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Operational Adaptation of Digital Filter Parameters when Changing the Discrete Time Step in the Problems of Forming the Control of Unmanned Vehicles Using Satellite Measurements

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Abstract. For the tasks of controlling unmanned vehicles using external information signals, in some cases, a change in the time parameters T_0 of periodic data receipt is typical. Such systems, the processing of incoming information in order to form digital control is implemented by digital filters (DF), based on discrete samples x[k] of some continuous signal x(t) at quantization times tk=k*T_0 Here T_0[seconds]- the period of discreteness in time, k=0,1,2,.. an integer variable, essentially a time counter. The properties of the filter are uniquely specified by its specific mathematical model - the discrete impulse transient function [1,3] of the filter (DITF), which characterizes the filter operation at a specific step $T_0 = 1/f_0$, where f_0 (hertz) is the frequency. When the time intervals T_0 of data arrival change, for example, due to a change in the transmission conditions in the radio channel, or due to a change in the location of the satellite constellation, the filter properties will change during operation [2], and if T_0 changes significantly, such filtering can lead to unsatisfactory results. In this paper, for the parameters of the digital filter, the adaptation problem is posed and solved, which ensures the invariance of the nature DITF, in contrast to the stabilization of the frequency properties of the digital filter, studied in [4]. To rebuild the numerical parameters of the filter, an algorithm is proposed that uses information about the time intervals obtained by direct measurement. At the stage of filter development, a special recalculation matrix is formed and when the filter is running in real time, the digital filter parameters are recalculated. For a model example, the calculation results are presented, which show good tuning accuracy, stable filter characteristics, as well as the simplicity of the required calculations on board with a significant change in time intervals.

1 Introduction

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In automation and in the technology of automatic control of moving objects, digital filters are widely used, which provide processing of discrete samples of a continuous signal x(t) at fixed times t= k, where (sec) is a certain step of discreteness in time, k = 0, 1,2.. is an integer variable that defines the dimensionless discrete time. The required properties of the filters are specified by a certain mathematical model - a finite-difference equation corresponding to the z-transfer function, or a discrete impulse transient function (DITF) [1,3,9] of the filter. It should be noted that these models determine the operation of the filter at a specific step value T_0 = 1/*f0,* where *f0* (hertz) is the frequency of periodic arrival of samples x[k] of a continuous signal x(t).

The time intervals for the arrival of signals in many technical problems can change significantly during operation, for example, as described in [7,9,10]. Such changes are especially characteristic for the control of unmanned objects, in which external (outside the side of the object) measurements of the movement coordinates necessary for control are used. In particular, this may be due to a change in the nature of transmission over the radio path [8], and also due to the conditions for receiving and transmitting information from continuous objects. The papers [7, 10] consider the changes associated with the satellite measurement system.

Regardless of the reasons for changing the time intervals, we will assume that the discrete period T_0= 1/*f0* during the operation of the system can change to the value TN=T_0·N, where N characterizes the multiplicity of the period change relative to the calculated one.

If the frequency f0 changes during operation, data arrives at the frequency fN=f0/N, the properties of the filter will change, and if the frequency changes significantly, such filtering can lead to unsatisfactory results if the device parameters are not rebuilt. This is especially critical for digital automatic systems [1], where digital filters operate in a closed control loop and a change in their properties can lead to a deterioration in the quality of control, and, possibly, to loss of stability of the digital system.

Let the processing of a discrete sample y[k] of a continuous signal y(t) be performed by a linear digital filter determined by the z-transfer function of the form:

$$D0(z)=\frac{A(z^{-1})}{B(z^{-1})}=\frac{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{n-3}+\cdots+b_{n-1}z^{-n+1}+b_nz^{-n}a_0}{1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n-1}z^{-n+1}+a_nz^{-n}} \quad (1)$$

where z = $e^{S\cdot T0}$, s is the Laplace transform parameter, A(z^{-1}), B(z^{-1}) are polynomials in z^{-1} determined by a specific set of coefficients (a_i, b_i). A digital filter of the described form, operating at a frequency f0 =1/T_0, we will call the reference digital filter. In the time domain, it corresponds to a difference equation that relates the input u[k] and the output y[k] of the digital filter, for time points t_k= k·To the equation takes the form:

$$y[k] =-a_1\cdot y[k-1]-\ a_2\cdot y[k-2]-\cdots-a_n\cdot y[k-n]+b_0\cdot u[k]+ b_1\cdot u[k-1]+..+b_{n-1}\cdot u[k-n+1]+\ b_n\cdot u[k-n]\ . \quad (2)$$

This difference equation defines one of the ways to implement a digital filter in the form of an algorithm for calculating the output coordinate y[k] from the past values of the output y[k-i] and the input u[k-i] delayed, i.e. stored in the memory of the calculator at the previous calculation steps. For a fixed frequency f0, a digital filter with z-transfer function (1) performs temporal signal processing.

