# On reachability of nominal aircraft flight dynamics by data-based control reconfiguration in case of actuator failures 

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#### Abstract

The paper proposes the data-based method for reaching and maintaining the nominal aircraft flight dynamics in in case of actuator failures. The method is based on aircraft control reconfiguration and uses input and output data of flight control system only. The novelty of the proposed method lies in the reachability of nominal dynamics not in one, but in several discrete steps from the moment the reconfiguration starts. This makes it possible to ensure the reachability of the desired states for any controllability index of the linear discrete model, as well as to reduce the norms of reconfigured control vectors.


## 1 Introduction

Aircraft actuator fault-safety is a priority requirement in the designing of advanced high-speed transport systems [1]. Along with the hardware redundancy [2], the functional redundancy is used [3] to accommodate failures of actuators [4]. The functional redundancy involves the control reconfiguration [5] between the remaining healthy actuators in such a way as to ensure flight safety and, if possible, maintain its nominal dynamics. It can be performed by model-based, knowledge-based, and data-based [6, 7] methods. The latter are the most versatile, because the use only data on input and output signals and do not require the designing the aircraft dynamics model or training the neural networks. A known databased control reconfiguration method [7] assumes the reachability of the desired state in one discrete step from the start of the reconfiguration. This is often fundamentally impossible because actuator mechanical restrictions. In this paper, a method for returning to nominal dynamics in several discrete steps, followed by a transition to one-step calculations is proposed, and recommendations are given for reducing the control vector norm.

## 2 The problem statement

Let the nominal aircraft dynamics is described by a linear model

$$
\sigma \mathbf{x}=\mathbf{A} \mathbf{x}+\mathbf{B u},
$$

[^0]where $\sigma$ is the discrete one step forward shift operator, $\mathbf{x}$ is the n-dimensional completely known state vector, $\mathbf{u}$ is the m -dimensional control vector, $\mathbf{A}$ is the state matrix, and $\mathbf{B}$ is the control matrix [8].

An emergency situation is considered, in which actuator failures occur at a discrete step $i_{f}$, leading to a change in the control matrix from $\mathbf{B}$ to $\mathbf{B}_{f}$. It is assumed that the pair of matrices $\mathbf{A}$ and $\mathbf{B}_{f}$ remains completely controllable [9]. The aircraft flight dynamics with the failed actuators is described by the state vector $\mathbf{x}^{f}$, the values of which differ from the values of the vector $\mathbf{x}$, starting from the discrete step $i_{f}+1$

$$
\begin{equation*}
\sigma \mathbf{x}^{f}=\mathbf{A} \mathbf{x}^{f}+\mathbf{B}_{f} \mathbf{u} \tag{2}
\end{equation*}
$$

A system of equations (2) at successive discrete steps (from $i_{f}$ up to $i_{f}+i$ ) can be written using block matrices in the form

$$
\left[\begin{array}{l:c}
-\mathbf{A} & \mathbf{I}_{n}
\end{array}\right]\left[\begin{array}{c}
\mathbf{X}_{i_{f}, i i_{f}+i}^{f}  \tag{3}\\
\sigma \mathbf{X}_{i_{f}, i_{f}+i}^{\prime}
\end{array}\right]=\mathbf{B}_{f} \mathbf{U}_{i_{f}, i_{f}+i},
$$

where $\mathbf{X}_{i_{f}, i_{f}+i}^{f}=\left[\begin{array}{l:l:l:l}\mathbf{x}_{i_{f}}^{f} & \mathbf{x}_{i_{f}+1}^{f} & \ldots & \mathbf{x}_{i_{f}+i}^{f}\end{array}\right], \quad \mathbf{U}_{i_{f}, i_{f}+i}=\left[\begin{array}{l:l:l:l}\mathbf{u}_{i_{f}} & \mathbf{u}_{i_{f}+1} & \ldots & \mathbf{u}_{i_{f}+i}\end{array}\right]$, and $\mathbf{I}_{n}$ is the identity matrix of order $n$. A sufficient condition for solvability [10] of this equation for the matrix $\left[\begin{array}{l:l}-\mathbf{A} & \mathbf{I}_{n}\end{array}\right]$

