# Algorithms for Selection and Combination of Sliding Intervals for Identification Based on Large Volumes of Flight Data 

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#### Abstract

This article considers approaches to processing large volumes of flight data using parametric identification methods. The problem is studied from the point of view of selecting informative areas based on a preliminary assessment of identification errors. The paper offers algorithms for combining estimates of pro-cessing intervals for the problem of signal recovery while ensuring its continuity. Their performance is presented by the processing of data obtained at a simulation bench.


## 1 Introduction

When analysing flight data, one has to deal with data defects such as heterogeneity, faults, sensor failures etc., which emphasizes the importance of preprocessing, as well as restoring lost data [1]. For successful identification careful selection of the most suitable data areas and the involvement of additional information from neighbouring areas is required to minimize estimation errors [2].

This article is devoted to research of possible criteria for selecting the most promising data for subsequent analysis. It compares the criteria based on their performance using as an example the problem of aircraft orientation angles recovery. The solution is based on the assumptions that the values of the projections of aircraft's speed (for example, according to measurements of a satellite navigation system) in the normal Earth's coordinate system and aircraft overloads in the body-fixed coordinate system are known.

The solution to the problem is found using direct methods of optimal control, where the angles fulfil the task of control signals. This approach is relatively new if compared with traditional methods of stochastic filtering [3]. The signals are approximated by cubic splines, the coefficients of which are found using Newton's method. In addition, the paper considers ways to combine the results from different processed areas. The data used in this work was obtained using a simulation bench.

## 2 Problem statement

[^0]Let us assume that the values of the projections of the aircraft's speed in the normal Earth's coordinate system and its overload in the body-fixed system are known. It is required to determine aircraft's angular position. This problem can arise either for miniature unmanned aerial vehicles that are not equipped with angle and angular velocity sensors, or when the operation of such sensors is disrupted.

Using relations between Earth velocity projections and fixed-body overloads [4] it is possible to determine angle values. To do this, we formulate orientation angles recovery as a problem of finding optimal control. Their values are determined using the direct method [5, 6], for which the signals are presented in the form of Hermitian cubic splines [7]. We use a modification of Newton's method to calculate spline coefficients.

For a problem formulated in this way, it is necessary to determine criteria that allow one to evaluate in advance the quality of the obtained solutions.

### 2.1 Mathematical model

In this problem, the mathematical model of the object [8] is formed by expressions relating overloads to accelerations in the body-fixed coordinate system

$$
a_{y}=g\left(n_{y}-\cos \vartheta \cos \gamma\right), \begin{gather*}
a_{x}=g\left(n_{x}-\sin \vartheta\right) \\
(1)  \tag{1}\\
a_{z}=g\left(n_{z}+\cos \vartheta \sin \gamma\right)
\end{gather*}
$$

where $\vartheta, \gamma$ - angles of pitch an roll,
equations of transition from the body-fixed system to the normal Earth's system [4], which also significantly depend on the values of pitch and roll, and the acceleration integral in the normal Earth's coordinate system

$$
\begin{equation*}
V_{g}=\int \vec{a}_{g} d t \tag{2}
\end{equation*}
$$

Note that the yaw angle is considered known.
The obtained speed values are compared with the given ones, for which we use a quadratic mismatch functional given below

$$
\begin{equation*}
J=\sum_{i=0}^{N}\left(\widehat{V}_{g}\left(t_{i}\right)-V_{g}\left(t_{i}, a\right)\right)^{\mathrm{T}}\left(\widehat{V}_{g}\left(t_{i}\right)-V_{g}\left(t_{i}, a\right)\right) \tag{3}
\end{equation*}
$$

where $\hat{V}_{g}$ - vector of measured values of three speed projections in normal Earth's coordinate system, $V_{g}\left(t_{i}, a\right)$ - vector of speed values recovered by the model, which depends on spline parameters; $a$ - vector of spline parameters which should be identified, $\mathrm{N}-$ number of measurements.

