

Design and research activity of students as a pedagogical problem

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Abstract. This article presents the origins of the design and research activities of students. A system of means that form the design and research activities of students has been developed. Four types of design and research activities of students (imitative, reproductive, partially exploratory and creative) and the mechanisms of their formation are described. The main ways of activating the project activity of students are revealed: the formation of learning motives, the use of problematic elements, the system of various types of project work, the connection of the studied material with the life and personal experience of the student. Work built on active cognitive activity brings students feelings of deep satisfaction, develops their imagination, develops a creative initiative, forms a worldview and moral character. We have made an attempt to analyze the theoretical foundations, methods and techniques of activating the project activity of students in the lessons of the course of mathematical analysis.

1 Introduction

The origins of students' design and research activities have deep roots that go back to the distant past. Even in ancient times, it was known that cognitive activity contributes to better memorization and deeper insight into the essence of objects, processes and phenomena.

In the natural scientific works of medieval thinkers of the Near and Middle East, there is a serious interest in various aspects of scientific knowledge, its principles, structure, criteria, the connection of scientific knowledge with human mental development and education.

A characteristic feature of the epistemological views of Khorezmi, Kindi, Farabi, Biruni, Ibn Sina, Omar Khayam and Tusi, their associates and followers, is that their attention was constantly attracted by the process of abstracting the image of an object in a person's mind, as a result of which the concept of the essence and specifics of this subject is being developed and formed.

What are the subject and sources of cognition, from which stages the process of cognition develops, what are the relations between cognitive and project - research activities, a far incomplete list of questions of cognition that attracted almost all the outstanding thinkers of the medieval East. Khorezmi [1] clearly distinguished cognition through "sensation" from cognition through "logical reasoning": the first cognizes the

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particular: "small", and the "logical" studies the essential, the significant, but they are interrelated. The germs of the theory of knowledge arose with the advent of science itself and are continuously developing. Khorezmi made a significant contribution to the development of this theory. He was one of the first to scientifically substantiate the experimental-observational and experimental method of cognition in the form of a tabular reflection of motion celestial objects, as well as spatial locations, ground points, concretized the principle of unity of the singular, special and general in the aspect of induction and deduction: moved from a special case in the equation to a special one (6 equations), but did not reach the general (canonical) form, developed an algorithmic method for solving mathematical problems, which is generally recognized nowadays in the general scientific method of cognition and action.

Human cognition, according to Kindi, is divided into sensual and rational. The object of sensory cognition has no stability and, although its nature is hidden from us, it is nevertheless very close to the sensually perceived and always possesses matter, because it is always a body and is in the body.

Thus, sensory cognition is conditioned by the object itself, its object is everything corporeal and material. If sensory cognition is a singular cognition, and then rational cognition is general, "Individual material things are amenable to sensory perception," Kindi wrote, "as for genera and species; they are not amenable to sensory perception and cannot serve as the subject of sensory cognition. But they are susceptible to the perception of that of the forces of the perfect, that is, the human soul, which is called the human mind." [2, p.62]. According to Kindi, sensory cognition provides only material for the mind. At the same time, the knowledge gained through sensory perception, Kindi subordinated positions in relation to rational cognition. He believed that the mind knows what is abstract. Groups of scientists and philologists who founded in the second half of the tenth century. The Brothers of Purity School greatly contributed to the dissemination of natural science and philosophical knowledge among the general population of the Arab Caliphate. Questions about labor and design and research activities, about the degree of dignity of crafts, about the work of the mind were considered in the so-called "Messages" of the members of this group: "Know that there are two types of activities peculiar to the human race: cognitive and labor. We assert that labor activity is the extraction of the form that a person has in his mind and its application to matter." Then they say that "knowledge is the images of what is known in the soul of the knower. Know that there is no knowledge without learning and assimilation of science. And learning is an urge that comes from the soul that knows the actual, to the soul that knows the potential. The assimilation of knowledge is the soul's perception of the forms of the known. Know that the soul perceives the forms of cognizable objects in three ways: first, through feelings, secondly, through arguments, and thirdly, through reflection and contemplation" [3, p.60].