$$
\mathbf{U}_{i_{f}: i_{f}+i}{\overline{\left[\begin{array}{c}
\mathbf{X}_{i_{f}: i_{f}+i}  \tag{4}\\
\sigma \mathbf{X}_{i_{f}, i_{f}+i}
\end{array}\right]^{R}}=\mathbf{0}, 0 .}^{f}
$$

is written in terms of the maximum rank right annihilator ( $\overline{\mathrm{Z}}^{R}$ for any matrix Z such $Z \bar{Z}^{R}=0$ ) and a zero matrix $\mathbf{0}^{\text {of suitable dimension. The condition (4) uses only the }}$ known signals on and can be used for data-based control reconfiguration.

Let the reconfigured state $\mathbf{x}^{r}$ and control $\mathbf{u}^{r}$ vectors define the model

$$
\begin{equation*}
\sigma \mathbf{x}^{r}=\mathbf{A} \mathbf{x}^{r}+\mathbf{B}_{f} \mathbf{u}^{r} \tag{5}
\end{equation*}
$$

In the known data-based reconfiguration method [10] the problem is to determine the control vector $\mathbf{u}_{i_{r}}^{r}$ that returns the aircraft state (5) to the nominal one at the next step $\mathbf{x}_{i_{r}+1}^{r}=\mathbf{x}_{i_{r}+1}$. Its solution (if it exists) is found from a condition similar to the equality (4)
where $\mathbf{X}_{i_{f} i_{r}-1}^{r}=\left[\begin{array}{l:l:l:l}\mathbf{x}_{i_{f}}^{r} & \mathbf{x}_{i_{f}+1}^{r} & \ldots & \mathbf{x}_{i_{r}-1}^{r}\end{array}\right], \mathbf{U}_{i_{f} i_{r}-1}^{r}=\left[\begin{array}{l:l:l:l}\mathbf{u}_{i_{f}}^{r} & \mathbf{u}_{i_{f}+1}^{r} & \ldots & \mathbf{u}_{i_{r}-1}^{r}\end{array}\right]$, according to the expression

$$
\begin{equation*}
\mathbf{u}_{i_{r}}^{r}=-\mathbf{U}_{i_{f}, i_{r}-1}^{r} \mathbf{r}_{1: k}^{[1]} / r_{k+1}^{[1]} . \tag{7}
\end{equation*}
$$

The expression (7) degenerates $\left(r_{k+1}^{[1]}=0\right)$ if the equation (5) for the step $j=i_{r}$

$$
\mathbf{B}_{f} \mathbf{u}_{j}^{r}=\mathbf{d}_{j}, \quad \mathbf{d}_{j}=\left[\begin{array}{l:l}
-\mathbf{A} & \mathbf{I}_{n}
\end{array}\right]\left[\begin{array}{c}
\mathbf{x}_{j}^{r}  \tag{8}\\
\mathbf{x}_{j+1}
\end{array}\right]
$$

turns out to be unsolvable for control $\mathbf{u}_{j}^{r}\left(\overline{\mathbf{B}}_{f}^{L} \mathbf{d}_{j} \neq \mathbf{0}\right.$, where $\overline{\mathbf{B}}_{f}^{L}$ is the maximum rank left-side annihilator of the matrix $\overline{\mathbf{B}}_{f}$ ) [11].

In addition, in the general case, the control (7) will not be optimal in the Euclidean norm [12] by solving the equation (8) $\mathbf{u}_{i_{r}}^{r *}=\mathbf{B}_{f}^{+} \mathbf{d}_{i_{r}}$, where $\mathbf{B}_{f}^{+}$is a pseudoinverse matrix. This also applies to further controls $\mathbf{u}_{j}^{r}\left(j>i_{r}\right)$. Their components can significantly exceed the limits [12], valid for actuators.

Next, we will consider how to reach the nominal dynamics $\mathbf{x}$ in case of actuator failure, when the equation (8) is unsolvable, and how to reduce the control vector norm $\mathbf{u}^{r}$.

