

# Synthesis of a Controller Built on the Principle of Inverse Dynamics and Identification of a Nonlinear Model of Longitudinal Motion in Offline Mode

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**Abstract.** This report is dedicated to the design of a controller based on the principle of inverse dynamics and the offline identification of the coefficients of a supersonic transport (SST) aircraft's linear longitudinal dynamics model based on the results of measuring the nonlinear mathematical model's responses. A landing task under the influence of atmospheric disturbances is considered.

## 1 Introduction

In 2020, the World-Class Research Center (WCRC) “Supersonic” was created in Russia, led by Central Aerohydrodynamic Institute (TsAGI), under which the development of a fundamental basis for the future creation of an advanced second-generation supersonic transport (SST) began.

Supersonic transports have specific dynamics due to their layout features and, as a result, flight modes that are very different from subsonic aircraft, especially during the approach and landing phases. Due to the landing phase itself being one of the most difficult stages of piloting [1-4], it is necessary to simplify the landing task for the pilot. A rational approach would be to create “comfortable” piloting conditions for the pilot. This can be achieved by reducing the controlled element dynamics to a simpler form, designing the control system based on the principle of “inverse dynamics”, which ensures the simplest nature of pilot action. Since to provide the correct performance of a control system based on inverse dynamics it is necessary to know the dynamics of the controlled element exactly, it is also necessary to provide the robustness of this system.

This also includes the so-called “friendly” interfaces – control inceptors and means of displaying visual information that do not prompt the pilot to take wrong actions, on the contrary, correcting their actions when the properties of the controlled element deteriorate.

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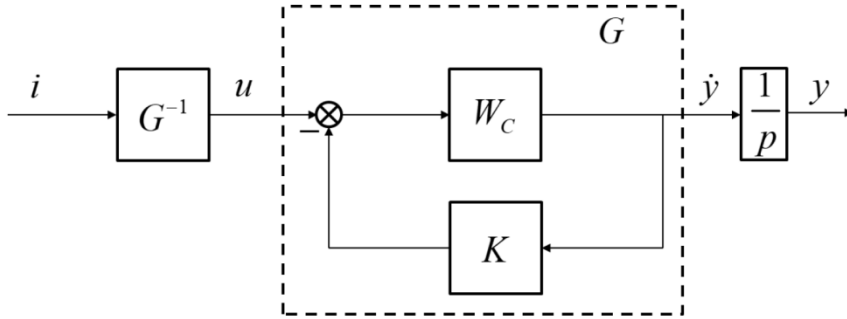
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## 2 Synthesis of a control system using the principle of inverse dynamics

### 2.1 Design of a control system based on the principle of inverse dynamics

The synthesis of a control system with inverse dynamics requires that the controlled element be stable. In this case, stability can be ensured by introducing feedback.



**Fig. 1.** Block diagram of a control system based on the principle of “inverse dynamics”.

Matrix  $G$  is quadratic, i.e., the number of input control actions is equal to the number of output coordinates. On the main diagonal of the matrix  $G(p)$  are the transfer functions that connect the controlled phase coordinates and the corresponding input signals. Outside the main diagonal are the transfer functions that describe the cross-coupling. Matrix  $G^{-1}$ , the inverse of matrix  $G$ , is an “inverse dynamics” filter matrix. A feature of the elements of this matrix is that the order of the denominators in the transfer functions of the elements of matrix  $G$  is greater than the orders of the numerators. This results in the problem of the feasibility of matrix  $G^{-1}$ . To solve it, it was proposed in [9, 10] to introduce a second-order link into each element of the matrix. This link equalizes the orders of the numerator and denominator. Having thus suppressed the dynamics with respect to the derivatives of the desired phase coordinates, the connections between the phase coordinates and pilot inputs will be described by integrators.