Farabi attached great importance to sciences as a necessary means of acquiring and accumulating knowledge, and considered mastering them an indicator of education. He believed that the senses, heart and brain are given to a person from birth, and all the rest – knowledge, various kinds of intellectual and moral properties: character traits, education, etc. – are acquired in the process of human life. Defining various character traits and the moral virtues of courage, courage, friendliness, generosity, wit, truthfulness, etc., he considered them the result of the upbringing and self-education of the individual. Thus, according to Farabi, the education of intellectual and moral qualities can be carried out in two ways (methods): the process of voluntary actions of the individual aimed at improvement and under coercion by force. In Farabi's works, the issues of didactics, theoretical problems of teaching, philosophical, physiological and psychological foundations of knowledge acquisition. For him, the interaction of object and subject, external existence with its variety of forms of existence and the human personality with its

complex physiological and mental processes and rich spiritual world is beyond doubt. For example, he believed that the highest stage of human spirituality is the soul, mind and thinking, acting as a specific form of human cognitive activity. Farabi has developed a number of detailed recommendations on organization of cognitive and project-research activities. "To be a good theorist," he said, "regardless of the science to which it relates, three conditions must be met: 1) it is good to know all the principles underlying this science; 2) to be able to draw the necessary conclusions from these principles and data related to this particular spider, i.e. to know the rules of reasoning; 3) to be able to refute erroneous theories and analyze the opinions of other authors in order to distinguish truth from lies and correct mistakes.

In the scientific heritage of Biruni, a sore place is occupied by the scientific method of studying and cognizing nature developed by him. The characteristic features of the basis of Biruni's scientific method were the objectivity and impartiality of the scientist, observation, experiments, the study of oral and written monuments, a critical approach to facts, comparing them in order to establish the truth, logical generalization of the form of conclusions, the transformation of conclusions into theory.

Ibn Sina (Avicenna) (X-XI centuries AD) gave a three-dimensional picture of various types of cognitive activity, presenting them as different forces of the Soul and removing a significant part of these from the power of the divine origin. He distinguishes three types of soul in accordance with the ancient tradition: vegetable, animal and intelligent. In the first, two forces are distinguished motor and perceiving; perceiving, in turn, is divided into perceiving from the outside and from the inside. The first one includes five or eight senses. Ibn Sina's scientific heritage is huge and covers all areas of human knowledge. The essence of these didactic principles is as follows: learning should go from easy to difficult. It should be carried out taking into account the inclinations and abilities of children, exercises should be feasible, and training should be combined with physical exercises. An important pedagogical principle of Ibn Sina is his idea that the human mind is able to influence the course of life, because a person differs from an animal precisely by the presence of reason, and therefore by the possibility of realizing the act he commits. An important premise in Ibn Sina's pedagogical views is the idea that the environment plays a leading role in the child's cognition.

2 Materials and methods

For the convenience of subsequent analysis from a modern point of view, we have divided the means of design and research activities used in higher education into three groups:

In the first, we include a means aimed at the development of the student's personality and his cognitive (and project-research) activities, in the second - mobilizing the existing cognitive forces of the student, in the third - means contributing to the activation of cognitive and project-research activities.

The means of forming the design and research activities of the first group, as was said by the thinkers of the East, Khorezmi, Kindi and others, are aimed at achieving two main goals: the development of design and research activities and the formation of the personality of students.

Since the development of students' personality is carried out in the process of forming their cognitive activity, it is not necessary to strictly distinguish these two sides, but such differentiation, in our opinion, allows a more purposeful approach to solving the problem of improving project research activities.

Design and research activity, like any other, has motivational and operational sides (internal and external forces according to Ibn Sina). The first assumes the presence of certain motives of activity, which are the motivating and guiding components of it, the

second formation of a number of knowledge, skills, design and research activities is possible in two directions: 1) development and improvement of the motivational sphere of activity, the formation of the need for knowledge, the desire to deepen and expand; 2) the formation of the operational side design and research activities, which include such means as the development of rational techniques, showing the teacher how to work with educational material, solving increasingly complex cognitive tasks, forming the ability to independently acquire knowledge, etc.

