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Development of algorithms for prediction of the technological process

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Abstract. This article discusses the development of algorithms for predicting and automatically controlling the process of microalgae cultivation. For the operational management of production, it is necessary to be able to evaluate the values of the criterion during the process for short periods of time and predict the influence of control actions on the optimality criterion. Since the cultivation process can be carried out in periodic or continuous modes, it is necessary to consider the possibilities and conditions for choosing the optimality criterion. For continuous mode, when at each moment of time the state of the process is determined only by the parameters of the state and does not depend on the state of the process at previous moments of time, an estimation instant can be used. In this case, the criterion will have the meaning of the instantaneous value of the process productivity, referred to profit. In this case, it is a criterion that is directly related to the profit of the considered class of objects. Therefore, it is expedient to choose an optimality criterion in the form of a target product maximization problem. It follows from this expression that for *N cultivators* connected in series, the total residence time $T = \frac{1}{\lambda^N}$ must be distributed equally among all cultivators, if individually they have the same volume.

1 Introduction

When solving optimization problems, as well as synthesizing control systems, it is necessary to select and substantiate the optimality criterion. It can be viewed as reaching an extremum of a certain value. Such a criterion can be a set of technical and economic indicators - such as process productivity, reduced costs, profitability, product quality, profit from product sales, etc.

As a criterion for conventional microbiological enterprises - a stable indicator that reflects almost all aspects of production activity [5]:

$$F(z) = S(z) * N(z) - \sum X_i(z) \tag{1}$$

where *F is* production profit;
N is the price of the product produced;
$\sum X\, i$ - total production costs;

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S - the amount of product produced for a certain time interval. [11, 16]

2 Research Methodology

For the considered group of objects, the problem of optimal control should be formulated as problems of profit maximization under restrictions on the quality of the product, the specific costs of raw materials and reagents:

$$F(z) \rightarrow \max, \quad x \geq x^{set}, \quad C_i \leq C_i^{set}, \tag{2}$$

where x_i and x^{set} are, respectively, the actual and set concentrations of the microorganism in the finished product;

C_i and C_i^{ass} are the actual and allowable specific rates of consumption of raw materials and reagents, respectively.

Expanding the quantities included in expression (1.1), rewrite it in the form

$$F = -З_n + (S_f N - З_f) - З_y \tag{3}$$

Here $З_n$, $З_f$ - the cost of preparing a nutrient medium and fermentation for a certain period of time;

$З_y$ - conditionally - fixed costs;

S is the amount of the target product produced for a certain period (productivity).

The profit of the enterprise with an increase in the amount of the target product produced tends to increase in indicators over the entire range of possible productivity values. As we can see, the problem of profit maximization in this case coincides with the problem of maximizing the productivity of a process unit. Thus, we are talking about the volume of the target product of the chlorella cultivation process, produced over a certain period of time, minus losses (which, to simplify the problem, are assumed to be constant).

S value in (1) is completely formed at the stage of fermentation and is taken as a criterion for controlling this stage under the following restrictions on the concentration of residual nutrients C_i in the culture liquid, the specific consumption of raw materials and reagents:

$$S_f(\overline{Y}; \overline{U}) = max; \quad b(\overline{Y}; \overline{U}) \leq B_{ext}; \quad C_i(\overline{Y}; \overline{U}) \leq C_i^{given}, \tag{4}$$

where $\overline{Y}$ и $\overline{U}$, respectively, are the vectors of state variables and control actions.

The productivity of a particular cultivator is determined both by the productivity of each production cycle and by the machine's turnover (i.e., it is determined by the frequency of these cycles). Therefore, the following expression can serve as a control criterion:

$$I = S_i \frac{\sum_{d=1}^{m} S_{ij} V_{ij}}{\sum_{d=1}^{m} (t_{ij}^{f} + t_{ij}^{prep} + t_{ij}^{dwn}}, \tag{5}$$

where $V_{ij,} S_{ij}$ is the volume of the medium in *i* – th cultivator and the concentration of microorganisms in it *j* -th fermentation;

t_{ij}^{f} – duration of *j* -th fermentation;

t_{ij}^{prep} – preparation time of the fermenter for execution *j* - Ouch operations;

t_{ij}^{dwn} – is the downtime of the apparatus in the cycles carried out during the period under consideration.

$$I = \frac{F}{T}$$

Here *F* - profit from the sale of the target product;

*T **is*** the time for which this profit was earned.

