E3S Web of Conferences **446**, 06005 (2023) https://doi.org/10.1051/e3sconf/202344606005
HSTD 2023

Synthesis of Control Law Based on Nonlinear Dynamic Inversion for Supersonic Aircraft in Presence of Noise and Uncertainties

Dhiman Gaurav[1,*], *Goro* Sekou[1] and *Moung Htang* Om[1]

[1]Moscow Aviation Institute, Moscow 125993, Russia

Abstract. In this paper, the approach for synthesis of control law based on nonlinear dynamic inversion (NDI) via feedback linearization combined with Proportional–integral–derivative (PID) controller in outer loop to control supersonic aircraft in its most challenging flight task, i.e., landing in the presence of uncertainty in knowledge of model, measurement noise is developed. The results obtained demonstrate higher control accuracy, lower pilot workload compared to PI in tracking task of pitch angle of a supersonic passenger aircraft in presence of uncertainties, noise and in case of control surface faults.

1 Introduction

The supersonic passenger aircraft as per previous experience has peculiar piloting issues. Maintaining the desired climb profile was a full time, high workload task due to the inherent flight path sensitivity to small pitch attitude changes, poor outside visibility, and the need for frequent pitch trim changes due to centre of lift shifting and CG adjustments as the Mach number increased. In addition, the location of the instantaneous centre of rotation near the cockpit deprived the pilot of motion cues due to pitch rate. Frequent reference to display was found to be essential for smooth control of climb and descent profiles [1].

To improve flying qualities qualitatively to level 1 for supersonic aircraft we compared various control methods. The dynamic inversion improves the controller performance qualitatively in tracking tasks, increasing tracking accuracy and decreasing pilot workload. To realize dynamic inversion based on filters is not considered, as it is a cumbersome process requires very accurate knowledge of model, requires gain scheduling and is based on a linear model. It also requires additional filters, as the aircraft in longitudinal channel is not invertible by nature.

To realize the advantages of dynamic inversion and overcome the disadvantages the authors used dynamic inversion via feedback on linear model, and nonlinear dynamic inversion via feedback linearization on nonlinear model. It keeps the advantages of inversion, eliminating the need of gain scheduling. However, it requires full state feedback and accurate

[*] Corresponding author: gauravdhiman96@gmail.com

knowledge of the model. Dynamic inversion is a control design methodology that uses a feedback signal to cancel inherent dynamics and simultaneously achieve a specified desired dynamic response [2].

Measurement noise and not accurate knowledge significantly hampers the control performance. Among the papers cited in this work, measurement noise is removed using the frequency filter but none of them considered Kalman filter to remove noise [3]. To manage with measurement noise the authors suggest using Kalman filtering to remove high frequency measurement noise.

2 Nonlinear Dynamic Inversion

The Nonlinear Dynamic Inversion (NDI) was developed in the late 1970s to provide control of nonlinear systems, being applicable to a class of systems known as feedback linearizable [4]. It allows to generate a control input using a state diffeomorphism such that, when applied to the system, all the relations between a virtual control and the outputs of the system are reduced to simple integrators. For the resulting linear system, a single linear control law can be adopted without the need for gain scheduling to tune the controller for different conditions of the nonlinear system. A detailed explanation of this technique is presented, in [4].

To exemplify the working principle of the NDI, consider a system of order n with the same number m of inputs u and outputs y and affine in the control inputs. Furthermore, the outputs coincide typically to the control variables and are assumed to be physically similar (for instance, three attitude angles). The extension of the theory to more complex systems is rather straightforward. This type of system can be mathematically represented by

$$\dot{x} = f(x) + G(x)u \quad (1)$$

$$y = h(x) \quad (2)$$

Where f and h are vector fields in Rn and Rm, respectively, and G is a nxm control effectiveness matrix.

The procedure to obtain the feedback linearization for the inversion of the system consists of consecutive time differentiations of y until an explicit dependence on u appears [5]. To each derivative, a new state vector is associated, and the derivative of the last state vector is given by a nonlinear expression (the virtual control) to complete the transformation. If r time differentiations are required, r:m≤n is known as the total relative degree of the system. Moreover, if r:m<n, there are n−r:m degrees of internal dynamics, unobservable to the input–output linearization and which must be Bounded-Input Bounded-Output (BIBO) stable in the region of interest to assure the effectiveness of the controller.

Assuming now $h(x) = x$, the first-order time-derivative of y is given by

$$\dot{y} = \dot{x} = f(x) + G(x)u \quad (3)$$

Since an explicit dependence on u was already found, the linear relation ν= x dot can be imposed if det $G(x)$ ≠0 by selecting:

$$u = G^{-1}(x)(v - f(x)) \quad (4)$$

Besides performing the linearization of the system, this input also allows to decouple the responses of the control variables since each component of ν only depends on the same component of x.