In the development of students' personality, two directions can also be distinguished, namely: the development of improving thinking and the formation of certain personality qualities. Since we consider the development of thinking to be a link in design and research activities, we will consider this group of tools in detail below. As for the formation of certain personality qualities, such means as project-independent work, solving research problems, creative work that are specifically aimed at the formation of independence, activity, creativity, passion and other personality qualities are used here.

The second group of funds, the design and research activities of the above-mentioned thinkers of the East, Farabi, Ibn Sina, is aimed at mobilizing the existing cognitive forces of students. This is achieved in two main ways. One of them involves forcing students to perform tasks by various means. This includes threats, punishments, and the creation of conditions that force students to work actively in the classroom and do homework. These means should be attributed to negative ones, because they, as correctly noted in modern didactics, give far-flimsy and profound learning results.

Another method involves the use of means that arouse students' interest in learning [4, 5, 6, 7, 8], which is achieved mainly by influencing the motivational and emotional sides of the student's personality. This includes creating an emotional background in the lesson, problem presentation of the material, demonstration of experiments at the beginning of the lesson, excursions, design and research work, etc.

The third group of funds achieves a certain effect in improving the design and research activities of schoolchildren by creating such learning conditions that remove unproductive time and create an optimal load on students' thinking, thereby intensifying design and research activities (Al-Biruni, Ibn Sina, Farabi, etc.).

This group includes various devices that facilitate the work of students (calculating machines, drawing devices, templates, stencils, and computers). This also includes tasks on a printed basis, selective input of answers to the task means that allow for a short period of time to give the necessary amount of diverse educational information (videos, filmstrips, sound recordings, posters, tables, mobile applications, etc.).

The main functions of the means of forming cognitive activity and improving the design and research activity are understood by thinkers of the East and in folk pedagogy: informational and motivational (obtaining new knowledge); controlling (reporting on the degree of assimilation of knowledge); and finally, to help organize independent project-individual work of students.

The following methods are used to solve these problems:

- analysis of psychological, pedagogical, mathematical and methodological literature on the research topic;
- analysis of the results of mastering knowledge on the problem of studying the topic of project activities of future mathematicians;
- conducting questionnaires, interviews, control works among teachers and students, analysis of their results;
- Analysis of university documents and familiarization with the experience of teachers.

3 Results

The project method has a long history. Such scientists as A.M. Vasilyeva [9], I. V. Chechel [10], V. A. Dalingier [11], T. K. Smykovskaya [12], E. S. Polat [13], V. M. Rozin [14], L. O. Filatova [15], I. A. Kolesnikova worked on these issues, M. P. Gorchakova-Sibirskaya [16], G. V. Narykova, Safronova T. M. [17], R.Ibragimov , A.Karataev, B. Kalimbetov, T. Kerimbetov [18].

It is impossible to characterize all the specific means of students' design and research activities due to their diversity. However, it is possible to name those means of design and research activities that are mainline. These include a problem-based approach to higher education and project-independent work of students, project-research work of students, business games, etc. These funds stimulate all aspects of design and research activities.

Based on the above concepts, we have developed a system of tools that form the design and research activities of students. When defining this system, we paid great attention to the works of thinkers of the East, folk pedagogy, folk wisdom, the so-called external and internal aspects of students' design and research activities. However, the content of these concepts is somewhat different here.

The main techniques used for the purposes of the internal side of the formation of students' design and research activities are:

- project-independent work of students;
- elements of self-control, mutual control;
- problem-based approach to teaching students;
- creative tasks (design and research);
- elements of an adaptive learning system, etc.

We will limit ourselves to giving below examples of several types of project work in the discipline of mathematical analysis that form the design and research activities of students. Providing students with project works of various types develop the level of their design and research activities. Therefore, we show samples of design works of theoretical and practical content.