## 3 Reaching the nominal dynamics in case of actuator failures

If the controllability index [13] of the system (5) is 1 , then the equation (8) is solvable and the control (7) is in any desired state $\mathbf{x}_{i_{r}+1}$, i.e. nominal dynamics is reached in 1 step from the moment $i_{r}$. Let the controllability index of the system (5) be equal to 2 , when $\operatorname{rank}\left[\mathbf{B}_{f}: \mathbf{A B}_{f}\right]=n$. If we introduce a shift by $j$ steps forward operator $\sigma^{j}\left(\sigma^{1} \equiv \sigma\right)$, then t . The relationship between the states $\mathbf{x}^{r}$ with a shift $\sigma^{2}$ follows from the model (5)

$$
\begin{equation*}
\sigma^{2} \mathbf{x}^{r}=\mathbf{A}\left(\mathbf{A} \mathbf{x}^{r}+\mathbf{B}_{f} \mathbf{u}^{r}\right)+\mathbf{B}_{f} \sigma \mathbf{u}^{r}=\mathbf{A}^{2} \mathbf{x}^{r}+\mathbf{A} \mathbf{B}_{f} \mathbf{u}^{r}+\mathbf{B}_{f} \sigma \mathbf{u}^{r} . \tag{9}
\end{equation*}
$$

A system of equations (9) at successive discrete steps (from $i_{f}$ up to $i_{f}+i$ ) can be written using block matrices in the form

$$
\left[\begin{array}{l:l}
-\mathbf{A}^{2} & \mathbf{I}_{n}
\end{array}\right]\left[\begin{array}{c}
\mathbf{X}_{i_{f, i}+i}^{r} \\
\sigma^{2} \mathbf{X}_{i_{f}, i_{f}+i}^{r}
\end{array}\right]=\left[\begin{array}{l:l}
\mathbf{A} \mathbf{B}_{f} & \mathbf{B}_{f}
\end{array}\right]\left[\begin{array}{c}
\mathbf{U}_{i_{f}, i_{f}+i}^{r} \\
\sigma \mathbf{U}_{i_{f}, i_{f}+i}^{r}
\end{array}\right] .
$$

A sufficient condition for the solvability of this equation for the matrix $\left[\begin{array}{l:l}-\mathbf{A}^{2} & \mathbf{I}_{n}\end{array}\right]$ by analogy with the condition (4), has the form

It is proposed to find controls $\mathbf{u}_{i_{r}}^{r}$ and $\mathbf{u}_{i_{r}+1}^{r}$, leading the system (5) in 2 steps from the moment $i_{r}$ to the nominal state $\mathbf{x}_{i_{r}+2}^{r}=\mathbf{x}_{i_{r}+2}$, from the condition

$$
\left[\begin{array}{c:c}
\mathbf{U}_{i_{f, i r}-2}^{r} & \mathbf{u}_{i_{r}}^{r}  \tag{10}\\
\sigma \mathbf{U}_{i_{f}, i_{r}-2}^{r} & \mathbf{u}_{i_{r}+1}^{r}
\end{array}\right]\left[\begin{array}{c}
\mathbf{r}_{1: k-1}^{[2]} \\
r_{k}^{[2]}
\end{array}\right]=0, \quad\left[\begin{array}{c}
\mathbf{r}_{1: k-1}^{[2]} \\
r_{k}^{[2]}
\end{array}\right]=\left[\begin{array}{c:c}
\mathbf{X}_{i_{f}, i_{i}-2}^{r} & \mathbf{x}_{i_{r}}^{r} \\
\sigma^{2} \mathbf{X}_{i_{f}, i_{r}+2}^{r} & \mathbf{x}_{i_{r}+2}
\end{array}\right]^{R}
$$

according to the formula

$$
\left[\begin{array}{c}
\mathbf{u}_{i_{r}}^{r}  \tag{11}\\
\mathbf{u}_{i_{r}+1}^{r}
\end{array}\right]=-\left[\begin{array}{c}
\mathbf{U}_{i_{f}, i_{i}-2}^{r} \\
\sigma \mathbf{U}_{i_{f}, i_{r}-2}^{r}
\end{array}\right] \mathbf{r}_{1: k-1}^{[2]} / r_{k}^{[2]} .
$$