### 2.2 Linearization of the SST nonlinear mathematical model

Further work required linearizing the used nonlinear model of SST spatial motion and approximating the obtained frequency characteristics of the aircraft by mathematical modeling. To measure them, we used a mathematical complex developed at Moscow Aviation Institute based on the universal method of Fourier coefficients, which is described in detail in [11, 12, 16]. This method involves the use of a polyharmonic input signal

$$i(t) = \sum_k A_k \cos \omega_k t$$

where  $\omega_k = k\omega_0$ . The input signal amplitudes are chosen by the method described in [16] so that the power distribution over the frequencies of such a signal corresponds to the power distribution of a signal described by the following form:

$$S_{ii}(\omega) = \frac{K^2}{(\omega^2 + \omega_i^2)^2}$$

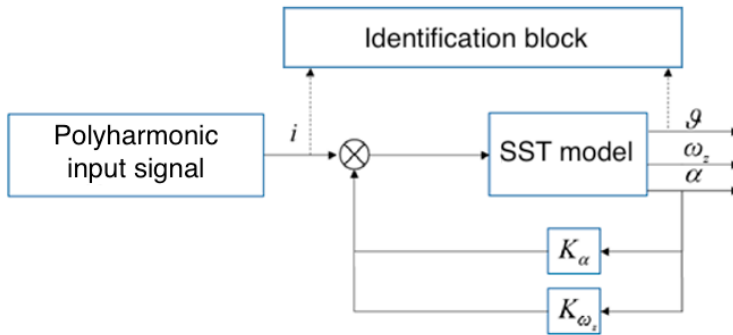
where  $k$  is an integer, and  $\omega_0 = \frac{2\pi}{T}$  ( $T$  being the duration of the experiment), allows, by the preliminary measurement of the Fourier coefficients

$$a_k^{(\bullet)} = \frac{2}{T} \int_0^T (\bullet) \cos \omega_k t dt;$$

$$b_k^{(\bullet)} = \frac{2}{T} \int_0^T (\bullet) \sin \omega_k t dt,$$

where  $(\bullet)$  is any of the measured signals, to determine with high accuracy the describing function of the operator, their noise  $S_{n_e n_e}$ , and all other characteristics of the pilot-aircraft system.

According to the described method, a polyharmonic input signal consisting of a sum of sinusoids at orthogonal frequencies is applied to the system input. As a result, the ratio of the Fourier transforms on a finite time interval of the signal  $i$  to the signal  $g$  at the frequencies of the input signal is determined.



**Fig. 2.** Block diagram of pitch angle frequency response identification.

The pitch angle frequency response obtained after identification was approximated by a link with the following structure:

$$\frac{x(1)(x(2)p+1)}{p(x(3)^2p^2+2x(3)x(4)p+1)} \quad (1)$$

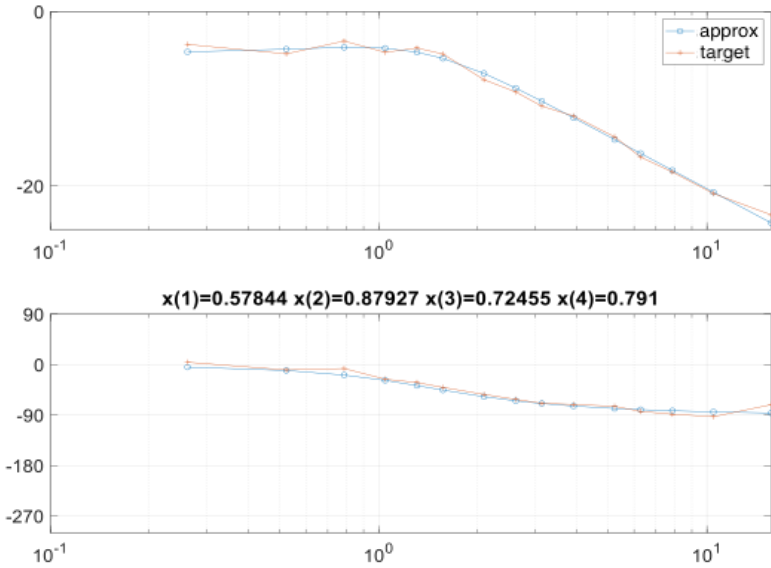
During the approximation, the problem of finding the global minimum of the functional  $J = \sum_{k=1}^n (A_{targ}^k - A_{appr}^k)^2 + \mu(\varphi_{targ}^k - \varphi_{appr}^k)^2$  is solved, where the parameters of the chosen structure are optimized.