### 2.2 Mathematical model

To determine the preliminary quality of solutions to the signal recovery problem, the following characteristics of a matrix composed of derivatives of the functional (3)

$$
\begin{equation*}
\frac{\partial V_{g}{ }^{T}}{\partial \vec{a}} \frac{\partial V_{g}}{\partial \vec{a}} \tag{4}
\end{equation*}
$$

were used

- matrix (4) diagonal elements that are proportional to the variances of the estimated parameters;
- condition numbers for the matrix (4) (here taken as the ratio of the largest eigenvalue to the smallest);
- determinant for the matrix (4)


## 3 Description of experiments

The research consisted of searching for patterns between the quality of the solution to the signal recovery problem and the values of the above-mentioned criteria.

To conduct computational experiment the values of overloads in the body-fixed coordinate system were fed into the model (1)-(2), velocity projections in the normal Earth's system were determined, based on which the mismatch functional was calculated according to formula (3).

Newton's method selected the values of the spline parameters that minimized the objective functional, and the angle values calculated using the spline formula were returned to the model.

One of the advantages of using Newton's method is that, due to its use of a matrix of second derivatives, the calculation of the selected criteria occurs during the working step of the algorithm. Calculations were performed with a zero initial approximation.

Under specified conditions, it was often possible to achieve very good signal recovery, as can be seen in Figure 1.


Fig. 1. Values of pitch angle on a processed interval (given - blue line, and recovered - violet one).
Comparison of different intervals shows that it is not possible to determine the quality of a solution based on condition numbers. Their estimate is too rough for considered problem.

Estimating proximity of the matrix (4) determinant to zero provides better results, which allows one to identify areas containing noticeable errors quite accurately, as can be seen from the comparison of two areas made in the Table 1.

Table 1. Values of matrix determinant for processed areas and errors in pitch angle values.

| Area <br> number | Value of <br> determinant (4) | Standard deviation of mismatch <br> for pitch angle, degree |
| :---: | :---: | :---: |
| 1 | 303.6 | 52 |
| 2 | $3 \cdot 10^{73}$ | 1.87 |

However, the value of the determinant gives only a general characteristic of the solution for the interval, providing no clue to the quality of the approximation for individual nodes. One has to consider the values of the diagonal elements of (4), as shown in Figure 2, in order to
obtain more detailed information about the solution accuracy.


Fig. 2. Estimates of diagonal elements of matrix (4) for pitch angle signal.


Fig. 3. Standard deviation of mismatch of pitch angle around spline node.
Comparison of Figures 2 and 3 shows that from the diagonal elements, one can obtain information about the qualitative nature of the mismatches (for example, an increase in the error towards the edges of the area or a larger error near the second or third node). Nonetheless, the errors themselves are not proportional to the diagonal elements.

As the experiment shows, the above-mentioned increase in error at the edges of the area occurs quite often. It can be overcome by shifting the processing interval so that the point of interest moves inside it. Due to the inherent properties of the spline, estimate of the function and its derivative for the neighboring node is sufficient to extrapolate the signal while maintaining its continuity and smoothness [7].

Thus, it is possible to obtain additional estimates of the values of the function and its derivative at selected points if the interval is processed with a sliding window with an offset selected so that the nodes at different windows coincide with each other. Out of several estimates, the best one can be selected based, for example, on the minimum value of the standard deviation, which can be derived from the corresponding diagonal element.

## 4 Conclusion

Under the studied conditions (absence of noise, known yaw angle), it is possible to recover the values of the aircraft orientation angles with sufficient accuracy.

Among the proposed criteria, the determinant of the matrix (4) and its diagonal elements are of greatest interest. The first criterion gives a generalized characteristic of the entire interval; the second criterion characterizes local differences in the accuracy of signal recovery. Although the use of criteria allows a preliminary assessment of the solution only at a qualitative level, it helps to localize sites of significant errors in the processed area.

Signal estimates for individual intervals, due to the properties of splines, can be easily extrapolated and brought into consistency. Adjusting the lengths of sliding windows and their offset ensures continuity of signal processing.

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