In this case, the equation (9) for the step $i_{r}$

$$
\underbrace{\left[\mathbf{A B}_{f}\right.}_{\mathbf{U}_{2}}: \mathbf{B}_{f}]\left[\begin{array}{c}
\mathbf{u}_{i_{r}}^{r} \\
\mathbf{u}_{i_{r}+1}^{r}
\end{array}\right]=\left[-\mathbf{A}^{2}: \mathbf{I}_{n}\right]\left[\begin{array}{c}
\mathbf{x}_{i_{r}}^{r} \\
\mathbf{x}_{i_{r}+2}
\end{array}\right]
$$

for any state $\mathbf{x}_{i_{r}+2}$ is solvable for the controls $\mathbf{u}_{i_{r}}^{r}$ and $\mathbf{u}_{i_{r}+1}^{r}$, because the matrix $\mathbf{U}_{2}$ has no left-side annihilators.

Let's generalize the multi-step scheme for reaching nominal dynamics for any controllability index $v$ when $\operatorname{rank}\left[\begin{array}{l:l:l:l}\mathbf{B}_{f} & \mathbf{A B}_{f} & \ldots & \left.\mathbf{A}^{v-1} \mathbf{B}_{f}\right]=n \text {. From (5) we obtain the }\end{array}\right.$ relationship between the values of the vector $\mathbf{x}^{r}$ with $\sigma^{v}$ shifts.

$$
\begin{equation*}
\sigma^{v} \mathbf{x}^{r}=\mathbf{A}^{v} \mathbf{x}^{r}+\mathbf{A}^{v-1} \mathbf{B}_{f} \mathbf{u}^{r}+\mathbf{A}^{v-1} \mathbf{B}_{f} \sigma \mathbf{u}^{r}+\ldots+\mathbf{B}_{f} \sigma^{v-1} \mathbf{u}^{r} \tag{12}
\end{equation*}
$$

The controls $\mathbf{u}_{i_{r}}^{r}, \mathbf{u}_{i_{r}+1}^{r}, \ldots, \mathbf{u}_{i_{r}+v-1}^{r}$, leading the system (5) in $v$ steps from the moment $i_{r}$ to the nominal state $\mathbf{x}_{i_{r}+v}^{r}=\mathbf{x}_{i_{r}+v}$, are found from the condition

$$
\left[\begin{array}{c:c}
\mathbf{U}_{i_{i}, i_{r}-v}^{r} & \mathbf{u}_{i_{r}}^{r}  \tag{13}\\
\sigma \mathbf{U}_{i_{f}, i_{r}-v}^{r} & \mathbf{u}_{i_{r}+1}^{r} \\
\vdots & \vdots \\
\sigma^{v-1} \mathbf{U}_{i_{f}, i_{r}-v}^{r} & \mathbf{u}_{i_{r}+v-1}^{r}
\end{array}\right]\left[\begin{array}{l}
\mathbf{r}_{1 / k-v+1}^{[v]} \\
r_{k-v+2}^{[v]}
\end{array}\right]=0, \quad\left[\begin{array}{l}
\mathbf{r}_{1 / k-v+1}^{[v]} \\
r_{k-v+2}^{[v]}
\end{array}\right]=\left[\begin{array}{c:c}
\mathbf{X}_{i_{f}, i_{i}-v}^{r} & \mathbf{x}_{i_{r}}^{r} \\
\sigma^{v} \mathbf{X}_{i_{f}, i_{r}-v}^{r} & \mathbf{x}_{i_{r}+v}
\end{array}\right]^{R}
$$

according to the expression

$$
\left[\begin{array}{c}
\mathbf{u}_{i_{r}}^{r}  \tag{14}\\
\mathbf{u}_{i_{r}+1}^{r} \\
\vdots \\
\mathbf{u}_{i_{r}+v-1}^{r}
\end{array}\right]=-\left[\begin{array}{c}
\mathbf{U}_{i_{i}, i_{r}-v}^{r} \\
\sigma \mathbf{U}_{i_{f}, i_{r}-v}^{r} \\
\vdots \\
\sigma^{v-1} \mathbf{U}_{i_{f}, i_{r}-v}^{r}
\end{array}\right] \mathbf{r}_{1: k-v+1}^{[v]} / r_{k-v+2}^{[v]} .
$$