1st project work. Natural numbers.

1. What is the basis of modern mathematics? (Numbers)
2. How did natural numbers come about?
3. What number do we get as a result of applying addition and multiplication operations to natural numbers once or several times?
4. Formulate the displacement and distribution laws of multiplication and addition of natural numbers, that is, specify the formulas of the basic arithmetic laws.
5. Give examples of comparing natural numbers with respect to their magnitude. if m and n are two natural numbers, how can they be distributed?
6. Observe how the natural numbers are distributed along the number line, and how they are distributed along each point of the number line?
7. Are the sets of natural numbers infinite?
8. Is it possible to write natural numbers in this way, distributing them into subsets?
 - 8.1 a) odd numbers
 - 8.2 a) prime numbers
 - 8.3 b) even numbers
 - 8.4 b) composite numbers
9. Tell us if you know prime numbers or not:
 - 9.1 Is there a general formula for prime numbers? What is the sieve of Eratosthenes?
 - 9.2 Give examples of decomposing a composite number into prime factors.
10. What do you know about the axiomatic structure of natural numbers?
11. Give examples of the axioms of addition and multiplication.

2nd project work. Integers

1. What are integers? Give an example of the axiomatic structure of integers.

1.1. What is the essence of the concept of a neutral element?

1.2 What is a symmetric element?

1.3 What scientists have conducted research on

2. Give examples of applying operations to integers.

2.1 $-a + (-b) = -a - b = -b - a$

$a + (-b) = a - b = -b + a$

$-a(b + c) = -ab - ac$

$a + 0 = a, \quad a \cdot 0 = 0$

3. What do we know about number systems?

4. What is writing numbers as a positional series?

4.1 In which number system are the following sets of numbers written?

$4796 = 4000 + 700 + 90 + 6 = 4 \cdot 10^3 + 7 \cdot 10^2 + 9 \cdot 10 + 6$

4.2 Is it possible to write any natural integer in general form as follows?

$z = a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + a_{n-2} \cdot 10^{n-2} + \dots + a_1 \cdot 10 + a_0$

4.3 Is the following definition of a rational number correct?

"If p and q are integers, then the ratio or fraction $\frac{p}{q}$ is called a rational number in mathematics"

4.4 Suppose that a, b, c, d - are any integers ($b, d \neq 0$), then $\frac{a}{b}, \frac{c}{d}$ are rational numbers. Do addition, subtraction, multiplication, and division of these numbers follow arithmetic laws?

4.5 Expressions like $\frac{1}{0}, \frac{5}{0}, \frac{-7}{0}$ don't make any sense. Are these numbers rational?

4.6 If the fractional parts 10, 100, 1000,etc. are such, then how do we call such fractions?

4.7 To decompose these rational numbers into a decimal fraction, you must divide the numerator of the fraction by the denominator. Consider all the events that occur there, and draw conclusions.

4.8 Give examples of periodic and non-periodic decimals.

4.9 Do you know the properties of rational numbers? Specify three of their properties.

5. Write down real numbers as infinite decimals.

3rd project work

Topic of project work: Applying operations to irrational numbers

Note: addition, subtraction, multiplication, and division operations can be applied to irrational numbers as well as to integers and fractions, but operations on irrational numbers have their own characteristics. First, it is necessary to define the concept of approximate values of irrational numbers. $\sqrt{5} = 2.2$ or $\sqrt{5} \approx 2.3$, so, respectively $2.2^2 = 4.84$ or $2.3^2 = 5.29$

Complete the following tasks:

1. The area of the square is 16.7 cm². Calculate its edges with an accuracy of 0.01 cm.

2. The circumference is 5.6 m². Calculate the diameter of the circle with an accuracy of 0.01 m. (let's assume $\pi = \frac{22}{7}$).

3. Find the value of the numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$. Calculate the resulting smallest and largest values with accuracy 0.1 ; 0.01; 0.001; 0.0001; 0.00001; 0.000001; 0.0000001

Follow these steps:

$\sqrt{2} + \sqrt{3} = 1.414 + 1.732 = 3.146$ (manual: addition with an accuracy of 0.001).

