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Methods of applying matrices in creating models of group pursuit

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Abstract. The article deals with the application of matrix modeling to the group pursuit of objects. The article applies matrix modeling to the group pursuit of objects based on models of actual objects' behavior and can be used in simulation modeling packages, virtual simulation of game processes, or transport logistics processes. The results of the article may be in demand in creating virtual reality models of delivery of postal goods by drones in creating an optimized hub - network. The author proposes to consider the model of group pursuit of multiple targets. Methods of pursuit by individual subjects are various modifications of methods of parallel approach, pursuit, and proportional approach. The author constructed a matrix reflecting the number of chasers and the number of targets. The conducted research contains a model of group pursuit of a set of targets. The model contains optimization based on the least time of simultaneous achievement of goals. These results are in the certificate of registration of programs for computers. Objectives assume that in the method of dynamic programming of matrices of distribution of pursuers, we will construct the matrix at each discrete moment, because the number of pursuers and objectives, and their strategies can change at any moment. The methods of forming matrices of distribution of pursuers and goals can be in demand in the design of virtual reality systems for game tasks. Such tasks simulate the process of group pursuit, running away, and evasion. The method of dynamic programming for the distribution matrix of pursuers on targets will allow us to move to an automated distribution process with optimization according to the specified parameters.

1 Introduction

This article forms the principles of automated distribution of pursuers to targets based on the selected target function. The model of the article proposes algorithms for modifying the trajectories of pursuers to reach targets simultaneously or according to a set schedule. The author considers the formation of a library of pursuit methods.

The works [1-4] raise the issues of the coordinated behavior of a group of pursuers and targets. The author considered works [5-9] on general theoretical and practical issues in solving problems. The author used sources [10-13] to analyze stalker targeting.

This article generates a matrix of pursuers' achievement of goals. Assigning goals to pursuers is carried out by the following principle. The pursuers achieved all combinations of

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goals, and the combination of a minimum value of criterion from the formed set with the maximum value was chosen.

Consider group pursuit of a set of goals: N pursuers catch up with M goals. Let's form a matrix of distribution of pursuers by goals:

$$\Psi_{ij}, where\ i = 1..N, j = 1..M.$$

Each cell Ψ_{ij} contains information about the phase coordinates of the i-th pursuer and j – target. The matrix Ψ_{ij} contains information about the method by which the i-th pursuer pursues the j-th target.

Based on the data stored in the cells of the matrix, it accesses the library of calculations of the chaser's control vectors.

Each cell of the matrix Ψ_{ij}can calculate, as an example, the predicted time for the i–ep pursuer to reach the j-th goal t_{ij}.

In each resulting sample $A_k = \{\Psi_{i_{1k}j_{1k}} \quad \cdots \quad \Psi_{i_{nk}j_{nk}}\}$, the maximum value of achievement times $t_k = Max\ \{t_{ij}\}$. Suppose in the sample of Table 1, we should find the maximum value of times $\{t_{21}, t_{23}, t_{32}, t_{41}\}$.

The maximum value of t_k in the sample A_k is dictated by the fact that all values of t_{ij}, which depend on vectors of velocities of pursuers and targets, as well as their admissible angular velocities, can be changed upward to the value of t_k.

In sample A_k it is possible to increase other values of times when pursuers reach targets t_{ij} up to value t_k, due to vectors of pursuers' and targets' velocities, values of angular velocities.

When the array of samples $\{A_k\}$ with corresponding values of times $\{t_{ij}\}$, is obtained, the minimum time $t_{min} = Min\ \{t_k\}$ should be found. Thus, the optimal time of group achievement of multiple goals simultaneously will be obtained.

2 Materials and methods

Algorithms for calculating the chaser's next step and estimating the time of the chaser reaching the target.

Fig. 1 shows a block diagram of the function algorithm for calculating the next step and the pursuer's velocity vector.