In this case, the equation (12) for the step $i_{r}$

$$
\underbrace{\left[\begin{array}{l:l:c:c}
\mathbf{A}^{v-1} \mathbf{B}_{f} & \mathbf{A}^{v-2} \mathbf{B}_{f} & \ldots & \mathbf{B}_{f}
\end{array}\right]\left[\begin{array}{c}
\mathbf{u}_{i_{r}}^{r} \\
\mathbf{u}_{i_{r}+1}^{r_{r}} \\
\vdots \\
\mathbf{u}_{i_{r}+\mathrm{v}-1}^{r}
\end{array}\right]=\left[-\mathbf{A}^{r}\right.}_{\mathbf{U}_{v}} \mathbf{I}_{n}]\left[\begin{array}{c}
\mathbf{x}_{i_{r}}^{r} \\
\mathbf{x}_{i_{r}+\mathrm{v}}
\end{array}\right]
$$

for any value $\mathbf{x}_{i_{r}+v}$ is solvable for the controls $\mathbf{u}_{i_{r}}^{r}, \mathbf{u}_{i_{r}+1}^{r}, \ldots, \mathbf{u}_{i_{r}+\mathrm{v}-1}^{r}$, because the matrix $\mathbf{U}_{v}$ has no left annihilators.

## 4 Maintaining nominal dynamics in case of actuator failures

According to the proposed methodology, control reconfiguration starts at step $i_{r}$. At step $i_{d}=i_{r}+v$ the desired nominal state is reached for the first time. But further, on the steps $j=i_{d}+i \quad(i=0,1, \ldots)$ it is most often impractical to calculate the controls $\mathbf{u}^{r}$ by formulas (11) or (14): each time the state vector will be reduced to the nominal value in several steps, and between them the values of the vector $\mathbf{x}^{r}$ may differ from the values of $\mathbf{x}$ considerably. Such "jumping" dynamics is unacceptable.

After reaching the nominal dynamics for maintaining it, we should return to the one-step reconfiguration scheme, similar to the formula (7):

$$
\begin{equation*}
\mathbf{u}_{j}^{r}=-\mathbf{U}_{j-\Delta: j-1}^{r} \mathbf{r}_{1: \Delta}^{[1]} / r_{\Delta+1}^{[1]}, \tag{15}
\end{equation*}
$$

where $\Delta$ is the number of steps sufficient to obtain a column-vector

$$
\left[\begin{array}{c}
\mathbf{r}_{[: L}^{[1]} \\
r_{\Delta+1}^{1]}
\end{array}\right]=\left[\begin{array}{c:c}
\mathbf{X}_{j-\Delta ; j-1}^{r} & \mathbf{x}_{j} \\
\sigma \mathbf{X}_{j-\Delta: j-1}^{r} & \mathbf{x}_{j+1}
\end{array}\right]^{R} .
$$

The proposed scheme is possible if $\operatorname{rank} \mathbf{B}_{f}=\operatorname{rank} \mathbf{B}$, then $\mathbf{x}_{j}^{r}=\mathbf{x}_{j}$, and the equation (5) unlike (8) is solvable for $\mathbf{u}_{j}^{r}$ for each step $j$ according to (1)

$$
\mathbf{B}_{f} \mathbf{u}_{j}^{r}=\mathbf{B} \mathbf{u}_{j}, \quad \mathbf{B} \mathbf{u}_{j}=\left[\begin{array}{l:l}
-\mathbf{A} & \mathbf{I}_{n}
\end{array}\right]\left[\begin{array}{c}
\mathbf{x}_{j}  \tag{16}\\
\mathbf{x}_{j+1}
\end{array}\right] .
$$