The NDI and DI controller relies on an accurate description of f and G to cancel all the nonlinearities of the system. Nevertheless, if inaccuracies exist, the exact cancellation of the nonlinearities becomes impossible [6].

3 Flight Control law synthesis

DI synthesis is a controller synthesis technique by which existing undesirable dynamics are cancelled out and replaced by desired dynamics. Cancellation and replacement are achieved through careful algebraic selection of the feedback function [7]. For this reason, this method is also called feedback linearization. It applies to both single-input, single-output (SISO) and MIMO systems, provided the control effectiveness function (in the SISO case) or the control influence matrix (in the MIMO case) is invertible. The method works for both full-state feedback (input-state feedback linearization) and output feedback (input-output feedback linearization). A fundamental assumption in this methodology, is that plant dynamics are perfectly modelled and can be cancelled exactly [8]. In practice this assumption is not realistic.

Theoretical introduction to dynamic inversion:

$$\dot{x} = f(x, u) \tag{5}$$

$$y = h(x) \tag{6}$$

x: State Vector, u: control vector, y: output vector (for system with small perturbation the above function f is linear in u.

The above equation can be re written:

$$\dot{x} = f(x) + g(x)u \tag{7}$$

f: is a nonlinear state dynamic function, g: is a nonlinear control function. By assuming $g(x)$ is invertible for all values of x, the control law can be obtained from the above equation by subtracting $f(x)$ from both sides of the above equation, before multiplying both sides by $g - 1\ (x)$

$$u = g - 1\ (x)[\ \dot{x}\ - f(x)] \tag{8}$$

The next step is to command our aircraft to specified states. Instead of specifying the desired states directly, we can specify the rate of desired states by swapping x ˙with x ˙des.

Methodology to create NDI/DI Control law:

Step 1: Describe full set of nonlinear equation for the variable to be controlled (e.g., angular velocity, angle of attack)

Step 2: Describe $f(x)$ and $g(x)$ as mentioned in equation (7) for a given flight envelope.

Step 3: Check whether $g(x)$ is invertible for the flight envelope for which the flight control law is being developed.

Step 4: If $g(x)$ is invertible NDI law can be developed successfully, by calculating required control input using equation (4)

Step 5: NDI law can be used as the inner loop controller, which linearizes the plant.

Step 6: NDI + Aircraft transfer function is equivalent to integrator, if a full state feedback is available. After that a PID Controller can be created in the outer loop, to achieve the desired response from a nonlinear plant.

Mathematical model used for simulations is mentioned in [9]. For nonlinear simulations the mathematical model developed in World-Class Research Center “Supersonic” in 2020–2025.[10]

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0110 & 0.0433 & 1.7295 & -7.1876 \\ -0.0691 & -0.06975 & -7.0678 & -54.8976 \\ 0.00011 & 0.00116 & -0.35407 & 0.0911 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -0.4412 \\ -12.388 \\ -0.58446 \\ 0 \end{bmatrix} [\eta]$$

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_u & x_w & x_q & x_\theta \\ z_u & z_w & z_q & z_\theta \\ m_u & m_w & m_q & m_\theta \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} x_\eta \\ z_\eta \\ m_\eta \\ 0 \end{bmatrix} [\eta]$$

$$A = \begin{bmatrix} -0.0110 & 0.0433 & 1.7295 & -7.1876 \\ -0.0691 & -0.6975 & -7.0678 & -54.8976 \\ -0.0001 & 0.0012 & -0.3541 & 0.0911 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.4412 \\ -12.3880 \\ -0.5845 \\ 0 \end{bmatrix}$$

Converting from state space model to equations of motion [11]:

$$\dot{u} = x_u u + x_w w + x_q q + x_\theta \theta + x_\eta \eta \tag{9}$$

$$\dot{w} = z_u u + z_w w + z_q q + z_\theta \theta + Z_\eta \eta \tag{10}$$

$$\dot{q} = m_u u + m_w w + m_q q + m_\theta \theta + m_\eta \eta \tag{11}$$

$$\dot{\theta} = q \tag{12}$$

Creating the pitch rate single input single output DI controller:

$$x = q \tag{13}$$

$$u = \eta \tag{14}$$

$$f = \dot{q} = m_u u + m_w w + m_q q + m_\theta \theta \tag{15}$$

$$g = m_\eta \tag{16}$$

m_η is constant for a linear time invariant system, the inverse of the of the function 'g' is obtained as constant = $\left(\frac{1}{m_\eta}\right)$.