### 2.3 Design of the controller for a specific flight mode

To provide the desired pitch angle dynamics close to the integrator  $\frac{1}{p}$  [11], the frequency response of pitch rate in the landing configuration was obtained using the mathematical complex developed at MAI based on the universal method of Fourier coefficients [11, 12] described earlier. Further, the approximation of this frequency response was carried out using the following structure:

$$\frac{x(1)(x(2)p+1)}{x(3)^2p^2+2x(3)x(4)p+1}$$

Consequently, the approximate frequency response of pitch rate was obtained.



**Fig. 3.** Approximate pitch rate frequency response in the landing configuration.

After that, an inverse dynamics controller for this flight mode was obtained, which has the following form:

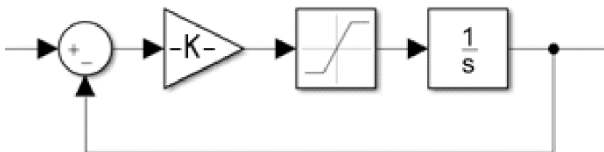
$$W_{G^{-1} * W_F} \Big|_{V=\frac{262 \text{ km}}{h}}(p) = \frac{0.525p^2 + 1.146p + 1}{0.005086p^3 + 0.1075p^2 + 0.6243p + 0.5784}$$

### 3 Ground-based simulation

#### 3.1 Studies performed using MAI’s ground-based simulator

The studies were carried out using MAI’s workstation with the pilot performing the task of compensatory tracking in the longitudinal channel while controlling the element which is described by the pitch angle transfer function. The goal of the operator was to minimize the current tracking error by moving the control inceptor. The input disturbance was a polyharmonic signal.

Since the mathematical model of the SST does not take into account actuator dynamics, a non-linear actuator model was introduced, shown in Figure 4.



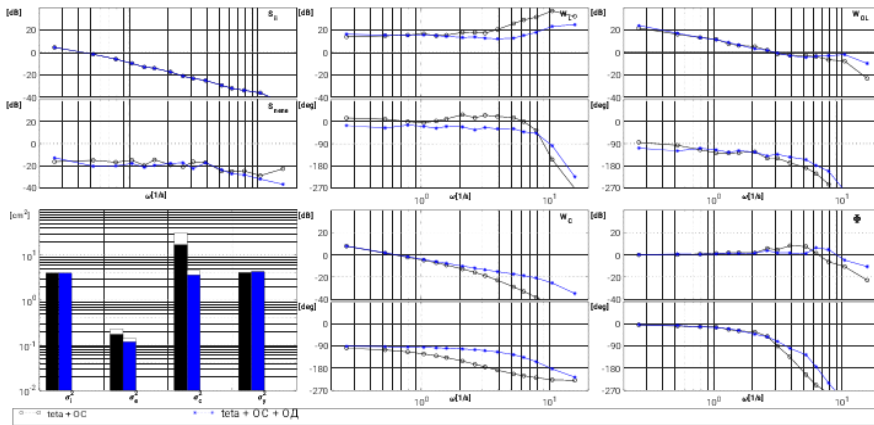
**Fig. 4.** Block diagram of the nonlinear actuator model.

Under the conditions of performing a high-precision piloting task with unsatisfactory controlled element dynamics, a number of characteristics of the pilot-aircraft system can become significantly degraded, which, as a result, can lead to a pilot-induced oscillation (PIO) event because of the actuator reaching the control surface deflection rate limits. Due

to this peculiarity, a number of experimental studies were carried out on the introduction of a nonlinear prefilter to prevent the PIO phenomenon described in [13]. The structure of the nonlinear prefilter is a simplified model of a nonlinear actuator, which takes into account the limits of control surface deflection, installed in the control system loop between the command input from the pilot and the control actuator.

The time processes obtained during the experiments revealed that the required deflection rates of the control surface reach high values that cannot be processed by the actuator, leading to PIO. The introduction of a prefilter makes it possible to reduce the required actuator deflection rates, the time to reach the actuator deflection rate limits from >20% to 1%, and the tendency for PIO.