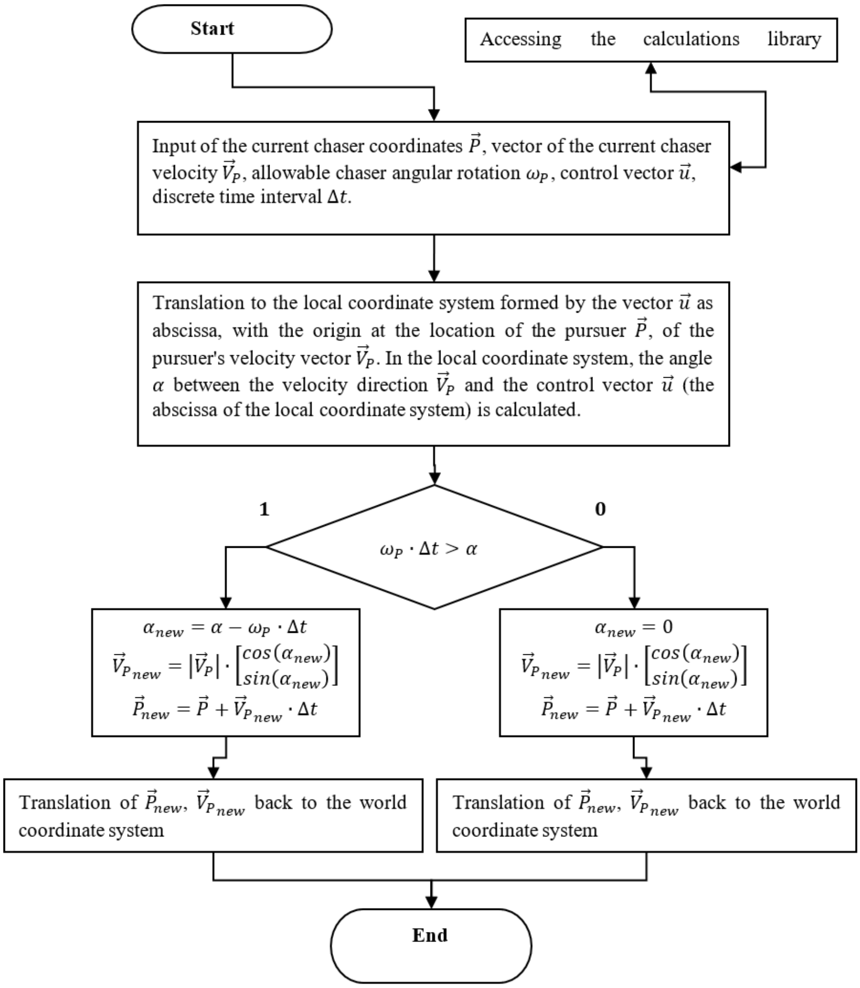


Fig. 1. Block diagram for calculating the phase coordinates of the chaser in the next step.

Fig. 2 shows a block diagram of the function for calculating the time and distance of the chaser reaching the target. The variable ε is the threshold value of the distance from the chaser to the target, then the targets reached.

When the target moves along a predetermined trajectory, the flowchart shown in Fig. 2 can give an estimate of the time t_{ij} for the i-th pursuer to reach the j-th target. The number of iterations of the pursuit process N_{it}.can serve as an output parameter of the function presented in Fig. 2. The value of N_{it}, the number of iterations, is the output parameter of the function for calculating the time and distance of the chaser reaching the target.

When the target takes retaliatory steps to avoid reaching it, we propose to estimate the time differently. This requires that the predicted trajectories be constructed as a composite of segments of straight lines, arcs of circles, square and cubic parabolas, and other known lines, so as not to solve boundary problems in the computational cycle.

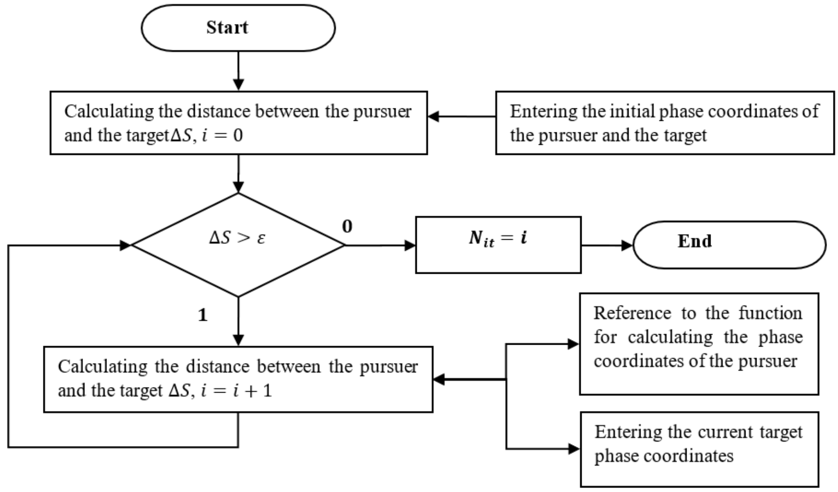


Fig. 2. Block diagram of the function for calculating the time and distance for the pursuer to reach the target.