Now for finding the required elevator input for desired pitch rate response, the equation of pitch rate should be inverted. The following equation shows the law for calculation of required elevator input.

$$\eta = [\dot{q} - (m_u u + m_w w + m_q q + m_\theta \theta)]/m_\eta \tag{17}$$

Substituting $\dot{q}$ for desired $\dot{q}_{\,des}$

$$\eta = [\dot{q}_{\,des} - (m_u u + m_w w + m_q q + m_\theta \theta)]/m_\eta \tag{18}$$

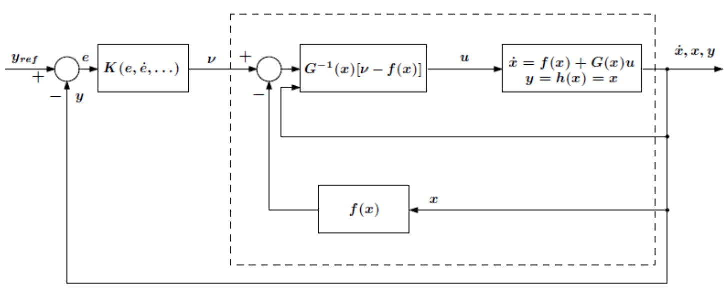


Fig. 1. Control scheme of NDI for multi-input multi output system.

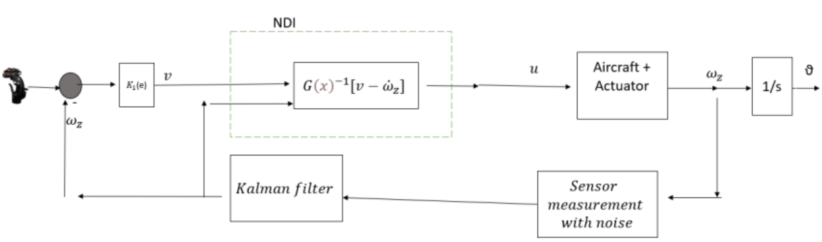


Fig. 2. Implementation of NDI + PI control for supersonic passenger aircraft.

The requirement of full state feedback in NDI + PI controller, made authors look at other variants of implementing the NDI controller [12]. The incremental nonlinear dynamic inversion was also considered.

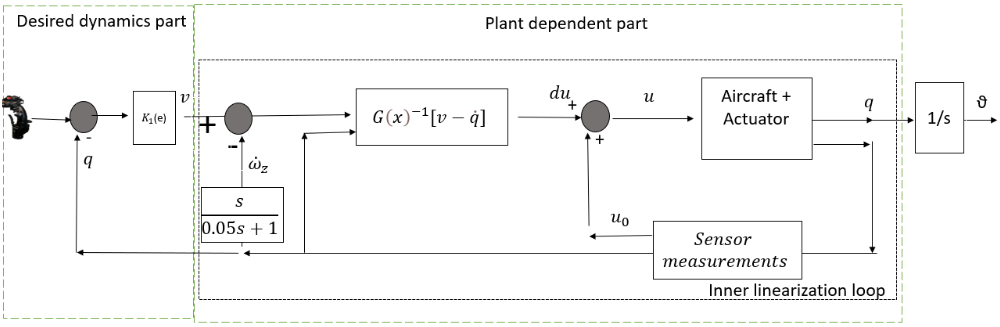


Fig. 3. Implementation of Incremental NDI + PI control for supersonic passenger aircraft longitudinal channel.

Incremental NDI overcomes the disadvantages of NDI, by not requiring full state feedback and reducing the dependence of control law on full mathematical model. [13]

4 Classic Kalman filter algorithm

To overcome practical problems using NDI due to measurement noise, the classic Kalman filter has been implemented within it.

The state space model of the system is presented as follows:

$$\begin{bmatrix}\dot{u}(k)\\ \dot{w}(k)\\ \dot{q}(k)\\ \dot{\theta}(k)\end{bmatrix} = F_k\begin{bmatrix}u(k-1)\\ w(k-1)\\ q(k-1)\\ \theta(k-1)\end{bmatrix} + B_k\begin{bmatrix}\eta(k)\\ \eta(k)\\ \eta(k)\\ \eta(k)\end{bmatrix} \quad (19)$$

The object model is defined as follows:

$$\dot{y} = f(y,u,t) \quad (20)$$

where,

$y^T = [\alpha, \vartheta, V]$- state vector

$u^T = [V, \omega_z]$ -control vector

The full observation model is defined as follows:

$$Z_1(t_i) = \alpha(t_i) + \varepsilon_\alpha(t_i) \quad (21)$$

$$Z_2(t_i) = \vartheta(t_i) + \varepsilon_\vartheta(t_i) \quad (22)$$

$$Z_3(t_i) = V(t_i) + \varepsilon_V(t_i) \quad (23)$$

$$Z_4(t_i) = \omega_z(t_i) + \varepsilon_{\omega_z}(t_i) \quad (24)$$

where,

$z^T = [Z_1(t_i), Z_2(t_i), Z_3(t_i)]$- sate vector

In the scope of this research work, the observation model is represented by (22)

Prediction Equations

The Kalman filter predicts next state of the system using these equations. They allow us to compute the new mean- the estimate of the state ($\hat{x}$) and the covariance (P) of the system.

$$\hat{x}_{k|k-1} = F_k\hat{x}_{k-1|k-1} + B_kU_k + \varepsilon_{k-1} \quad (25)$$

$$P_{k|k-1} = F_kP_{k-1|k-1}F_k^T + Q_k \quad (26)$$

$\boldsymbol{\hat{x}_{k|k-1}}$-the estimate for the state (mean) ($\hat{x}$) at step k given the estimate from step k-1.