$\sqrt{5} - \sqrt{3} = 2.236 - 1.732 = 0.504$ (Manual: subtraction with an accuracy of 0.001).

$\sqrt{3} \times \sqrt{2} = 1.732 \times 1.414 = 2.449$

4. Draw the length of the segment $\sqrt{3}$ cm.

5. Draw the following numbers on the numeric line: $-\sqrt{2}; \sqrt{3}; \sqrt{2} + \sqrt{3}; \sqrt{5} - \sqrt{3}$.

6. Follow these steps: 1.1414123...+3.1213456...;

$2.71+2.7123; \quad 2.71...+3.14...$

Try answering the following questions:

1. Give examples of irrational numbers.
2. What new numbers will we add to the fractional numbers to get a set of irrational numbers? If we add irrational numbers to fractional numbers, what set of numbers do we get?
3. What other name can you give to infinite decimals?
4. What is the name given to a set that has infinite periodic decimals?
5. Give examples of pure and mixed periodic decimals. Is it possible to represent them as an ordinary fraction? Is it possible to convert infinite decimals to an ordinary fraction?

4th project work.

Project work is carried out by dividing a group of students into 5-6 small groups. Each team leader is assigned to manage the project work. Each manager is given the following problem tasks, reports, and questions of various contents. The end result is an analysis of all the relationships of limits corresponding to the progression topics covered in high school. Reports will be prepared on the topics "Finding the sum of progressions and relationships between function limits" and "Applying limits to finding the sum of arithmetic progressions". Protection of reports will be organized (with the replacement of group leaders by new leaders).

Project topic: Calculating the sum of terms of an arithmetic progression using limits.

Goal of the project work: Using limits to find the sum of arithmetic progressions

1. Study of theoretical material.

1. What is an arithmetic progression?
2. Write a formula for finding the common term of an arithmetic progression.
3. write a formula for finding the sum of the terms of an arithmetic progression.
4. specify the characteristics of the progressions:
 - a) find the sums of terms of arithmetic and geometric progressions;
 - b) find the sums of terms of an infinitely decreasing geometric progression.
5. Write the appropriate formulas for geometric progressions.
 - a) Write numerical sequences that are examples of geometric progressions.
 - b) Find the multiplicity of a geometric progression, give examples of finding the difference of an arithmetic progression; write a formula for finding their common term

1. Fragment of project work execution:

$$a_n = a_1 + d(n-1), \quad S_n = \frac{1}{2}(a_1 + a_n) \cdot n,$$

where a_n is the general term, S_n - n is the partial sum

a_1 - the first term, d - the difference, n - the number of the obtained term.

1. Write the formula for the general term of the geometric progression:

a) $a_n = a_1 q^{n-1}, \quad S_n = \frac{a_1 + a_n q}{1 - q}$

a_n - general term, a_1 - first term, $S_n - n$ - partial sum, q multiplicity of the progression, $n \rightarrow \infty, \quad q < 1, \quad S = \frac{a_1}{1 - q}$.

2. The following sequences should be described verbally:

a) 2,3,5,7,11,...

b) 2; 2,2; 2,23; 2,236; 2,2361;

3. $\left\{ \frac{n-1}{n+1} \right\}$ a convergent sequence, and its limit is 1. Check that the sequence limit definition has been completed.

The main attention is paid to students' performance of the following tasks:

Ideal 1. Determine the sum of the terms of an infinitely decreasing geometric progression:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{3 \cdot 2^{n-1}} = \sum_{n=1}^{\infty} \frac{1}{3 \cdot 2^{n-1}}$$

Solution: Here $a_{11} = \frac{1}{3}$, $an_n = \frac{1}{3 \times 2^{n-1}}$, $q = \frac{1}{2}$, so $q < 1$, $S_n = \frac{\frac{1}{3}}{1 - \frac{1}{2}} = \frac{1}{3} \times$

Determine the sum of the following series:

1. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{n(n+2)} + \dots = \sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

Decision: Let's apply the mathematical induction method:

$$\begin{aligned} S_n &= \frac{1}{2} \left\{ \frac{1}{1} - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{n-1} - \frac{1}{n+1} + \frac{1}{n} - \frac{1}{n+2} \right\} = \\ &= \frac{1}{2} \left\{ 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right\} - \frac{1}{2} \left\{ \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} \right\} = \\ &= \frac{1}{2} \left\{ 1 + \frac{1}{2} + \frac{1}{n+1} + \frac{1}{n+2} \right\}; \quad S = \lim_{x \rightarrow \infty} \frac{1}{2} \left\{ 1 + \frac{1}{2} + \frac{1}{n+1} + \frac{1}{n+2} \right\} = \frac{3}{4} \end{aligned}$$

2. Find the amounts shown below:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} + \dots = \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$$

3. $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \dots + \frac{2n+1}{n^2 \cdot (n+1)^2} + \dots = \sum_{n=1}^{\infty} \frac{2n+1}{n^2 \cdot (n+1)^2}$

4. $1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \dots - \frac{1}{2^n} - \dots = \sum_{n=1}^{\infty} \frac{1}{2^n}$

Necessary convergence condition

If a series of numbers converges, then $n \rightarrow \infty$, $a_n \rightarrow 0$

$$\lim_{n \rightarrow \infty} a_n = 0.$$

If $\lim_{x \rightarrow +\infty} a_n \neq 0$, then the series of numbers converges.

Check that the required convergence condition is met in the following series:

5. $\sum_{n=1}^{\infty} \cos \frac{1}{n}$; 5. $\sum_{n=1}^{\infty} \frac{n}{(n+1)^3}$; 6. $\sum_{n=1}^{\infty} n \cdot \arctg \frac{1}{n}$; 7. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2}$.

5th project work

Topic of the project work: Functional sequences and series.

The aim of the project work is to develop students' ability to study the convergence of functional series. Give students tasks to solve independently.

Content of the project work:

Students work independently on project work to find answers to the following questions, define them, and make rules for them.

Problematic issues:

1. What is a functional series?
2. Define the convergence of a functional series.
3. Zone of accumulation of functional series.
4. Convergence interval of the functional series.
5. Determination of the functional sequence.
6. Name the convergence conditions for the functional sequence.
7. The Weierstrass attribute.

Fragment of project work execution.

A functional series is a sequence whose members consist of the following functions:

$$u_1(x) + u_2(x) + \dots + u_n(x) + \dots = \sum_{n=1}^{\infty} u_n(x).$$

The convergence of functional series is formulated as follows:

any x_0 in its numerical value is converted to a u_n ($u_n(x_0)$) number, so the functional series is converted to a numerical series, and convergence at a point is studied by the convergence signs of the numerical series.

Definition 1. The set of all x , that converge in a series is called the domain of convergence of a functional series.

If the members of the functional series are defined in a certain interval, then the following definition is accepted.

Definition 2. When a function series (a, b) converges as a numerical series at each point, this is called convergence of the intermediate function series.

Between functional series (a, b) , partial sums give the functional sequence

$$S_1(x) = u_1(x), \quad S_2(x) = u_1(x) + u_2(x), \dots, S_n(x) = \sum_{n=1}^{\infty} u_n(x).$$

Definition 3. If in the interval (a, b) the numbers in each value of x converge as a sequence, we are talking about the convergence of the functional sequence $\sum_{n=1}^{\infty} u_n(x)$.

If all numbers converge as a sequence $x \in (a, b)$ of ones $\sum_{n=1}^{\infty} u_n(x)$, then the given limit holds

$$\lim_{n \rightarrow \infty} S_n(x) = S(x),$$

where $S(x) = \sum_{n=1}^{\infty} u_n(x)$ is called the limit function of a sequence of functions and it is equal to the sum of the function series:

$$\sum_{n=1}^{\infty} u_n(x) = S(x)$$

Example. The segment $[0, 1]$ contains a functional series: $\sum_{n=1}^{\infty} \frac{1}{x+n}$

Define the functional sequence and limit function.