Let us further assume that the samples arrive not with the calculated interval To, but with a changed one - TN= T_0·N. Here, the number N determines the multiplicity of change T_0. The important point is that the new quantization period T_0·N should also ensure the transmission of the properties of a continuous signal by discrete samples, which, according to the Nyquist-Shannon theorem, corresponds to the relation ω_max < π/(T_0*N), where ω_max(rad/sec) is the maximum frequency present in the spectrum of a continuous signal. When changing the quantization period TN= T_0·N, the mathematical models and properties will change - in particular, the z-transfer function of the filter, which is determined by a formula of the form:

D0N(z,TN)= D0(z^N) .

This transfer function D0N(z,TN) leads to a change in the properties of the temporal characteristics of the filter, relative to the properties of the reference filter: the transition function scales along the time axis, it is possible that the characteristics of the temporal processing by the digital filter may deteriorate, while, since the new quantization moments t_k= *k·To·N* do not match the calculated *k·To*, choose a “similar filter”, i.e. some digital filter D0N(*z,To·N*), of the same order, which would completely repeat the properties of the reference digital filter when the frequency f0 changes to *fN=f0/N* is impossible.

The paper considers the problem of forming the restructuring of the digital filter parameters, i.e. constants $\{a_i, b_i\}$, which determine the digital filter operation algorithm, so that when the frequency fN of information arrival changes, the main temporal properties of the digital filter remain unchanged, in the sense of a certain constancy w[ti], otherwise, the equivalence of temporal characteristics for different TN.

Let us explain the term "certain constancy" in more detail due to the non-obviousness of the concept. We will consider the digital filters equivalent, in the sense of ensuring the constancy of the DIFT, in the case when the points of the DIFT of different filters lie on the same curve corresponding to the impulse response function (IRF) of a continuous filter with a given characteristic. So, in Fig.1 for filters of the 2nd order are shown: h0(t) - reference IRF - dotted line, h05[t] - DITF for TN=0.5 - points with symbol o, h02[k] – DPTF for TN=0.2– step function points. Although the characteristics of the filters are different, but they lie on the same curve w(t) - such characteristics will be considered equivalent.

Based on the research conducted using the methods of z-transformation and modal control, an algorithm for the operational restructuring of the numerical parameters of the filter based on information about the time intervals of information receipt has been developed. A technique for calculating the restructuring algorithm based on the preliminary formation of special recalculation matrices is proposed. At the stage of real-time filter operation, the matrix data is used to recalculate the filter parameters, i.e., algorithm constants (2). A model example shows that the proposed approaches make it possible to ensure the constancy of the temporal characteristics of the digital filter with a high accuracy with a significant change in the periods of information receipt. At the same time, the implementation of the proposed approaches does not require significant computing resources on board and can be carried out on 8–16-bit microprocessor devices.

2 The theory of constructing a customizable digital filter

Let us form the design structure of a customizable digital filter adjusted to the model of a continuous reference dynamic system that sets the basic properties of signal processing.

Let the reference transfer function We(s) of a continuous filter be given, the discrete impulse response of which must be implemented in the digital filter W(z,TN). Then, for a physically implemented filter (the degree m of the numerator l is not greater than the degree n of the denominator), such a filter can be represented as a continuous dynamic system with one input u and one discrete output y, and described by the equations of state and observation in the form:

$$\dot{x}=A\cdot x+B\cdot u, \quad (3)$$
$$y=C\cdot x+G\cdot u,$$

where x is an n-dimensional state vector, u is a scalar input signal, constant numerical matrices A, B, C, G have sizes (n×n), (n×1), (1×n) and (1×1) respectively and are determined by pre-selected constant coefficients a_(ij) and bj , c_(i), g .

We choose matrices A, B, and C so that {A,B} form a controlled pair, i.e., for them, the conditions of complete controllability [1] were satisfied, and the matrices {A,C} constitute an observable pair, i.e. obey the conditions of observability [1,5].