$P_{k|k-1}$-the state covariance matrix (P) at step k given the state covariance matrix from step k-1

F_k-the state transition matrix at step k (for a given time step), B_k is the control function at step k and U_k is the control input at step k; $B_k U_k$ is the contribution of the controls to the state after the transition.

Q_k- is the process noise covariance matrix at step k; for Kinematic systems that we model using Newton's equations, we assume that the acceleration is constant for each discrete time but in reality, it's not constant due to external and unmodeled forces; for that we assume that the acceleration changes by a continuous time zero-mean white noise w(t). As known, the noise is changing continuously but we are using a discrete time interval, so we will need to integrate in order to get the discrete noise [14];

One of the ways to easily model the process covariance noise is to assume that the acceleration is constant for the duration of each time period, but it differs for each time period, for that case Q is computed as [14]:

$$Q = \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix} \sigma_v^2, \quad (27)$$

where,

σ_v^2 is the variance of the noise

$k = 1$…n, where n is the length of the measurement

Update equations

Measurement equation:

$$z_k = Hx_k + v_k \quad (28)$$

System uncertainty:

$$S_k = H_k P_{k|k-1} H_k^{\mathrm{T}} + R_k \quad (29)$$

$H_k P_{k|k-1} H_k^{\mathrm{T}}$- projects the covariance matrix into measurement space in order for Kalman filter to work; where $P_{k|k-1}$ is the state covariance we calculated during the prediction and H_k^{T} is the measurement function.

Once the covariance matrix is transited to the measurement space then we add the sensor noise, which is represented by R_k

Kalman gain:

$$K_k = P_{k|k-1} H_k^T S_K^{-1} \quad (30)$$

$\mathrm{K_k}$ is the Kalman gain at step k, a real number between 0 and 1[15]. Its value performs the selection of a mean (the estimate) somewhere between the prediction and the measurement [16]. This selection is based on the certainties about the measurement and the prediction, which are respectly represented by the covariance matrices R_k and Q_k [17,18]; so, if we have more certainty about the measurement (small variances in R_k) the estimate will be close to it but if we have more certainty about the prediction (small variances in Q_k the estimate will be close to the prediction.

Residual:

$$\hat{y}_k = z_k - H_k \hat{x}_{k|k-1} \quad (31)$$

We perform the difference between the measurements and the values being predicted (predictions)

State update:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \hat{y}_k \tag{32}$$

We update our estimate (calculate the new estate at step k) by adding the last estimate (the estimate at step k) to the residual scaled by the Kalman filter.

Covariance update:

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{33}$$

We update the covariance by multiplying the last covariance matrix by the factor$(I - K_k H_k)$, where I is an identity matrix.

The main objective of our Kalman filter: to estimate the pitch rate.

Based on the aircraft longitudinal model, and using the previous value of PITCH RATE, the prediction step of the Kalman filter gives us a prediction of the PITCH RATE. Based on the mathematical model of supersonic aircraft, on the aircraft model and on the previous PITCH RATE estimate a measurement comes in from PITCH RATE sensor. That measurement is going to be used to refine the prediction previously calculated. The second step, or correction step, compares the measurement with the measurement prediction calculated by the mathematical model. There has to be a mechanism in place to give less "weight" to an inaccurate measurement and more "weight" to a very good (compared to the model) and valid measurement: this is what the Kalman gain (computed by the Kalman filter) does. The previously calculated angular velocity is then corrected by a factor equal to the Kalman Gain multiplied by the difference between the estimated value and the measured value.

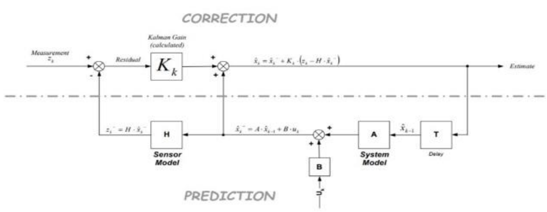


Fig. 4. Block diagram of Kalman filter algorithm.

5 Simulation

Simulations are conducted in MATLAB environment. The following simulations were conducted:

- Incremental NDI + PI Controller (referred in text as INDI or Incremental NDI).
- PI controller.
- Incremental NDI + PI Controller performance in presence of uncertainties.
- Incremental NDI + PI controller performance in presence of measurement noise.
- Incremental NDI + PI with Kalman filter to filter measurement noise.
- Incremental variant NDI + PI- experimental results.