## 5 Reducing the norm of the control vector

At any step $j \geq i_{r}$, the control $\mathbf{u}_{j}^{r}$ satisfies the equation (8), which has many solutions [11]

$$
\begin{equation*}
\mathbf{u}_{j}^{r}=\mathbf{B}_{f}^{+} \mathbf{d}_{j}+\overline{\mathbf{B}}_{f}^{R} \boldsymbol{\omega}, \tag{17}
\end{equation*}
$$

where $\boldsymbol{\omega}$ is an arbitrary vector, which have the minimal Euclidean norm solution

$$
\mathbf{u}_{j}^{r *}=\mathbf{B}_{f}^{+} \mathbf{d}_{j}=\mathbf{B}_{f}^{+} \mathbf{B}_{f} \mathbf{u}_{j}^{r}=\left(\mathbf{I}_{m}-\overline{\mathbf{B}}_{f}^{R} \overline{\mathbf{B}}_{f}^{R+}\right) \mathbf{u}_{j}^{r},
$$

where $\overline{\mathbf{B}}_{f}^{R}$ is the right-side maximum rank annihilator of the matrix $\mathbf{B}_{f}$, and $\mathbf{u}_{j}^{r}$ is a control vector from the set (17) [11]. Even some of $\tilde{\mathbf{B}}_{f}^{R}$ the annihilator columns $\overline{\mathbf{B}}_{f}^{R}$ will decrease the control norm $\mathbf{u}_{j}^{r}$. Indeed, let

$$
\begin{equation*}
\tilde{\mathbf{u}}_{j}^{r}=\left(\mathbf{I}_{m}-\tilde{\mathbf{B}}_{f}^{R} \tilde{\mathbf{B}}_{f}^{R+}\right) \mathbf{u}_{j}^{r}, \tag{18}
\end{equation*}
$$

where $\tilde{\mathbf{B}}_{f}^{R+}=\left(\tilde{\mathbf{B}}_{f}^{R T} \tilde{\mathbf{B}}_{f}^{R}\right)^{-1} \tilde{\mathbf{B}}_{f}^{R T}$ is the pseudo-inverse of the full-rank matrix. Then
$\tilde{\mathbf{u}}_{j}^{r T} \tilde{\mathbf{u}}_{j}^{r}=\mathbf{u}_{j}^{r T}\left(\mathbf{I}_{m}-\tilde{\mathbf{B}}_{f}^{R}\left(\tilde{\mathbf{B}}_{f}^{R T} \tilde{\mathbf{B}}_{f}^{R}\right)^{-1} \tilde{\mathbf{B}}_{f}^{R T}\right) \mathbf{u}_{j}^{r}=\mathbf{u}_{j}^{r T} \mathbf{u}_{j}^{r}-\left(\tilde{\mathbf{B}}^{R T} \mathbf{u}_{j}^{r}\right)^{T}\left(\tilde{\mathbf{B}}^{R T} \tilde{\mathbf{B}}^{R}\right)^{-1} \tilde{\mathbf{B}}^{R T} \mathbf{u}_{j}^{r}$,
$\tilde{\mathbf{u}}_{j}^{r T} \tilde{\mathbf{u}}_{j}^{r} \leq \mathbf{u}_{j}^{r T} \mathbf{u}_{j}^{r}$ (the non-negative definite quadratic form is subtracted).
Before the reconfiguration, ( $i_{f} \leq j \leq i_{r}-1$ ) the control $\mathbf{u}_{j}^{r}=\mathbf{u}_{j}$ does not correspond to the zero equalities (6), (10), (13). Therefore, according to the solvability condition of the equation (3) for $\left[\begin{array}{l:l}-\mathbf{A} & \mathbf{I}_{n}\end{array}\right]$, if the matrix

$$
\tilde{\mathbf{B}}_{f}^{R}=\mathbf{U}_{i_{f}, i_{r}-1}^{r}\left[\begin{array}{c}
\mathbf{X}_{i_{f}, i_{r}-1}  \tag{19}\\
\sigma \mathbf{X}_{i_{f}, i_{i}-1}
\end{array}\right]^{R}
$$

exists, it is the right annihilator (possibly not of maximum rank) of the matrix $\mathbf{B}_{f}$.
If reaching the nominal dynamics is carried out according to a one-step scheme (6), then at the first appearance of the right-side annihilator, the reconfigured control is calculated. The right side of the expression (19) is set to zero.