An example of such a choice is the choice of the modal matrix A=((diag λ_(i))) and nonzero components of the matrix B and C. Another approach uses the controllability form [1] of equations (3), which, for example, for the 3rd order, is given by such a structure matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C=[c_1, c_2, .. c_n].$$

where $a_i \neq 0$ are the coefficients of the characteristic equation of the matrix A. In this case, the transfer function of the system is defined as:

$$We(s) = D0(s) = \frac{b_0 + b_1 \cdot s + b_2 \cdot s^2 + b_3 \cdot s^3 + \dots + b_{n-1} \cdot s^{n-1} + b_n \cdot s^n}{1 + a_1 \cdot s + a_2 \cdot s^2 + \dots + a_{n-1} \cdot s^{n-1} + a_n \cdot s^n}.$$

The solution of differential equations of state for constants A and B will take the form:

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B \cdot u(\tau) \cdot d\tau, \quad (4)$$
$$y(t) = C \cdot x + g \cdot u(\tau).$$

If we consider the Dirac impulse function δ(t) as an input signal u(t), and consider the initial conditions x(0) to be zero, then the solution y(t) from (4) with such an input signal will correspond to the impulse response w(t) filter:

$$w(t) = C \cdot e^{A \cdot t} B + g \cdot \delta(t) \quad . \quad (5)$$

In what follows, we will consider the filter to be inertial, i.e. n>m, whence in (5) we get g=0.

The Laplace image L{w(t)} is W(s) - transfer function, which has the form:

$$W(s) = C \cdot (E \cdot s - A)^{-1} \cdot B = L\{C \cdot e^{A \cdot t} \cdot B\}. \quad (6)$$
$$w(t) = L^{-1}\{C \cdot (E \cdot s - A)^{-1} \cdot B\}.$$

The lattice impulse function, or DIFT, from (6) corresponds to a set of points w(t) at quantization times t=t_k=k·N ·T_0 , it is determined by the expression:

$$w[k \cdot TN] = C \cdot e^{A \cdot T_0 \cdot N \cdot k} \cdot B. \quad (7)$$

By setting successively integer values k=0, 1, 2, .. you can get a set of specific DIFT values:

$$w[0] = C \cdot B, \quad w[TN] = C \cdot e^{A \cdot T_0 \cdot N} \cdot B, \; \dots \; w[k \cdot TN] = C \cdot e^{A \cdot k \cdot T_0 \cdot N} \cdot B. \quad (8)$$

On the other hand, the DIFT can be defined in terms of the discrete (z-) transfer function of the discrete filter We(z), which is calculated as the z-transform of the lattice impulse transition function. Such a relationship is defined by the expression:

$$We(z) = Z\{ w[k \cdot T_0]\} = Z\{ L^{-1}\{C \cdot (E \cdot s - A)^{-1} \cdot B\} \; when\; substituting\; t = k \cdot TN\}. \; (9)$$

We will form a digital filter at an arbitrary frequency fN=1/TN so that the points of its DIFT lie exactly on the ITF of the continuous reference filter, i.e. digital filter to match the desired time response at the current TN value. Let the transfer function of such a filter have the form:

$$WeN(z, TN) = D0N(z) = \frac{AN(z^{-1})}{BN(z^{-1})} = \frac{bN_0 + bN_1 \cdot z^{-1} + bN_2 \cdot z^{-2} + \dots + bN_{n-1} \cdot z^{-n+1} + bN_n \cdot z^{-n}}{1 + aN_1 \cdot z^{-1} + aN_2 \cdot z^{-2} + \dots + aN_{n-1} \cdot z^{-n+1} + aN_n \cdot z^{-n}}, (10)$$

Then such a filter for times t_k=k·TN corresponds to a finite difference equation of the form (2) with the notation $\{a_i, b_i\}$ replaced by $\{aN_i, bN_i\}$ for the coefficients (2).

The task of generating a tuning algorithm to ensure the reference nature of the time characteristic (or the coefficients z of the transfer function that uniquely determine the DIFT) - this complete task is divided into 2 related subtasks:

Ensuring a given type of free movement corresponding to certain roots of the characteristic equation, i.e., z-transfer function poles is a typical modal control problem.

Ensuring a given type of lattice DITF.

Synthesis of modal control. Here, task 1 described above is the most important, since the arrangement of the poles provides the basic dynamic properties, including the stability property, as well as the stability margins of the digital filter.

We will formulate this problem as a problem of synthesis of modal control.