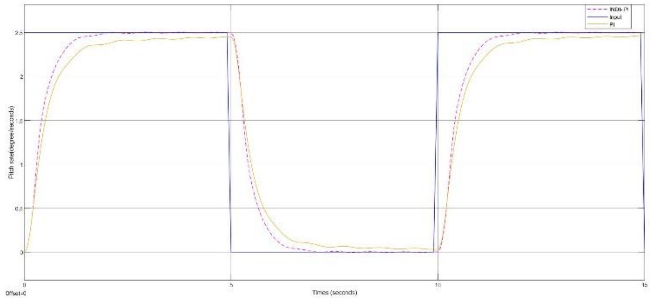


Fig. 5. INDI vs PI controller.

The simulation result highlights the Incremental NDI control law achieved the desired response.

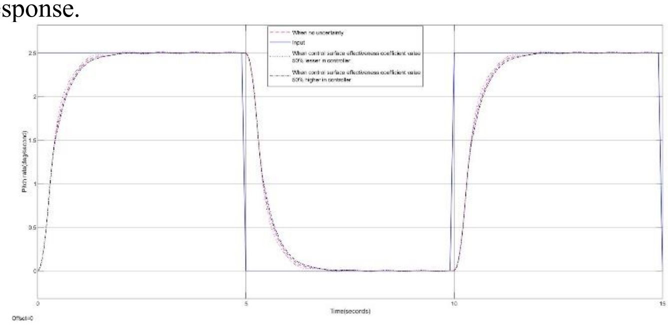


Fig. 6. Performance of INDI Controller in presence of uncertain knowledge of coefficients.

When the value of coefficients has more than 50% difference in the aircraft and in the controller, still the control error is not more than 5%.

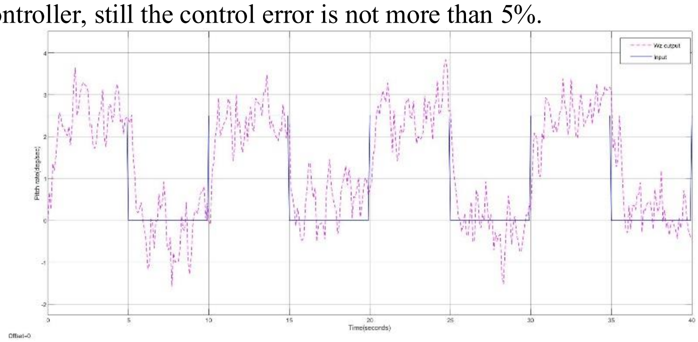


Fig. 7. Performance of INDI Controller in presence of measurement noise in angular velocity.

In presence of measurement noise, the controller performance is significantly diminished, and robustness is lost. The steady state error is more than 20%.

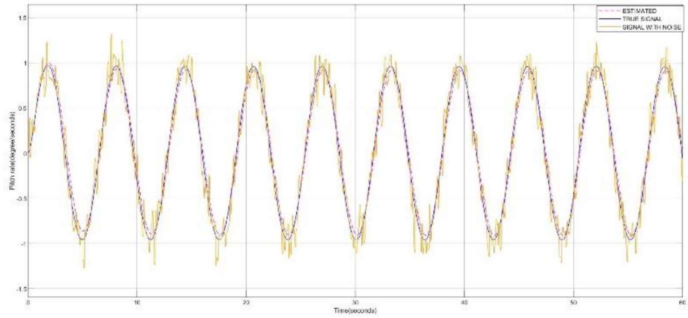


Fig. 8. Performance of Kalman filter to filter angular velocity in pitch measurement noise.

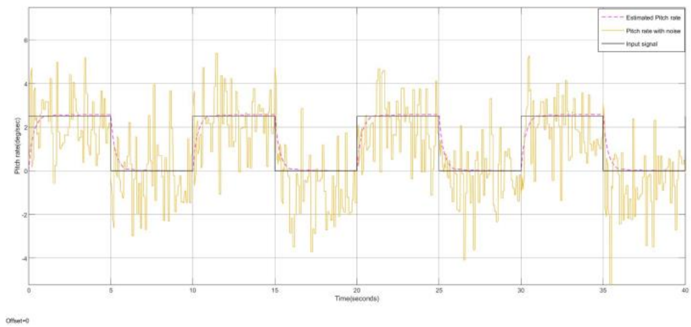


Fig. 9. Performance of Kalman filter to filter angular velocity in pitch measurement noise.

The use of Kalman filter to filter noise results in filtering of measurement noise, but leads to a small-time delay. For practical scenario to manage with measurement noise, the spline method needs to be researched further in comparison with Kalman filter [9].

6 Experimental Assessment

Fig. 10. MAI Ground Based Simulator.