Forming the library of control vector calculations.

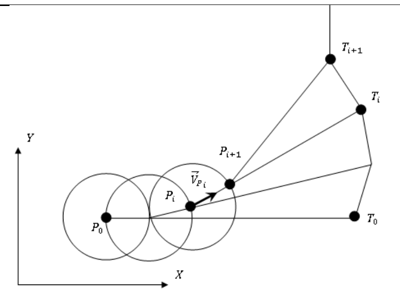
It is assumed that the distribution matrix Ψ_{ij}, where $i = 1..N, j = 1..M$ of pursuers on targets will be constructed at each discrete time interval. Each cell of matrix Ψ_{ij} will store information about the pursuit method. Based on this information, the library of functions for calculating control vectors $\vec{u}$ (Table 1) will be accessed.

This library of control vector calculations contains methods of pursuit on the plane, in space, and on the surface. Parallel convergence methods are calculated on the plane, in space, and on the surface. Proportional approach methods are calculated on the plane and in space. Three-point methods are computed on the plane and in space. Changed chase methods are computed on the plane and in space when the chaser can be controlled by changing the allowable curvature of trajectories. Modified methods of parallel approach are computed on the plane and in space.

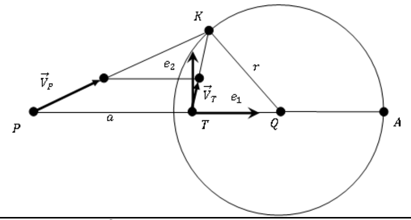
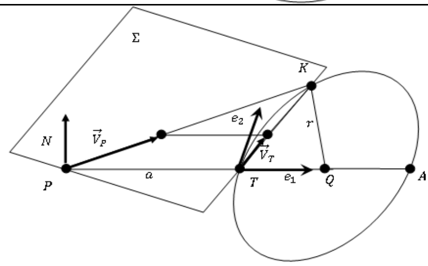
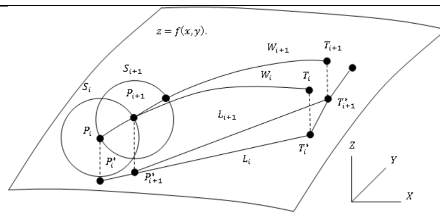
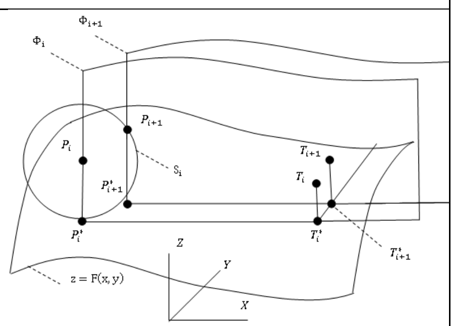
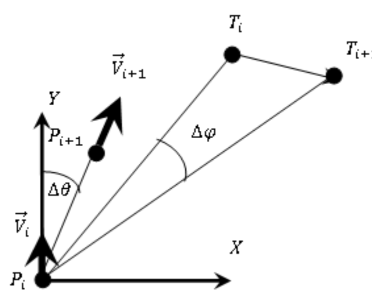
Modification of the parallel approach and chase methods allows us to construct a network of predicted trajectories that admit different boundary conditions.

Table 1 contains the materials of the author of the article, based on the video of his YouTube channel @alexdubanov5999 (Alexander Dubanov).

Table 1. Methods of pursuing a target moving along a certain trajectory.

Chase method on the plane and space: $\vec{u}_i = \frac{\vec{T}_i - \vec{P}_i}{\lvert\vec{T}_i - \vec{P}_i\rvert}$, $\vec{T}_i$ - target position, $\vec{P}_i$ - pursuer's position.	

Continued of Table 1.