Decision: 1) if $n = 1, 2, \dots$, then the functional sequence is written as follows:

$$\frac{1}{x+1}, \frac{1}{x+2}, \dots, \frac{1}{x+n}, \quad S_n(x) = \sum_{n=1}^{\infty} \frac{1}{x+n}.$$

2) We find the limit function from the above expression.

$$S(x) = \lim_{n \rightarrow \infty} S_n(x) = \sum_{n=1}^{\infty} \frac{1}{x+\infty} = 0, \quad S(x) = 0.$$

Definition 4. in an interval (a, b) , a functional sequence $S_n(x)$ converges to a limit function uniformly if $\varepsilon > 0$ the following condition is met in any number, in any interval $n \geq N(x)$ $x \in (a, b)$ выполняется следующее условие $|S_n(x) - S(x)| < \varepsilon$.

this is called the normal convergence condition.

This condition is written as follows:

$$S(x) - \varepsilon < S_n(x) < S(x) + \varepsilon, \quad (n = 1, 2, \dots)$$

If the functional sequence $S_n(x)$ converges uniformly to the limit function $S(x)$ in the interval (a, b) , then the sequence $S_n(x)$ is located between $[S(x) - \varepsilon]$ and $[S(x) + \varepsilon]$.

Definition 5. If in the interval (a, b) the functional sequence $S_n(x)$ converges uniformly to the limit function $S(x)$, then the functional series $\sum_{n=1}^{\infty} u_n(x)$ also converges uniformly in this interval.

Theorem 1. (Weierstrass sign). Let the functional series $\sum_{n=1}^{\infty} u_n(x)$ be given in the interval (a, b) . If we find a convergent series $\sum_{n=1}^{\infty} b_n(x)$ with a positive sign, and the condition is met

$$|u_n(x)| \leq b_n, \quad n = 1, 2, 3, \dots$$

then the function-channel series $\sum_{n=1}^{\infty} u_n(x)$ also converges absolutely and uniformly, and $\sum_{n=1}^{\infty} b_n$ is called a majorant.

Example. Prove uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$.

Decision: Since $|\sin nx| \leq 1$, therefore, the majorant $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (it is convergent) as $\sum_{n=1}^{\infty} \left| \frac{\sin nx}{n^2} \right| < \sum_{n=1}^{\infty} \frac{1}{n^2}$, in theorem 1, it is uniformly convergent in the series $(-\infty, \infty)$.

Determine the range of convergence of the series in the following problem:

1. $\sum_{n=1}^{\infty} e^{-nx}$. Solution: Apply the Cauchy attribute

$$\lim_{n \rightarrow \infty} \sqrt[n]{e^{-nx}} = e^{-x} = \begin{cases} < 1, \text{ if } x > 0, \\ \text{convergent } > 1, \text{ if } x < 0, \text{ non-convergent} \end{cases}$$

at the point $x=0$, the series is converted to a numerical series, $1 + 1 + \dots + 1 + \dots$ it does not converge, so the convergence region is $(0 < x < +\infty)$.

2. $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{|x|}{x}\right)^n$. Solution: the members of the functional series are not defined at a point $x = 0$, but are defined and available at other points.

If $x < 0$, then $\frac{|x|}{x} = \frac{-x}{x} = -1$, hence in negative x characters:

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{|x|}{x}\right)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}.$$

They converge according to the Leibniz criterion.

If $x > 0$, then $\frac{|x|}{x} = \frac{x}{x} = 1$, hence in positive x characters:

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{|x|}{x}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}.$$

The harmonic series does not converge, so the area of conditional convergence is $(-\infty, 0)$.