The experiments were conducted in pilot vehicle lab of Moscow Aviation Institute. These experiments were conducted to assess the controller performance for tracking task of pitch angle. [19]

MAI Ground based simulator is used to study manual control tasks.

The workstation consists of the following elements:

- a control stick and simulator connected to a computer by an analog-to-digital converter;
- software.

During the experiment, information about the current task execution error is displayed on the screen. tracking (Figure 11).

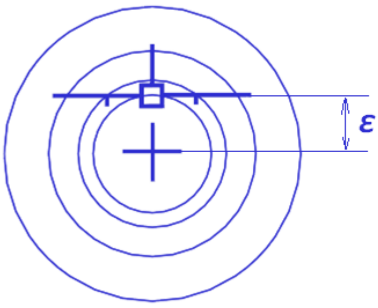
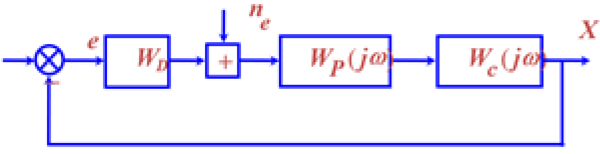


Fig. 11. Information about the current tracking error displayed on the screen in real time.

The control stick is a side control handle with variable characteristics (stiffness and damping).



$W_p = Pilot\ describing\ function$

$W_c = Aircraft\ dynamics + control\ system$

$n_e = Remnant$

Fig.12. Scheme with Pilot in the loop.

The following set of experiments were carried out on the nonlinear model developed recently in the process of research carried out within the Program for the Development of the World-Class Research Center "Supersonic" in 2020–2025.[10]

First experiment was conducted for tracking task via PI controller

Second experiment was conducted with the INDI controller

Third experiment was conducted when half of the section of stabilizer stopped working for PI Controller

Fourth experiment was conducted when half of the section of stabilizer stopped working for INDI controller

 https://doi.org/10.1051/e3sconf/202344606005

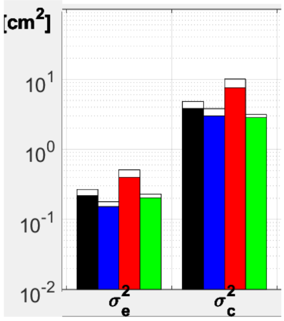


Fig. 13. Black: PI controller; Blue: Incremental NDI + PI; Red: PI with faults; Green: Incremental controller + PI with faults.

The variance of control stick usage, clearly shows, that in experiments when there is no fault the nonlinear controller required less usage of stick implying(about two times less(σ_c^2)) less pilot workload. When a section of stabilizer stopped working, the usage of control stick for INDI is almost the same, while for PI controller the use of stick usage variance has increased 6 times after error(σ_e^2). It clearly shows the INDI control combined with PI is more robust.

The tracking error variance clearly shows, in case when there's no fault INDI controller performance is almost 2 times (σ_e^2) better than PI controller. After a section of stabilizer stopped working, the tracking error for INDI controller has not increased significantly about 0.2 times compared to without fault, while for PI controller it has significantly increased about 3 times.

The variance results obtained from experiments with pilot in the loop show the robustness and tracking accuracy of incremental variant of nonlinear dynamic inversion method.

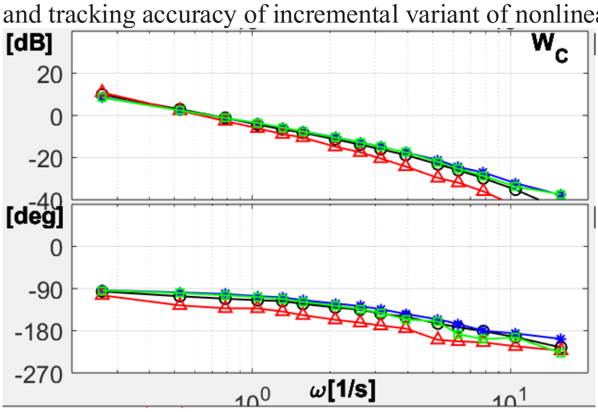


Fig. 14. Frequency response: Aircraft + Controller.

The aircraft + controller open loop frequency response shows that after fault, the PI controller has significantly reduced stability margin, whereas the incremental NDI controller + aircraft has no difference in stability margin.

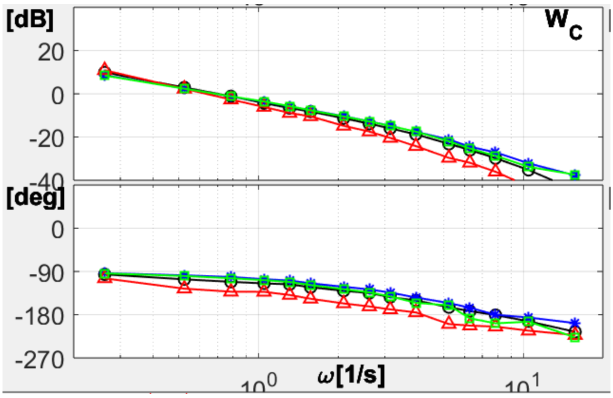


Fig. 15. Frequency response: Aircraft + Controller.

