Physics: Conference Series: Materials Science and Engineering 714: 012002 doi.org/10.1088/1757-899X/714/1/012002
7. Testoedov NA, Nagovitsin VN, Permyakov MY (2016) The use of sandwich honeycomb structures in spacecraft. Vestnik SibGAU 17: 200-211
8. Akimov DV, Larionov IF, Klimenko DV et. al . (2019 Mathematical Modeling and Investigation of the Stress-Strain State of Compartments of Space Rockets. Kosmicheskaya Tekhnika. Raketnoe Vooruzhenie (Space Technology. Missile Armaments) 117: 21–27
9. Razin AF (2018) The Problem of Optimum Design of Composite Housings of Solid Propellant Rocket Engines,. Mech Solids 53; 418-426
10. Azarov AV (2018) The problem of Designing Aerospace Mesh Composite Structures. Mech. Solids 53: 427-434
11. Bakulin VN , Nedbay AY (2022) Parametric Resonance of a Three-Layer Cylindrical Composite Shell, Supported by Longitudinal Ribs and a Cylinder, Under The Action of a Time-Varying Axial Force. Doklady Physics 67: 209-214
12. Bakulin VN, Nedbay AY (2021) The Dynamic Stability Of Three-Layered Cylindrical Shell, Reinforced Ring Ribs And Hollow Cylinder Under External Pressure Pulsing. Doklady Physics, 66: 175-181
13. Gusev EL, Bakulin VN (2022) Generalized durability models and their application to solving problems on predicting the defining characteristics of composites. Mechanics of Composite Materials. 58(3):355–364; doi:10.1007/s11029-022-10034-1
14. Gusev EL, Bakulin VN (2020) The use of generalized models in the variational formulation of the prediction tasks defining characteristics of composite materials. J. Phys.: Conf. Ser. Collection of Materials of the VIII Int. Conf. 1431: 012017.
15. Gusev EL, Bakulin VN (2018) Variation formulations of inverse problems in forecastting the residual life of composites. Doklady Physics 63: 388-392.
16. Gusev EL, Bakulin VN (2021) Variational Methods of Solving Problems on Control of the Intensity of a Temperature Field. Journal of Engineering Physics and Thermophysics 94: 1117–1123
17. Bakulin VN, Ostrik AV (2014) The combined thermal and mechanical effect of radiation and shock waves on a multilayer orthotropic shell with a heterogeneous coating. J. Appl. Math. Mech. 78: 155-162
18. Gusev EL, Bakulin V N (2020) The use of generalized models in the variational formulation of the preiction tasks defining characteristics of composite materials. J. Phys.: Conf. Ser. Collection of Materials of the VIII Int. Conf. 1431: 012017
19. Urzhumtsev US, Chersky IN (1985) Scientific foundations of engineering climatology of polymer and composite materials. Mechanics of composite materials 4: 708-714