Further, as a result of a series of experiments with and without a built-in inverse dynamics controller, the integral and frequency response characteristics of the pilot-aircraft system were obtained.



**Fig. 5.** Frequency response of a system with and without a built-in inverse dynamic controller.

Based on the results of ground-based simulation, it can be concluded that the use of an inverse dynamics controller in the pitch control system improves piloting accuracy by 1.6 times and reduces the control stick force by 6.8 times.

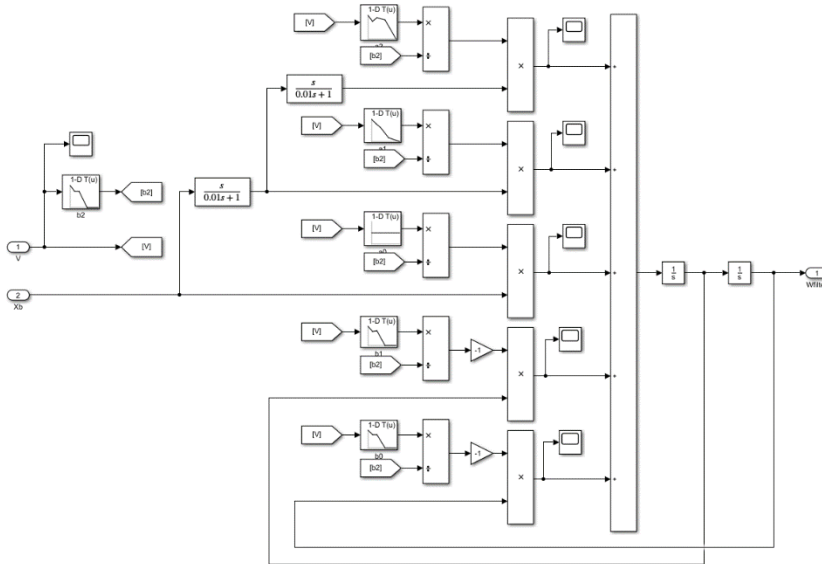
### 3.2 Experimental studies performed using the MAI ground-based simulator

Research involving a ground-based simulator and a nonlinear model requires using an inverse dynamics controller with variable coefficients, which needs to be determined in various flight conditions.

To design a controller with variable coefficients, it is also required to obtain the frequency response using the Fourier coefficient method [11, 12]. In addition, based on the results of ground-based simulation, a structure was chosen that most closely describes the controlled element dynamics, which has the following form:

$$\frac{x(1)}{x(2)^2 p^2 + 2x(2)x(3)p + 1}$$

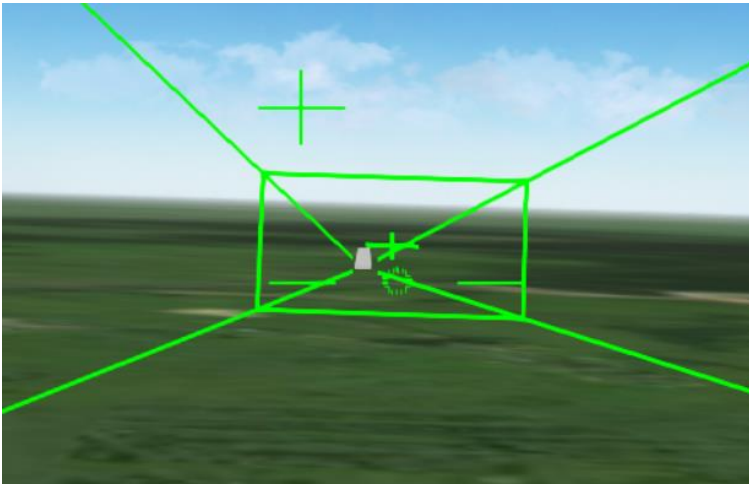
Next, transfer functions were obtained and a system was formed for the implementation of a controller with variable coefficients (Figure 6).