3. Solution: Based on the Cauchy attribute

$$\lim_{n \rightarrow \infty} \sqrt[n]{(3-x^2)^n} = \lim_{n \rightarrow \infty} (3-x^2) = 3-x^2 < 1, \quad 3-x^2 < 1, \quad 2 < x^2, \quad \pm\sqrt{2} < x, \\ (-2, -\sqrt{2}) \quad (2, \sqrt{2}).$$

4. $\sum_{n=1}^{\infty} \frac{1}{x^n}$; 5. $\sum_{n=1}^{\infty} \ln^2 x$; 6. $\sum_{n=1}^{\infty} (2-x^2)^n$;

7. $\sum_{n=1}^{\infty} \left[\frac{x(x+n)}{n}\right]^n$; 8. $\sum_{n=1}^{\infty} \frac{1}{x^{n+1}}$.

4 Discussion

By project activity we will understand such activity, which is based on the activation of cognitive and practical components, as a result of which the student produces a product that has a subjective (sometimes objective) novelty.

Project-based learning is the organization of the educational process aimed at solving educational tasks by students on the basis of independent collection of information based on these signs and interpretation, mandatory justification and adjustment of subsequent productive educational activities, its self-assessment and presentation of the result. At the same time, learning takes on a great personal meaning, which significantly increases the motivation of the actual teaching.

The main form of project-based learning is the design method, which is defined differently in the literature.

Proceeding from this, we believe that one of the central places should be given to the activity of the teacher, which is associated with the organization of a system of means that form the design and research activities. Not only for the transfer of students' knowledge, but also for the organization and management of the aspects of students' design and research activities and mainly independent cognitive activity, at all stages of training, to reveal the content of the means of control, mutual control, self-control, mutual learning and self-education.

As a result of the mutual influence of specific goals and means of design and research activities on the methods of cognitive activity, it can be noted that the specific weight of methods forming different levels of cognitive activity of students is increasing. Therefore, at the initial stage, such methods of design and research activities are used, which ensure that students are brought to the realization of the need for new knowledge, for this purpose, the teacher can rely on the capabilities of such means of design and research activities as a textbook, teaching instruments, tables, maps, diagrams, diagrams, use methodological techniques such as presenting logical tasks, creating problem situations; apply a frontal conversation or project-independent work, use game situations.

In concrete terms, the system of means forming the design and research activities of students must necessarily be adequate to the goal of the initial stage-the formation of a cognitive motive. However, even at this stage, there should be an impact on other components of the design and research activity (forms, content, methods, techniques, components should be applied to other methods, self-control and self-awareness).

Thus, the means that form the design and research activity will only act as a system when their selection is carried out taking into account the specific purpose of each stage of educational cognition and in their unity they affect each component of the design and research activities of students. This is the essence of the main starting point for the construction of a didactic system of tools that form the students' research activities.

The elements of the system developed by us are a set of tools aimed at forming all components of design and research activities, taking into account the specific purpose of this stage of design and research activities. In our experimental study, as a set of tools that form the design and research activities of students, we took: educational content; specific methods and techniques, methods of design and research activities; organizational forms of design and research activities. Consequently, the specific means that form the students' pre-research activities appear in unity and interconnection.

The system we are considering is holistic.

Thus, we have characterized the generalized model of the system of means forming the design and research activities of students, considering it from the standpoint of the main provisions of the theory of activity, the theory of thinkers of the East, folk pedagogy, goals, stages of educational cognition, the unity of educational goals, methods and forms of

education and, finally, from the standpoint of the basic requirements of a systematic approach. All this ensures the reliability of the didactic justification of the system of means forming the design and research activities of students.

5 Conclusions

The analysis of university practice and the above study allow us to state that the effectiveness of training is directly dependent on the level of cognitive activity of the student. The activity of students in the process of learning ensures the development of their creative abilities, skills of project activities. An analysis of experience and theoretical studies show that the teacher plays a leading role in organizing students' active project activities. His personality, knowledge, attitude to work and students, methodical skill - all this largely determines success in solving the problem under consideration. At the same time, other factors also act: the content of knowledge, the level of development of students, etc. All these factors work in unison. It is the teacher who, structuring the educational material, determining the forms and methods of teaching, organizes the learning process, taking into account the abilities and capabilities of each student.

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