In multi-step schemes (10), (13) more steps go through before the start of reconfiguration than in the scheme (6). Therefore, the non-zero columns formed on the right side of the expression (19) are actually the right annihilators of the matrix $\mathbf{B}_{f}$.

## 6 Numerical examples

Consider a linearized longitudinal dynamics model of a Boeing 747-100/200 [13] of the form (1),
where

| $\mathbf{A}=\mathbf{I}_{5}+0.01$ |  | $\left[\begin{array}{c}-0.4861 \\ 0 \\ 1.0053 \\ 1 \\ 0\end{array}\right.$ | 0.000317 -0.0199 -0.0021 0 0 | $\begin{gathered} -0.5588 \\ 3.0796 \\ -0.5211 \\ 0 \\ -92.6 \end{gathered}$ | $\begin{gathered} 0 \\ -9.8048 \\ 0 \\ 0 \\ 92.6 \end{gathered}$ | $\left.\begin{array}{c} -2.4 \cdot 10^{-6} \\ 8.98 \cdot 10^{-5} \\ -9.3 \cdot 10^{-6} \\ 0 \\ 0 \end{array}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}=-0.01$ | $\left[\begin{array}{c:c}b_{1,1} & b_{1,1} \\ 0 & 0 \\ b_{3,1} & b_{3,1} \\ 0 & 0 \\ 0 & 0\end{array}\right.$ | $\begin{array}{c:c}b_{1,3} & b_{1,3} \\ 0 & 0 \\ b_{3,3} & b_{3,3} \\ 0 & 0 \\ 0 & 0\end{array}$ | $\begin{array}{c:c}b_{1,5} & b_{1,6} \\ b_{2,5} & b_{2,6} \\ b_{3,5} & b_{3,6} \\ 0 & 0 \\ 0 & 0\end{array}$ |  | (e:c ${ }_{1}{ }_{1,6}$ |
|  | $\begin{aligned} & b_{1,1}=-0.1455, \quad b_{3,1}=-0.0071, \quad b_{1,3}=-0.1494, \quad b_{3,3}=-0.0074, \quad b_{1,5}=-1.286, \\ & b_{2,5}=-0.3122, \quad b_{3,5}=-0.0676, b_{1,6}=0.0013, b_{2,6}=0.1999, \quad b_{3,6}=-0.0004, \quad b_{1,7}=0.0035 . \end{aligned}$ |  |  |  |  |  |

The controllability index of this system is 2 .
Let the hypothetical control vectors values $\mathbf{u}$ be set for the nominal flight mode, and the 2 nd actuator is jammed in trim position at the step of $i_{f}=5$

$$
\mathbf{B}_{f}=\mathbf{B} \operatorname{diag}\left[\begin{array}{l:l:l:l:l:l:l}
1 & 0 & 1 & 1 & 1 & 1 & 1
\end{array} 1 / 1\right] .
$$

The simulation showed that the control reconfiguration (10) starts at step $i_{r}=16$, and after 2 steps the nominal dynamics is reached successfully. The norms of reconfigured control vectors (11) and (15) are more than nominal ones. The norm is reduced by means of a correction (18).

For the graphs of state and control vectors (Fig. 1), channels were selected that clearly show reaching the nominal dynamics in case of failure.

## 7 Conclusions

The study shows that the data-based method of reconfiguration in case of failures of the aircraft actuators can be modified to reach the nominal flight dynamics in the emergency mode. The modification based on the fact that the data matrix is formed from pairs of state vector values not at successive discrete steps, but after a number of steps equal to the closed-loop system controllability index after failures. The same number of steps passes from the moment the reconfiguration starts until the nominal dynamics is reached. Further, to maintain the nominal dynamics, the reconfiguration proceeds according to a one-step algorithm, if the rank of the control matrix has not decreased as a result of failures.


Fig. 1. States and controls of Boeing 747-100/200 longitudinal dynamics model without failures, with failure, and with reconfiguration.

The modification also allows, by determining a part of the linearly independent right annihilators of the control matrix after failures, without its complete identification, to reduce the norm of the reconfigured control vectors to meet the given restrictions on their deviations.

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