3 Ensuring a given type of DITF time characteristic

To perform this task, it is necessary to calculate the specific values of the coefficients of matric C, which determine the output signal of the digital filter, with the calculated coefficients $\{aN_i\}$ of the denominators of the z-transfer function. To do this, one should first calculate the coefficients $\{bN_i\}$ of the numerators using the difference equation (2) of a linear discrete filter of the nth order. Considering in (2) the pulse signal at the input under zero initial conditions y[-n]= y[-n+1]=..= y[0]=0 and using the recurrent solution [1,6], we successively obtain specific numerical values of the DITF w[k] . The system of equations for calculating the coefficients $bN_i$of the numerator of the digital filter transfer function from the known values of the IRF at times k·TN and the known coefficients $aN_j$of the denominator can be reduced to the following Gaussian form:

$$bN_0 = w[0]\,,$$
$$bN_1 = w[1] + aN_1 \cdot w[0],$$
$$..\qquad (11)$$
$$bN_n = w[n] + aN_n \cdot w[0] + aN_{n-1} \cdot w[1] + .. + aN_2 \cdot w[n-2] + aN_1 \cdot w[n-1]\,.$$

Such an algebraic system of linear (with respect to unknown parameters $\{bN_i\}$) is simply solved by sequential substitution. On the other hand, the specific values of the points w[i] of the time characteristic for the moments of time t_k=k·TN can be calculated for a given value of the discrete step TN and the known transition matrix ΦN=$e^{A\cdot TN} = e^{A\cdot T_0\cdot N}$according to formula (7) in the form:

$$w[k \cdot TN] = C \cdot e^{A\cdot k\cdot TN} \cdot B, \qquad (12)$$

where C, B - are matrices defining a continuous standard (8).

Thus, by calculating the value w[k·TN] of the time function and substituting these values into system (11), we can determine the desired parameters bN_i of the numerator z, the transfer function of the digital filter.

Based on the calculated parameters $\{ aN_i, bN_i\}$ a specific digital filter can be implemented. An economical implementation using the direct programming method [1,6] is determined by the equations for converting the input signal u[k] into the output signal y[k] using auxiliary coordinates x_i in the following form:

$$x_1[k] = x_2[k-1],$$
$$x_2[k] = x_3[k-1],$$
$$\ldots \qquad (13)$$
$$x_{n-1}[k] = x_n[k-1],$$
$$x_n[k] = -aN_1 \cdot x_n[k-1] - aN_2 \cdot x_{n-1}[k-1] .. - aN_n \cdot x_1[k-1] + bN_n \cdot u[k-1]\quad .$$

The output signal y[k] is calculated (for b_n=0) by the expression:

$$y[k] = (bN_1) \cdot x_n[k] + (bN_2) \cdot x_{n-1}[k-1] .. + (bN_n) \cdot x_1[k-1]. \qquad (14)$$

This implementation method requires TN (2·n+1) multiplications, 2·n additions and several transfers to be performed at each step.

4 Technique of formation of the law of restructuring of filter parameters

To implement the restructuring of the digital filter parameters when the step T_0 of the discreteness changes to the current value TN, a sequence of steps is proposed, consisting of two stages: 1) preliminary calculations at the design stage,
2) operational calculations when the filter is running in real time.

Stage 1. The required model of a continuous analog of a digital filter is formed in the form of equations of state (1) - matrices A, B, C, G, based on the specific purpose and requirements for the dynamic characteristics of the filter - this is a standard task for designing digital filters [6,9]. The value of the interval T_0 is selected, corresponding to the middle of the possible range: T_0=(maxTN+minTN)/2 . A matrix model of the reference digital filter for step T_0 is built, based on the calculation of the matrix exponent in the form of expansion in terms of eigenvalues λ_i For this, the Sylvester formula [5] is used, which defines the representation of a function from a matrix as a composition of functions from the eigenvalues λ_i. This approach makes it possible to reduce the complexity of calculations on board at the 2nd stage. For simple $\lambda_i \neq 0$ we get.

$$\Phi 0 = e^{A\cdot T_0} = \sum_i^n F_i \,.\, e^{\lambda_i \cdot T_0},\ D0 = \sum_i^n (F_i \,.\, e^{\lambda_i \cdot T_0} - E)\cdot A^{-1}\cdot B = \sum_i^n (G_i \,.\, e^{\lambda_i \cdot T_0} - L).$$

Where $F_i = \Pi_{j\neq i}^n \dfrac{(A-\lambda_j * E)}{(\lambda_j - \lambda_i)}, G_i = F_i * A^{-1} * B, L = A^{-1} * B$ - are constant matrices (n×n), (n×1), (n×1) respectively and are defined by a set of n^2+2n numbers.

The n coefficients $a_i$of the characteristic polynomial of the reference filter are calculated. All computed numeric data - n^2+3n numbers are loaded into the calculator's memory.