The aircraft + controller frequency response shows after fault, the PI controller has significantly reduced stability margin, whereas the incremental NDI controller has no difference in stability margin.

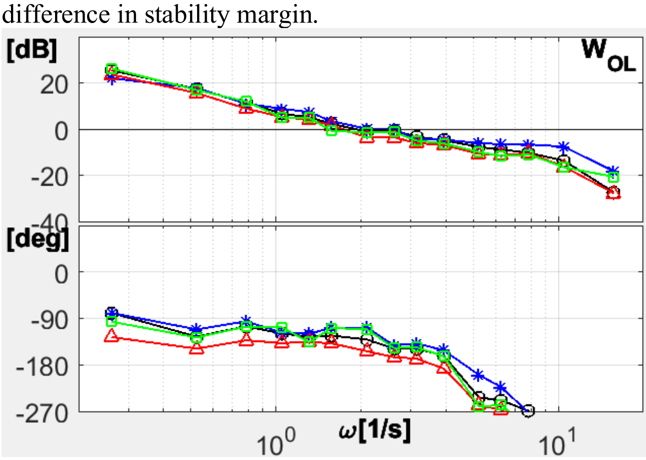


Fig. 16. Open Loop frequency response (Pilot + Aircraft(including controller)).

After the fault the PI controller with pilot in the loop the phase margin and gain margin of aircraft with PI controller has reduced and is on the verge of instability, whereas for the Incremental NDI controller there's no considerable change.

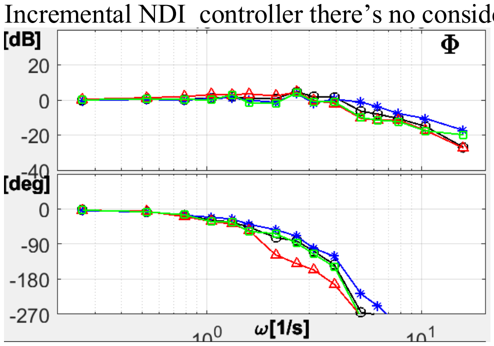


Fig. 17. Closed Loop frequency response (Pilot + Aircraft (including controller)).

In closed loop response, it can be observed after the fault, aircraft with PI controller has as significantly reduced closed loop system bandwidth with pilot in the loop.

Actuator model used:

For practical executions, we consider first-order actuator dynamics represented by the following transfer function

$$G(p) = \frac{K_a}{T_a p + 1}$$

$\frac{1}{T_a} \gg$
$> sufficiently\, fast, and\, its\, bandwidth\, is\, higher\, than\, control\, system\, closed\, loop\, bandwidth$

To avoid actuator saturation a pseudo control hedging based approach needs to be used in further works. [20]

7 Conclusions

NDI is a well-known control method. The realization of NDI control law usually is done by using two control loops which complexifies the control law. The authors suggested approach to control only fast dynamics by incremental variant of NDI and the use of PID Controller in outer loop is significantly simpler to practically realize, it combines the benefit of dynamic inversion control accuracy and PID controller robustness in presence of uncertainties. The overall controller was demonstrated for tracking tasks in context of longitudinal motion of supersonic aircraft- in the landing approach. The aircraft is unstable in longitudinal channel, and the pitch rate feedback in outer loop makes it stable, and the inner loop of incremental variant of NDI results in dynamic inversion.

The INDI approach has shown practical results as tested in flight tests.[21]

The incremental NDI controller has a practical issue of requiring actuator feedback, which in reality cannot be obtained. The authors suggest to use system identification of actuator model, and use a pseudo model as in [22]. The requirement of angular acceleration measurements is another problem which needs to be practically resolved as filtering a differentiated signal adds a time delay, which can impact overall performance. [23, 24, 25]

The use of rotational accelerometers are routinely used in missile systems, there must be concers regarding signal noise generated by aircraft structure, engines etc. [26]

The use of Kalman filter to filter sensor noise, and pre filter to deal with actuator saturation makes the overall combination practically realizable. The robustness of system in presence of faults, higher tracking accuracy, lower dependence of control law on mathematical model of object and lower pilot workload clearly show the potentialities of the control law.

We would like to express our deep appreciation to our teammates, without their contributions, this research would not have been complete.