The method of parallel convergence on the plane: $\vec{u} = \frac{K-P}{\lvert K-P \rvert}$, T - target position, P - point of the pursuers' position, K - point on the circle of Apollonius, uniquely defined by the points P , T and the velocity vector of the target $\vec{V}_T$.	
The method of parallel approach in space: $\vec{u} = \frac{K-P}{\lvert K-P \rvert}$,$T$ - target position, P - pursuer's position point, K - point on the circle of Apollonius, the circle of Apollonius lies in the plane Σ, formed by the points P, T and the velocity vector of the target $\vec{V}_T$. This example shows the case when the velocity vector of the pursuer is directed arbitrarily. After some time, the pursuer's velocity is directed to a point on the Apollonius circle.	
Surface pursuit method: $\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$, where P_{i+1} - is the result of the intersection of the surface $z = f(x,y)$, plane $P_i P_i^* T_i$ and spheres S_i centered at P_i and radius $\lvert \vec{V}_T \rvert \cdot \Delta t$. Point P_i^* - orthogonal projection of a point P_i on the plane XY. Expressing a control vector is necessary to produce a unified reference to the library.	
The method of parallel approach on the surface: $\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$,where P_{i+1} is the result of intersection of surface $z = f(x,y)$, plane $P_{i+1}P_{i+1}^*T_{i+1}$ and sphere S_i with center in point P_i and radius $\lvert \vec{V}_T \rvert \cdot \Delta t$. Point P_{i+1}^* - is the orthogonal projection of the point P_{i+1} on the plane XY. It is necessary to express the control vector in order to make a unified reference to the library. A one-parameter family of planes Φ_i parallel to each other.	
Proportional convergence method: $\frac{d\theta}{dt} = k \cdot \frac{d\varphi}{dt}$,$\Delta\varphi = arccos\left(\frac{\lvert T_i \rvert^2+\lvert T_{i+1} \rvert^2-\lvert T_i-T_{i+1} \rvert^2}{2\cdot\lvert T_i \rvert\cdot\lvert T_{i+1} \rvert}\right)$, $\Delta\theta = k \cdot arccos\left(\frac{\lvert T_i \rvert^2+\lvert T_{i+1} \rvert^2-\lvert T_i-T_{i+1} \rvert^2}{2\cdot\lvert T_i \rvert\cdot\lvert T_{i+1} \rvert}\right)$, $P_{i+1} = \begin{bmatrix} V_P \cdot \Delta t \cdot cos\left(k \cdot arccos\left(\frac{\lvert T_i \rvert^2+\lvert T_{i+1} \rvert^2-\lvert T_i-T_{i+1} \rvert^2}{2\cdot\lvert T_i \rvert\cdot\lvert T_{i+1} \rvert}\right)\right) \\ V_P \cdot \Delta t \cdot sin\left(k \cdot arccos\left(\frac{\lvert T_i \rvert^2+\lvert T_{i+1} \rvert^2-\lvert T_i-T_{i+1} \rvert^2}{2\cdot\lvert T_i \rvert\cdot\lvert T_{i+1} \rvert}\right)\right) \end{bmatrix}$,$\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$	

Continued of Table 1.

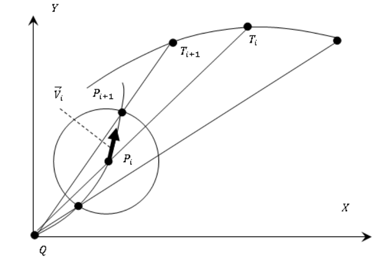
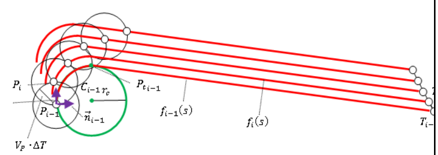
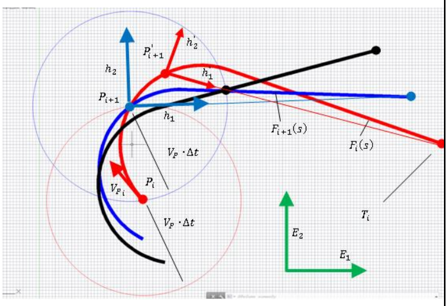
Three-point approach method: $(P_{i+1} - P_i)^2 = \left(\vec{V}_i \cdot \Delta t\right)^2$ $,\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$. $P_{i+1} = (1-\tau) \cdot Q + \tau \cdot T_{i+1}$ The application of the method is convenient when the target moves along a ballistic trajectory.	
A modification of the parallel convergence method on the plane: a network of parallel lines: $f_{i+1}(s) = f_i(s) + T_{i+1} - T_i$, where s - line arc length, T_i - an array of reference points of the target trajectory. Solving the equation $(f_{i+1}(s) - P_i)^2 = (V_P \cdot \Delta t)^2$ relative to the parameter s will allow us to find the value s^*, which will correspond to $P_{i+1} = f_{i+1}(s^*)$. $\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$. The family of lines $f_i(s)$ can be lines of arbitrary configuration.	
Modification of the parallel approach method on the plane: the network $f_i(s)$, where s is the arc length of the line, T_i is the array of reference points of the target trajectory. The condition is fulfilled that the end of line $f_i(s)$ passes through point T_i, and point P_i is incident to line $f_i(s)$. It is used as a template. Solving the equation $(f_{i+1}(s) - P_i)^2 = (V_P \cdot \Delta t)^2$ with respect to the parameter s will find the value of s^*, which will correspond to $P_{i+1} = f_{i+1}(s^*)$. $\vec{u}_i = \frac{P_{i+1}-P_i}{\lvert P_{i+1}-P_i \rvert}$. The family of lines $f_i(s)$ can be lines of arbitrary configuration.	