It can be seen from (5) that, in addition to the parameters determined by the fermentation itself, the objective function is also affected by the indicators of preparatory operations and equipment downtime.

In order to achieve criterion (4), this value should be maximized at each stage of the technological cycle.

Now the problem of managing the fermentation cycle can be formulated as follows: it is necessary to determine such control actions from the range of permissible ($\overline{u}$) that would deliver the maximum optimality criterion under given initial conditions, as well as subject to restrictions on the content of residual nutrient salts in the medium, on specific costs raw materials and reagents [7, 8].

In the symbols of set theory, this problem can be formalized as follows:

$$\max\{I[\overline{Y}(t_k), t_k] \,/ G[\overline{Y}(t), t] = 0;$$

$$\overline{Y}(t) \in \Omega_{\overline{Y_0}}\, \overline{U}(t) \in \Omega_{\overline{U_i}} \quad b(\overline{Y}, \overline{U}) \le b_{ext};$$

$$C_i(\overline{Y}, \overline{U}) \le C_i^{given}\}$$

Here, $\overline{G}$denoted by the vector of dependencies of the mathematical model of the process, the components of which are functions of the vector of state variables $\overline{Y}$, the vector of their productivity in time - $\overline{Y}$, the vector of control actions $\overline{U}$ and the current time *t.* The set $\Omega_{\overline{U}}$defines the area of acceptable initial conditions for the process. The set $\Omega_{\overline{Y_0}}$ defines the range of allowable values of control actions. The parameter means t_k the end time of the process [3, 4].

3 Research results

For the operational management of production, it is necessary to be able to evaluate the values of the criterion during the process for short periods of time and predict the influence of control actions on the optimality criterion. Since the cultivation process can be carried out in periodic or continuous modes, it is necessary to consider the possibilities and conditions for choosing the optimality criterion. For continuous mode, when at each moment of time the state of the process is determined only by the parameters of the state and does not depend on the state of the process at previous moments of time, an evaluation instant can be used. In this case, the criterion will have the meaning of the instantaneous value of the process productivity, referred to profit. For periodic processes, when the output of the finished product takes place only at the moment of completion of the technological cycle, the evaluation of the criterion makes sense only at the moment of the completion of the technological cycle. For this case, the time interval T in formula (5) take on the meaning of the duration of the technological cycle, and the criterion is the average per cycle productivity of the apparatus in relation to profit. The criterion in the form (1) or (3), although it is a generalized indicator, but sometimes, when the target product does not yet have a final presentation, is more sensitive to control parameters. In this case, it is a criterion that is directly related to the profit of the considered class of objects. Therefore, it is advisable to choose the optimality criterion in the form of the problem of maximizing the target product

$$I = \frac{\mu x}{D} \tag{6}$$

This criterion is directly related to the previously considered (3) and (5), since an increase in the yield of the target product leads to an increase in productivity and, thus, to an increase in profit. The limitation is the residence time of microorganisms in the cultivator. [5,9,10]

$$0 \le D \le \mu_0. \tag{7}$$

In what follows, when solving problems of technological optimization and optimal control, we will use relations (6) and (7) as the main criteria and the conditions necessary for the process of microalgae cultivation.
For the considered multistage processes, in order to obtain the maximum concentration of microalgae, an optimality criterion of the form
Where:

$$R_1 = \sum_{i=1}^{N} x_1, \tag{8}$$

Where
$$x_1 = \frac{D_1 - \mu_i}{D_i X_{i-1}} \tag{9}$$

μ_i – specific growth rate of microalgae in the i -th reactor;
X_i - concentration of microalgae in the i-th reactor
For the case I = 1 we have

$$X_1 = \frac{D_1 - \mu_1}{D_1 X_0} \tag{10}$$

For a cascade, reactor a is defined as follows:

Z \u003d 0, i \u003d 1.2, ..., N -1; Z $_n$ = x $^{(N)}$ (11)