Stage 2. Directly in the course of work, upon receipt of information signals intended for processing by a digital filter, the time interval TN of data receipt is measured, for example, as described in [4], the multiplicity factor N= TN/T_0 . If it differs exceeds 3±5% from 1, restructuring is performed in the following sequence. According to this coefficient, the values of the matrix exponent are recalculated:

$$\Phi NT = e^{A\cdot T_0 N} = \sum_i^n F_i \,.\, e^{\lambda_i \cdot T_0 \cdot N}, \quad DNT == \sum_i^n (G_i \,.\, e^{\lambda_i \cdot T_0 \cdot N} - L).$$

Here the calculations are: n calculations $e^{\lambda_i * T_0 * N}$, n multiplications (nxn) of a matrix by a number, n additions (nxn) of matrices, n multiplications of a vector by a number, n additions of vectors. The obtained data determine the matrix difference equations of the digital filter.

To simplify calculations at each data processing step, it is advisable to reduce the filter program to a compact calculation system (13), (14). In this case, the rearranged coefficients aN_i of the difference equation (2) of the filter are calculated using the Leverrier-Faddeev algorithm [5], (n-matrix multiplications (nxn), n-matrix trace calculations, n-matrix additions (nxn)). Then the values of n points w[k] of the lattice DITF are determined using formulas (8). The obtained point values are used to calculate the coefficients bN_i of the numerator of the z-transfer function of the digital filter. When replacing the old values of the coefficients with new ones - { aN_i, bN_i } the filter is rebuilt and these values of the coefficients are used further in calculations when processing the input signal according to algorithms (13), (14).

5 Model calculation example

These approaches were used to construct the restructuring law in the problem of controlling the lateral movement of an unmanned aircraft of an aircraft scheme. For implementation, a 6th order digital filter was chosen that implements a given DITF (Fig. 1).

This filter provided the suppression of the high-frequency component and the implementation of a proportionally differential (PD) control law. To calculate the restructuring law at stage 2, it was necessary to use 6 recalculation matrices - 216 16-bit numbers, in the process of one-time adaptation, 108 multiplications, 10 additions of 16-bit floating-point numbers were performed, with the accuracy of representing the results - 4 decimal digits. The results of the restructuring of the digital filter with good calculation accuracy are illustrated in Fig. 1b, where the dotted line defines a continuous ITF, the points marked with the symbol **x** lie on the DITF for T_0=0.1, the symbol **o** defines TN=0.5, the symbol + corresponds to TN=0.25. As can be seen from the figures, tuning with high accuracy ensures the constancy of the temporal characteristics of the digital filter with a significant (5 times) change in the periods of information receipt.

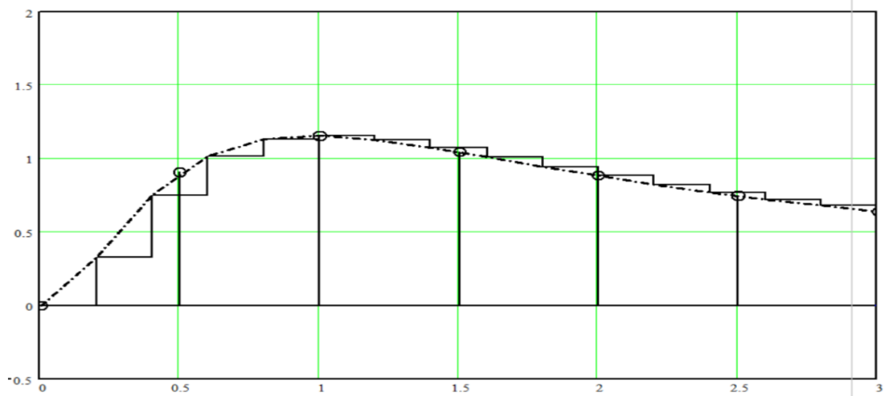

Fig.1. Transition functions: reference filter (dash-dotted line), discrete filter TN=0.2 s (stepped line), discrete filter TN=0.5 s (points **0**).

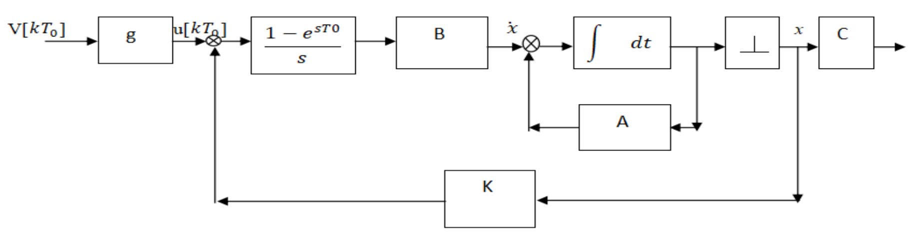

Fig. 2. The equivalent structure of a digital filter.

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