References

1. R. A. Rivers, E. Bruce Jackson, C. Gondon Fullerton, Timothy H. Cox, Norman H.Princen, *A qualitative piloted evaluation of the Tupolev Tu-144 Supersonic Transport*, February 2000
2. D. Ito, D. T. Ward, J. Valasek, Robust dynamic inversion controller design and analysis for the X-38, 2001 AIAA Guidance, Navigation and Control Conf. and Exhibit, Montreal, Canada. AIAA-2001-4380

3. H. Khorramabadi, P. R. Gray, High-frequency CMOS continuous-time filters, IEEE J. Solid-State Circuits Soc., **19**(6), 939-948, 1984, doi: 10.1109/JSSC.1984.1052249
4. J.-J. E. Slotine, W. Li, *Applied Nonlinear Control* (Prentice Hall, New Jersey, 1991)
5. H. K. Khalil, *Nonlinear systems*, 3rd ed. (Prentice Hall, New Jersey, 2002)
6. C. J. Miller, Nonlinear dynamic inversion baseline control law: Flight-test results for the full-scale advanced systems testbed F/A-18 airplane, 2011 AIAA Guidance, Navigation, and Control Conf., Portland, Oregon, USA AIAA-2011-6468
7. J. Reiner, G. J. Balas, W. L. Garrard, Robust Dynamic Inversion for Control of Highly Maneuverable Aircraft, J. Guid. Control Dyn., **18**(1), 18–24 (1995)
8. G. Looye, Design of robust autopilot control laws with nonlinear dynamic inversion, Automatic sierungstechnik, **49**(12), 523531 (2001)
9. A.J. Steer, Flight Control for advanced supersonic transport aircraft handling quality design, November 2001
10. A.V. Efremov, E.V. Efremov, M.S. Tiaglik , I. Kh Irgaleev , A.I. Shcherbakov ,Z. Mbikayi, Adaptive flight control system for flight safety improvement in reentry and other high-velocity vehicles. Acta Astronautica, **204**, 900-911 2023 https://doi.org/10.1016/j.actaastro.2022.10.056.
11. M.V. Cook, *Flight dynamics principles*, 2nd Ed., Elsevier, 2007
12. C.J. Miller, Nonlinear dynamic inversion baseline Control law: Architecture and performance predictions, 2011 AIAA Guidance, Navigation, and Control Conference, Portland, Oregon, USA. AIAA-2011 6467
13. R. C Van't Veld, Incremental Nonlinear Dynamic inversion control, Thesis 2016
14. R. J. Meinhold, N.D. Singpurwalla, Understanding the Kalman Filter, Am. Stat., **37**(2), 123-127 (1983) DOI: 10.1080/00031305.1983.10482723
15. A.L. Barker, D.E. Brown, W.N. Martin, *Bayesian Estimation and the Kalman Filter* (Technical Report. Charlottesville, VA, USA, 1994)
16. T. Babb, How a Kalman filter works, in pictures | bzarg. 2018. https://www.bzarg.com/p/how-a-kalman-filter-works-in-pictures/ Accessed: 2018-11-30
17. U. Lagerblad, H. Wentzel, A. Kulachenko, Study of a fixed- lag Kalman smoother for input and state estimation in vibrating structures, Inverse Problems Sci. Eng., **29**(9), 1260–1281 (2021) https://doi. org/10.1080/17415977.2020.1845669
18. M. Khodarahmi, V. Maihami, A Review on Kalman Filter Models, Arch. Computat. Methods Eng., **30**, 727–747 (2023) https://doi.org/10.1007/s11831-022-09815-7
19. A.V. Efrenov, A.V. Ogloblshn, A.N. Predtechenskii, V.V. Rodchenko, *Pilot as a dynamic system* (Mashinostroenie, Moscow, 1992)
20. T.J.J. Lombaerts , G.H.N. Looye, Q.P. Chu, J.A. Mulder, Pseudo Control Hedging and its Application for Safe Flight Envelope Protection, AIAA Guidance, Navigation, and Control Conf. 2 - 5 August 2010, Toronto, Ontario Canada, AIAA 2010-8210
21. F. G1ondman, G.H.N. Looye, R. Kucha1, Q.P. Chu, E. J. van Kampen, Design and flight testing of incremental nonlinear dynamic inversion based control laws for a passenger aircraft, 2018 AIAA Guidance, Navigation, and Control Conf., Kissimmee, Florida, USA AIAA- 2018-0385
22. F. Binz, D. Moormann, Actuator modelling for attitude control using incremental nonlinear dynamic inversion, MAV 2019–Original Research Article

23. S. J. Ovaska, S. Va" liviiia, Angular acceleration measurement: A review, IEEE Trans. on Instrumentation and Measurement, **47**(5), 1211-1217 (1998)
24. A. Isidori, *Nonlinear control systems*, 3rd ed. (Springer, 1995)
25. G. Ducard, H.P. Geering, Stability analysis of a dynamic inversion based pitch rate controller for an unmanned aircraft
26. P.R. Smith, A simplified approach to non linear dynamic inversion based flight control, AIAA Atmospheric Flight Mech. Conf., AIAA-98-4461, 762-770 (1998)