Let the control variables of process D $_i$ be subject to constraints

$$D^{(n)} = \frac{1}{\sum_{i=1}^{n} \frac{1}{D_i}} \tag{12}$$

To evaluate the optimality criterion for each flock, new expressions are formed:

$$Z^* = \lambda D_i; \quad r^* = \lambda * D_n + x^{(N)}$$

With the introduction of the indefinite Lagrange multiplier, the original criterion is modified

$$R_i{}^* = x^{(n)} + \frac{\lambda}{\sum_{i=1}^{N} 1/D} = R_1 + \frac{\lambda}{\sum_{i=1}^{N} 1/D_1} \tag{13}$$

The mathematical form of the principle of optimality for the last cultivator of a multistage process depends on the recurrent Wellman formula

$$f\left(x^{(N-1)}, \lambda\right) = D_n \max\left\{\lambda D_n + \frac{D_i - \mu_i}{D_i X_{N-1}}\right\} \tag{14}$$

The change in the concentration x $_i$ in the technological process of continuous growth of microalgae is described by the equation

$$\frac{dx_i}{dt} = D_i(x_{i-1} - x_i) + \mu_i * x_i \tag{15}$$

the time derivatives in this formula (1.15) equal to zero:

$$D_i(x_{i-1} - x_i) + \mu_i * x_i = 0 \tag{16}$$

The value x_i from formula (16) and the optimal value of D_n for the last reactor are determined from the following condition:

$$\frac{\delta}{\delta D_N}\{\lambda D_N + \frac{D_N - \mu_N}{D_N * x_{N-1}}\} \tag{17}$$

we get

$$\lambda + \frac{\mu_N}{{D_N}^2 x_{N-1}} = 0$$

Its result is:

$$D_n = \frac{\sqrt{\mu_N}}{\lambda * x_{N-1}} \tag{18}$$

Using the formula (16), we substitute the obtained values in the formula (14) and get

$$f_1(x^{N-2}, \lambda) = D_{N-1} \max\left\{\lambda D_{N-1} + \lambda\sqrt{\frac{\alpha_N X_N}{\lambda X_{N-1}}} + x^{(N-1)} + \frac{\sqrt{\alpha_{N-1} x_{N-1} - \mu_N}}{\sqrt{\frac{\alpha_{N-1} x_{N-1}}{\lambda x_N x_{N-1}}}}\right\} \tag{19}$$

Based on equation (11), we write the Bellman recurrence relation for (N -1) reactor

$$f_2\left(x^{(N-2)}, \lambda\right) = D_{N-1} \max\{\lambda D_{N-1} + \lambda\sqrt{\frac{\alpha_N}{\lambda X_{N-1}}} + x^{(N-1)} + \frac{\sqrt{\alpha_{N-1} x_{N-1} - \mu_N}}{\sqrt{\frac{\alpha_{N-1}}{\lambda x_N x_{N-1}}}}\} \tag{20}$$

Where in

$$x^{(N-1)} = x^{(N-2)} + \frac{D_{N-1} - \alpha_{N-1}}{D_{N-1} x_{N-2}} \tag{21}$$

Substituting the value $x^{(N-1)}$ into equation (20), we get

$$f_2\left(x^{(N-2)}, \lambda\right) = D_{N-1} \max\{\lambda D_{N-1} + \lambda\sqrt{\frac{\alpha_{N-1}}{\lambda X_{N-1}}} + x^{(N+2)} + \frac{D_{N-1} - \alpha_{N-1}}{D_{N-1} x_{N-2}} + \frac{\alpha x_N x_{N-1}}{\sqrt{\frac{\alpha_{N-1} x_{N-1}}{\lambda}}}\} \tag{22}$$

Similarly $f_2\left(x^{(N-2)}, \lambda\right)$, we find D_{N-1} from the condition (17):

$$D_{N-1} = \sqrt{\frac{\alpha_{N-1} x_{N-1}}{\lambda}} \tag{23}$$

$$f_2\left(x^{(N-2)}, \lambda\right) = \lambda\sqrt{\frac{\alpha_N x_N}{\lambda}} + x^{(N-2)} + \frac{\alpha x_N x_{N-1}}{\sqrt{\frac{\alpha_{N-1} x_{N-1}}{\lambda}}} + \frac{\alpha_N x_N}{\sqrt{\frac{\alpha_N x_N}{\lambda}}} \tag{24}$$