**Fig. 6.** Block diagram of an inverse dynamic's controller with variable coefficients.

Studies involving the performance of a landing task under atmospheric disturbances [14] were carried out using the ground-based simulator of MAI's Pilot-Vehicle Lab with a motion cue simulating system. Dryden's turbulence model [15] was used to simulate wind disturbances.

For additional support of the pilot during the approach, a configuration using a “pursuit and predictive” display developed in the Pilot-Vehicle Lab [5] was considered, shown in Figure 7.

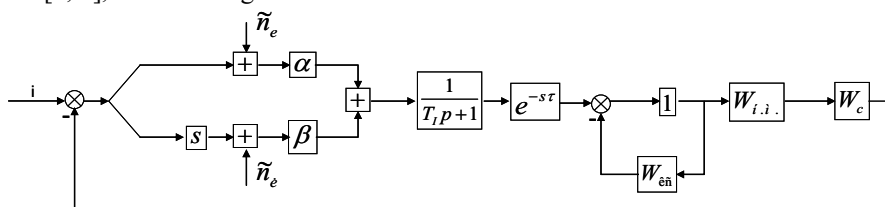


**Fig. 7.** Predictive display.

The “pursuit and predictive” display shows additional information on the screen, which includes the velocity vector indicating the aircraft's direction of flight, as well as a 3D corridor with the aircraft's velocity.

The choice of display parameters must be carried out based on an analysis of the pilot-aircraft system, by performing mathematical modelling using a model of pilot control input

characteristics. The mathematical modelling of the system was done using a refined Hess model [5, 6], shown in Figure 8.



**Fig. 8.** Refined Hess model used in mathematical modelling.

When using the “pursuit and predictive” display, the controlled element is the velocity vector projected onto the predictive plane.

Recently, a number of studies have been carried out in MAI’s Pilot-Vehicle Lab [7, 8], which have shown that for the aircraft under consideration, the following law of sighting angle control should be chosen:

$$\varepsilon = \theta_{pr} + \frac{\Delta H}{L}$$

## 4 Conducting experimental studies and analysing of the obtained data

When performing experimental studies in the landing phase, 4 configurations of the controlled element with different control systems were considered: a controlled element with a control system with feedback without/with an integrated “pursuit and predictive” display with/without a feedback dynamics controller under atmospheric turbulence conditions. After a series of experiments, an analysis of the obtained data was carried out.

**Table 1.** Compares the calculated RMSE values of various configurations.

Parameters	RMSE wind	RMSE wind + ID	RMSE wind + display	RMSE wind + ID + display
$\alpha$	0.731	0.412	0.326	0.304
$\gamma$	2.374	1.535	0.951	1.229
$\vartheta$	1.001	0.561	0.352	0.308
H	3.560	2.651	0.349	0.319
Z	4.943	4.506	0.439	0.379
V	3.589	2.944	2.145	2.098
ny	0.216	0.096	0.084	0.089
Thrust lever	0.050	0.028	0.034	0.034
X <sub>o</sub>	1.787	0.876	0.756	0.829
X <sub>B</sub>	1.068	0.348	0.549	0.491

It is not hard to see from the obtained results that when comparing control system configurations designed using only feedback with a system that includes an inverse dynamics controller, all RMSE values are reduced by a factor of more than 2, which indicates an improvement in accuracy, as well as piloting comfort.

With the additional use of a “pursuit and predictive” display in conjunction with an inverse dynamic’s controller, tracking accuracy improved by a factor of 11.2 and 13 in height and lateral coordinate, respectively.

## 5 Conclusion

In this paper, it was shown that the most important direction in aviation development is flight safety improvement. An analysis of the statistics of aircraft accidents showed that the most difficult and dangerous phase of flight is the approach and landing stages, especially in conditions of atmospheric turbulence.

The results of experimental studies showed that:

- Using the designed controller based on inverse dynamics principle brings the dynamics of control element close to the integrator.
- Built-in nonlinear prefilter reduces deflection rates of control actuator and as a result tendency of PIO.
- Additional using a perspective “pursuit and predictive” display with inverse dynamics controller improve accuracy and comfort of piloting under atmospheric disturbances.

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