Table 1 does not reflect all methods of calculating control vectors. The implication is that this will be an open, replenishable library of functions.

An example of application of matrix modeling to group pursuit.

Consider the example of group pursuit shown in Fig. 3.

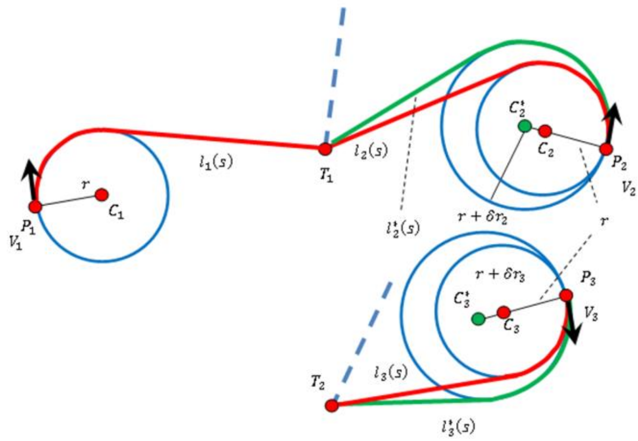


Fig. 3. A scheme of group pursuit of multiple targets.

It is necessary to form matrices Ψ_{ij}, where $i = 1..3, j = 1..2$, corresponding to samples $A_k, k = 1..6$ (Table 2). This is followed by circulation to find the maximum value of $t_k = Max\ \{t_{ij}\}$. After calculating the attainment times, it is found that the longest attainment time has the chaser P_1 catching the target T_1 from sample A_2.

Table 2. Samples corresponding to the distribution of pursuers by target.

		targets		targets		targets		targets		targets		targets	
Pursuers		1	2	1	2	1	2	1	2	1	2	1	2
	1	x		x		x			x		x		x
	2		x	x			x	x		x			x
	3		x		x	x		x			x	x	
Samples		A_1		A_2		A_3		A_4		A_5		A_6	

The situation in Fig. 3 shows all pursuers reach the target using the changed parallel approach method, which corresponds to row 8 of Table 1.

The example in Fig. 3 shows a case where there was a calculation of simultaneous goal achievement. The curvature of the trajectory in the pursuit model specified in row 8 of Table 1 must not be greater than a certain value. The initial radius of curvature of the trajectory increases in the model for pursuers P_2 and P_3 (Figure 3).

In the sample A_k the pursuer P_i catches up with T_j. Then the primary estimation of the time of reaching t_{ij} takes place. The estimation of the time t_{ij} in the length's calculation of the length of the rectilinear segment to thein the arc's calculation length of the contiguous circle of acceptable radius. Then, the maximum value $t_k = Max\{t_{ij}\}$ is chosen. The increase in time t_{ij} to the value t_k is due to the increase in the pursuer P_i of the radius of the conjugate circle from the value r_i to the value $r_i + \delta r_i$.

Figure 4 is supplemented with an animated image, which shows the process of group pursuit of a set of targets (based on the YouTube video @alexdubanov5999, Alexander Dubanov).