For (N -2) - th cultivator, the following equations can also be derived in the same way:

$$D_{N-2} = \sqrt{\frac{\alpha_{N-2} x_{N-2}}{\lambda}} \tag{25}$$

$$f_3\left(x^{(N-3)},\lambda\right)=\lambda\sqrt{\frac{\alpha_{N-2}x_{N-2}}{\lambda}}+\lambda\sqrt{\frac{\alpha_{N-2}x_{N-2}}{\lambda}}+\lambda\sqrt{\frac{\alpha_N x_N}{\lambda}}+x^{(N-3)}+\frac{\alpha x_{N-2}x_{N-2}}{\sqrt{\frac{\alpha_{N-2}x_{N-2}}{\lambda}}}+\frac{\alpha x_N x_{N-1}}{\sqrt{\frac{\alpha_{N-1}x_{N-1}}{\lambda}}}+\frac{\alpha_N x_N}{\sqrt{\frac{\alpha_N x_N}{\lambda}}} \quad (26)$$

From the equations (18), (25), and also taking into account the expressions (19, (24), and (26) for an arbitrary i -th reactor, the following are derived formulas:

$$D_i=\sqrt{\frac{\alpha_i x_i}{\lambda}} \quad (27)$$

$$f_{N-i+1}\left(x^{(i-1)},\lambda\right)=\lambda\sum_{j=0}^{N-i}\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}+x^{(i-1)}+\sum_{j=0}^{N-j}\frac{\alpha_{N-j}x_{N-j}}{\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}} \quad (28)$$

Using formulas (27) and (28) for the first cascade cultivator with i =1 we get

$$D_i=\sqrt{\frac{\alpha_i x_i}{\lambda}} \quad (29)$$

$$f_N\left(x^{(0)},\lambda\right)=\lambda\sum_{j=0}^{N-i}\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}+x^{(0)}+\sum_{j=0}^{N-j}\frac{\alpha_{N-j}x_{N-j}}{\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}} \quad (30)$$

Substituting the values from (30) into formulas e (11) for *i =1* , we obtain

$$x_1=\frac{D_1-\mu_1}{\lambda_1 x_0}=\frac{\sqrt{\mu_1/\lambda x_1}-\mu_1}{\sqrt{\mu_1/\lambda x_1}x_0}$$

According to equation (29), we calculate the optimal value for the case *i = 2*

$$D_2=\sqrt{\frac{\mu_2}{\lambda x_1}}$$

Substituting the quantities obtained as functions D_i from expression (29) into condition (12), we determine the values λ. In this case, the formula takes the form

$$\lambda_i=\lambda^{(N)}N \quad (31)$$

4 Conclusions

The maximum value of the concentration of chlorella *X* at the outlet of the last cultivator can be expressed as (13), if we bear in mind that the maximum value of R * is reflected by the relation (31)

$$f_N\left(x^{(0)},\lambda\right)=\lambda\sum_{j=0}^{N-i}\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}+x^{(0)}+\sum_{j=0}^{N-j}\frac{\alpha_{N-j}x_{N-j}}{\sqrt{\frac{\alpha_{N-j}x_{N-j}}{\lambda}}}$$

$$D^{(N)} = \frac{1}{\sum_{j=0}^{N} * \frac{1}{D_i}} = \frac{1}{\sum_{j=0}^{N} * \frac{1}{\sqrt{\frac{\mu_i}{\lambda x_i}}}},$$

Where:

$$\lambda = \frac{1}{\left(D^{(N)} \sum_{j=0}^{N} * \frac{1}{\sqrt{\frac{\mu_i}{\lambda x_i}}}\right)^2}$$

the obtained values λ into equations (29) and obtain the dilution rate for an arbitrary i - th reactor in the form

$$D_i = D^{(N)} \sqrt{\alpha_i * x_i} \left(\sum_{j=0}^{N} * \sqrt{\frac{x_i}{\mu_i}}\right) \tag{32}$$

At $\mu_i = const$and$x_i = const$ formula (32) takes the following form:

$$D_i = D^{(N)} * N \tag{33}$$

or

$$D_i = \frac{1}{T} N, \;\; i = \overline{1, N} \tag{34}$$

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