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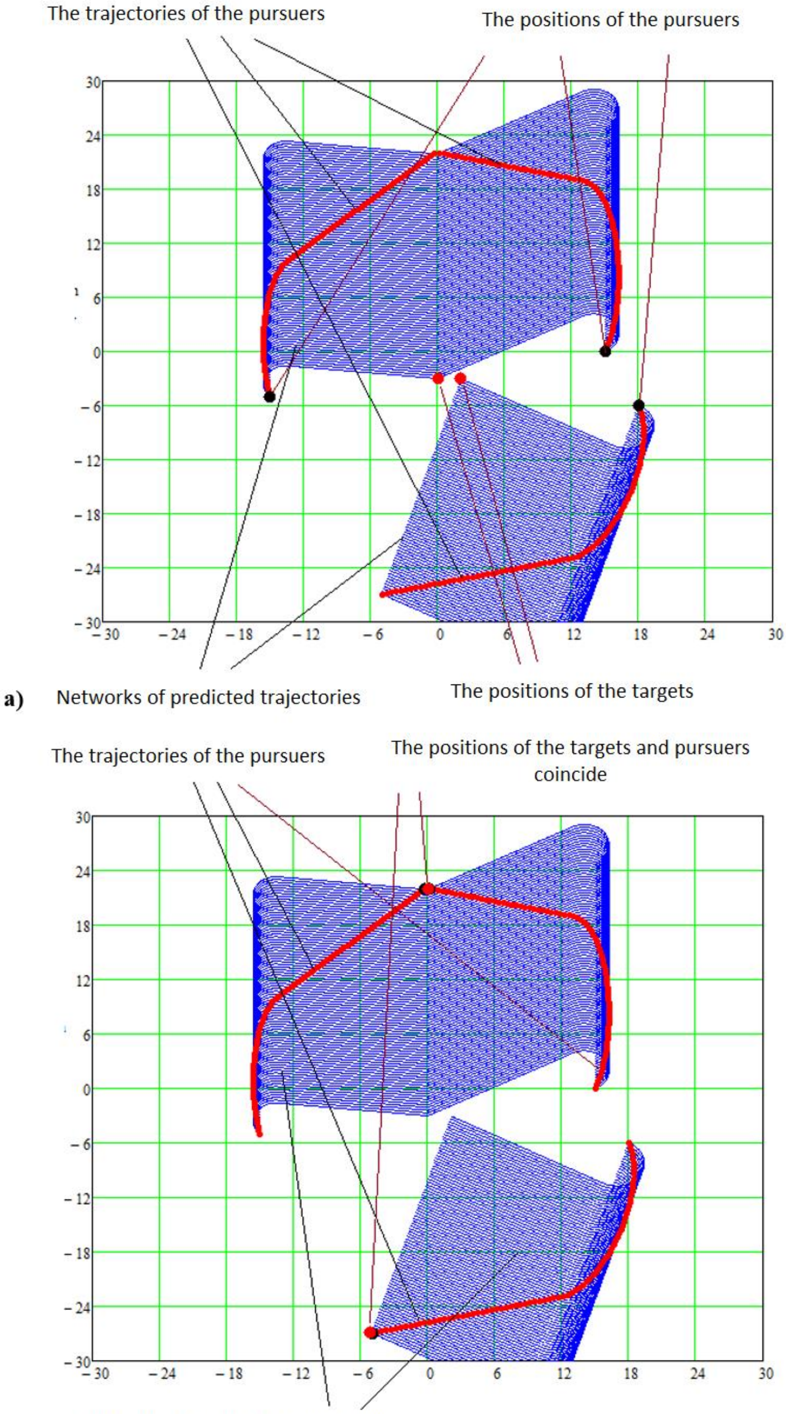

Fig. 4. Schemes of the phases of group pursuit: a - initial phase; b - final phase.

3 Research results

The author created a program of group pursuit of several targets based on the results of the study. We got a certificate of software registration (Software Registration Certificate No. 2021618920, Model of parallel convergence on the plane of a group of pursuers with simultaneous target attainment).

4 Discussion and conclusion

A party representing targets that sets itself the goal of not being reached can use modeling of the distribution matrix of pursuers on targets.

The goal chaser distribution matrix will be generated at each point in time. Goals and pursuers can disappear, and new ones can appear.

The method of forming a matrix of distribution of pursuers on targets can be in demand in the design of virtual reality systems for game tasks, in which the simulation of the process of group pursuit, escape, evasion will be performed.

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